Search with Costs and Heuristic Search

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UBC CS 322 – Search 3

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Textbook §3.5.3, 3.6, 3.6.1

Today's Lecture

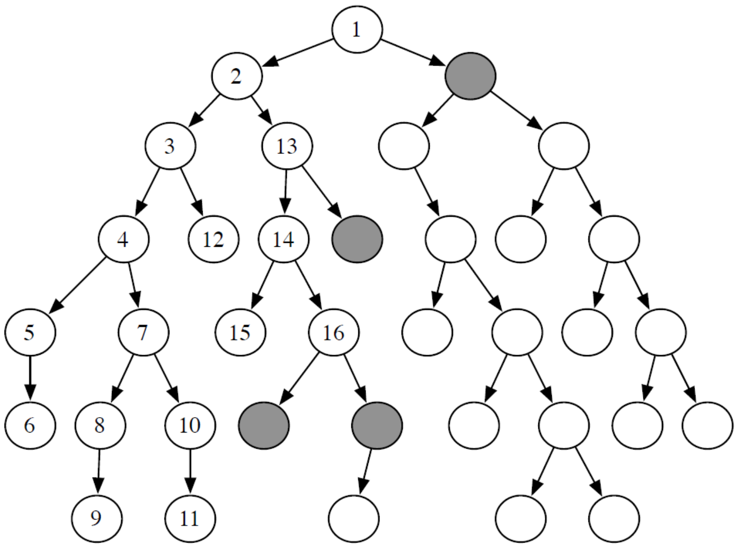
- ⇨ Recap from last lecture, combined with AIspace demo
- Search with costs: Least Cost First Search
- Heuristic Search: Best First Search

Learning Goals from last class

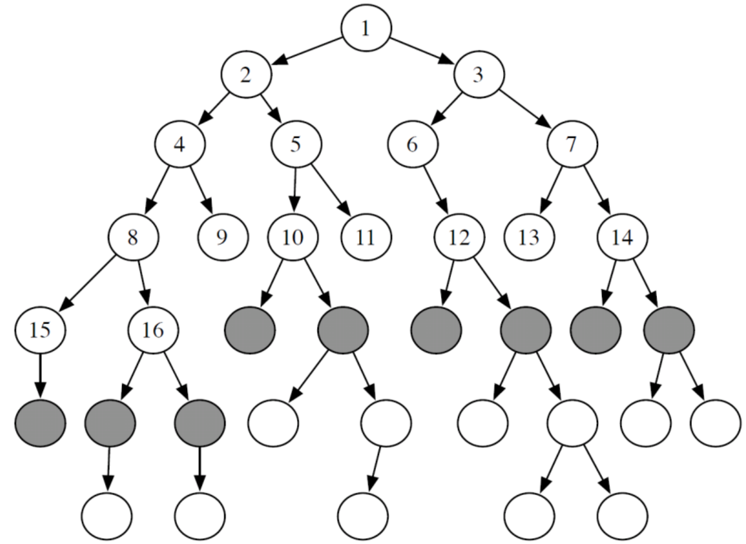
- Apply basic properties of search algorithms:
  - completeness
  - optimality
  - time and space complexity of search algorithms

- Select the most appropriate search algorithms for specific problems.
  - Depth-First Search vs. Breadth-First Search

DFS and BFS



Depth-First Search, DFS



Breadth-First Search, BFS

Let’s look at these algorithms in action: AIspace

Comparing Searching Algorithms: Will it find a solution? The best one?

Def. : A search algorithm is complete if
whenever there is at least one solution, the algorithm is guaranteed to find it within a finite amount of time.

- BFS is complete, DFS is not

Def.: A search algorithm is optimal if
when it finds a solution, it is the best one

- BFS is optimal, DFS is not

Comparing Searching Algorithms: Complexity

Def.: The time complexity of a search algorithm is the worst-case amount of time it will take to run, expressed in terms of

- maximum path length m
- maximum forward branching factor b.

- Both BFS and DFS take time $O(b^m)$ in the worst case

Def.: The space complexity of a search algorithm is the worst-case amount of memory that the algorithm will use (i.e., the maximal number of paths on the frontier).

- BFS: $O(b^m)$ $O(m^b)$ $O(bm)$ $O(b+m)$
- DFS: $O(b^m)$ $O(m^b)$ $O(bm)$ $O(b+m)$

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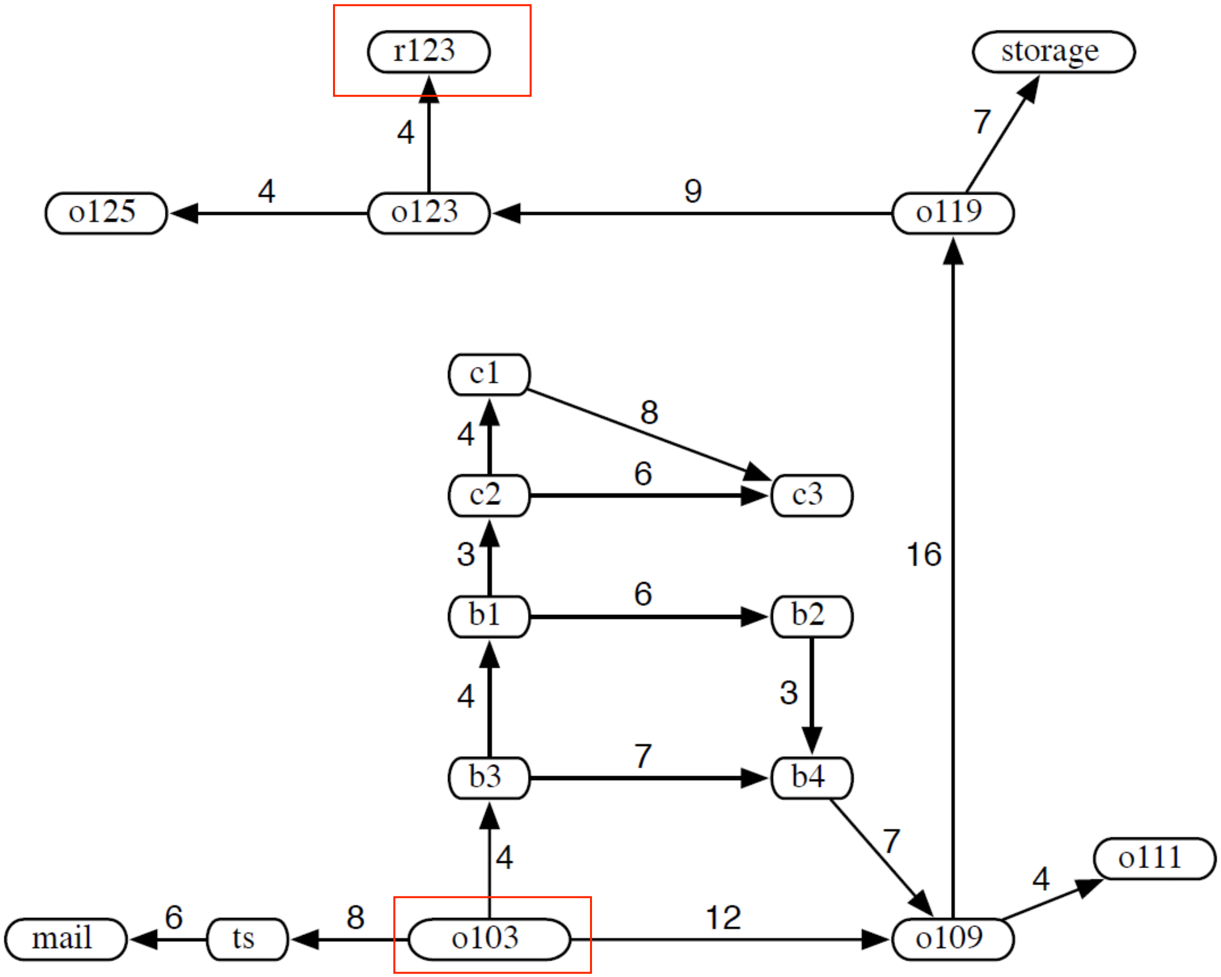
Example: edge costs in the delivery robot domain



Search with Costs

- Sometimes there are costs associated with arcs.

Def.: The cost of a path is the sum of the costs of its arcs

$$\text{cost}\left(\langle n_0, \ldots, n_k \rangle\right) = \sum_{i=1}^{k} \text{cost}(\langle n_{i-1}, n_i \rangle)$$

- In this setting we often don't just want to find any solution
  - we usually want to find the solution that minimizes cost

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Def.: A search algorithm is optimal if
when it finds a solution, it is the best one:
it has the lowest path cost

Lowest-Cost-First Search (LCFS)

- Lowest-cost-first search finds the path with the lowest cost to a goal node
- At each stage, it selects the path with the lowest cost on the frontier.
- The frontier is implemented as a priority queue ordered by path cost.

Let's look at this in action: AIspace

When arc costs are equal, LCFS is equivalent to…

DFS

BFS

IDS

None of the above

Analysis of Lowest-Cost First Search

- Is LCFS complete?
  - Not in general: a cycle with zero cost, or negative arc costs could be followed forever
  - Yes, as long as arc costs are strictly positive $\geq \mathcal{E} > 0$ and branching factor is finite.
- Is LCFS optimal?

YES **NO** **IT DEPENDS**

  - Not in general: arc costs could be negative: a path that initially looks high-cost could end up getting a 'refund'.
  - Yes, as long as arc costs are guaranteed to be strictly positive $\geq \mathcal{E} > 0$ and branching factor is finite.

Analysis of Lowest-Cost First Search

- What is the time complexity of LCFS if the maximum path length is *m* and the maximum branching factor is *b*?

$\tilde{O}(b^m)$ | $O(m^b)$ | $O(bm)$ | $O(b+m)$

  - Answer: $\tilde{O}(b^m) = O(\log(b^m) b^m)$
  - Implement priority queue as a heap
  - Knowing costs doesn't help here; worst case: all nodes
- What is the space complexity?

$O(b^m)$ | $O(m^b)$ | $O(bm)$ | $O(b+m)$

E.g. uniform cost: just like BFS, in worst case frontier has to store all nodes m-1 steps from the start node

‘Uninformed Search’: DFS, BFS, LCFS

- Why are all these strategies called uninformed?
  - Because they do not consider any information about the states and the goals to decide which path to expand first on the frontier
  - They are blind to the goal

- In other words, they are general and do not take into account the specific nature of the problem.

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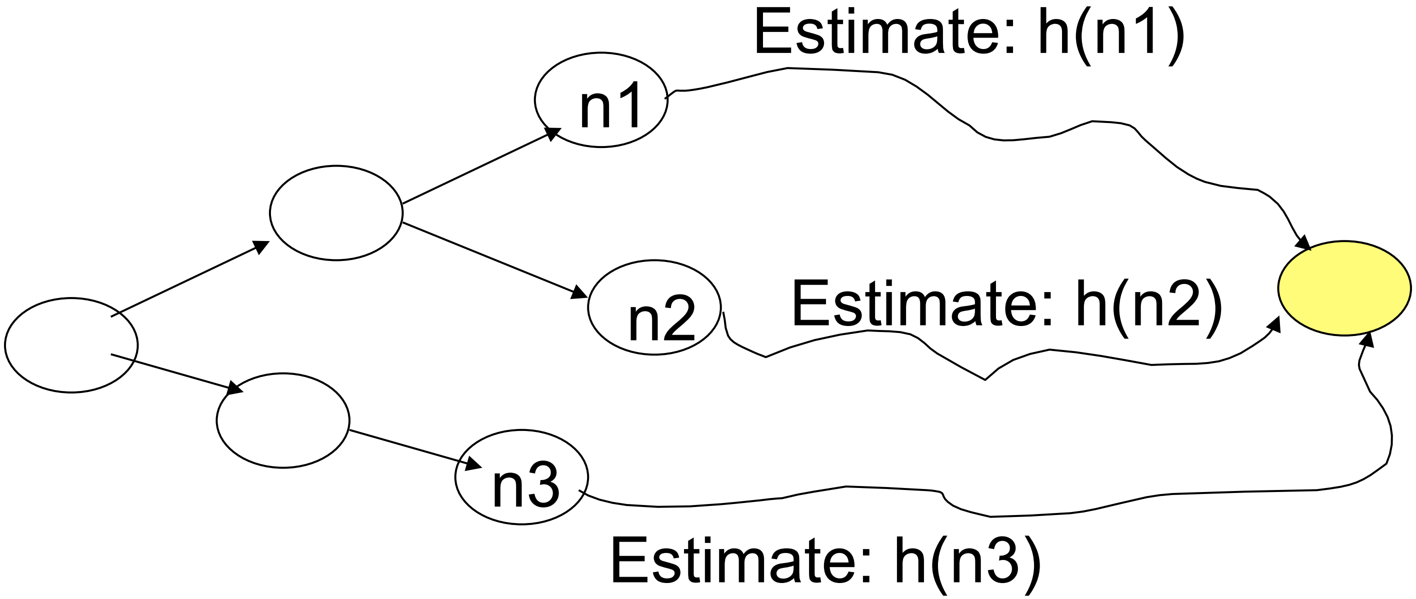
Heuristic Search

- Blind search algorithms do not take into account the goal until they are at a goal node.
- Often there is extra knowledge that can be used to guide the search:
  - an estimate of the distance/cost from node *n* to a goal node.
- This estimate is called a search heuristic.

More formally

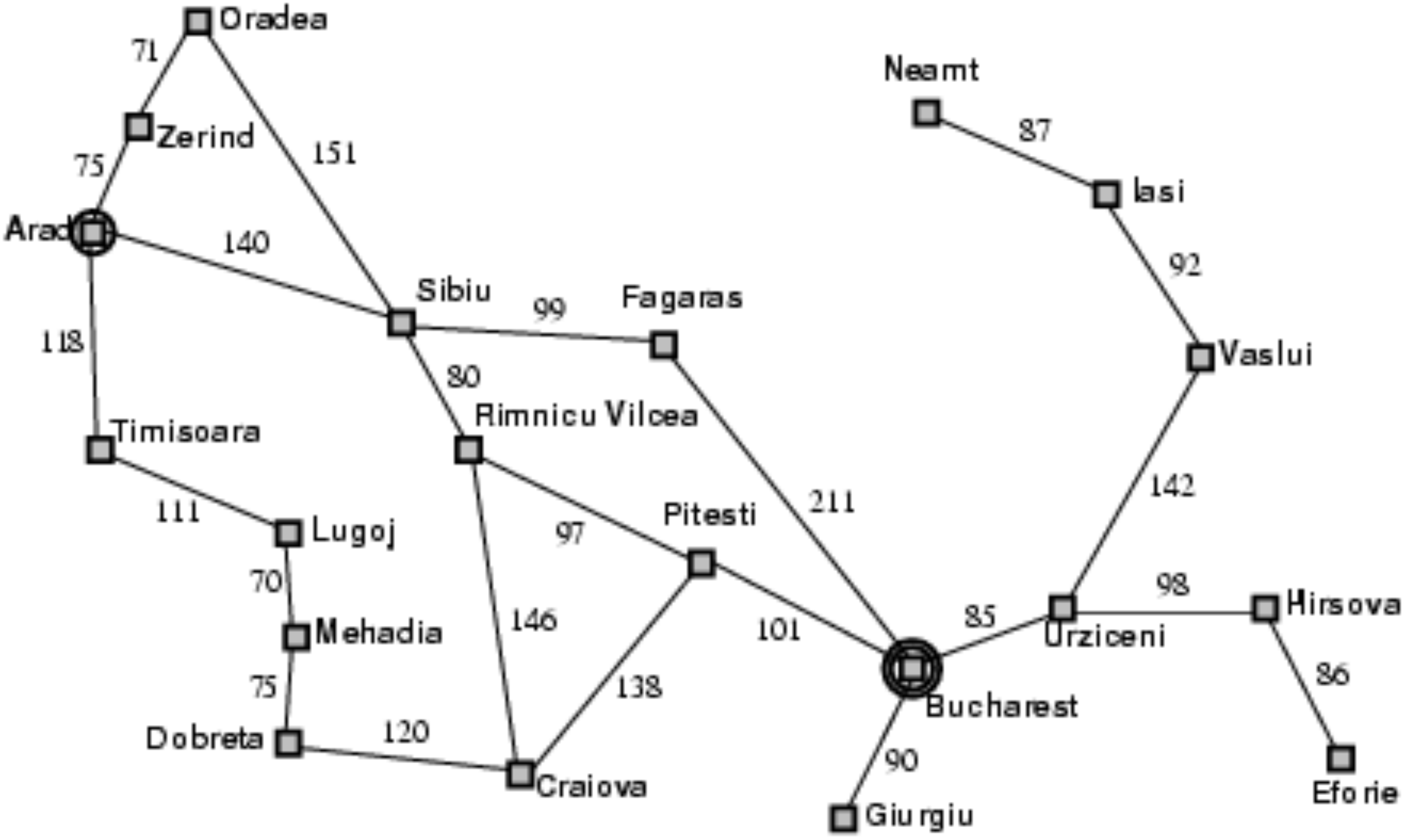
Def.:

A search heuristic *h(n)* is an estimate of the cost of the optimal (cheapest) path from node *n* to a goal node.



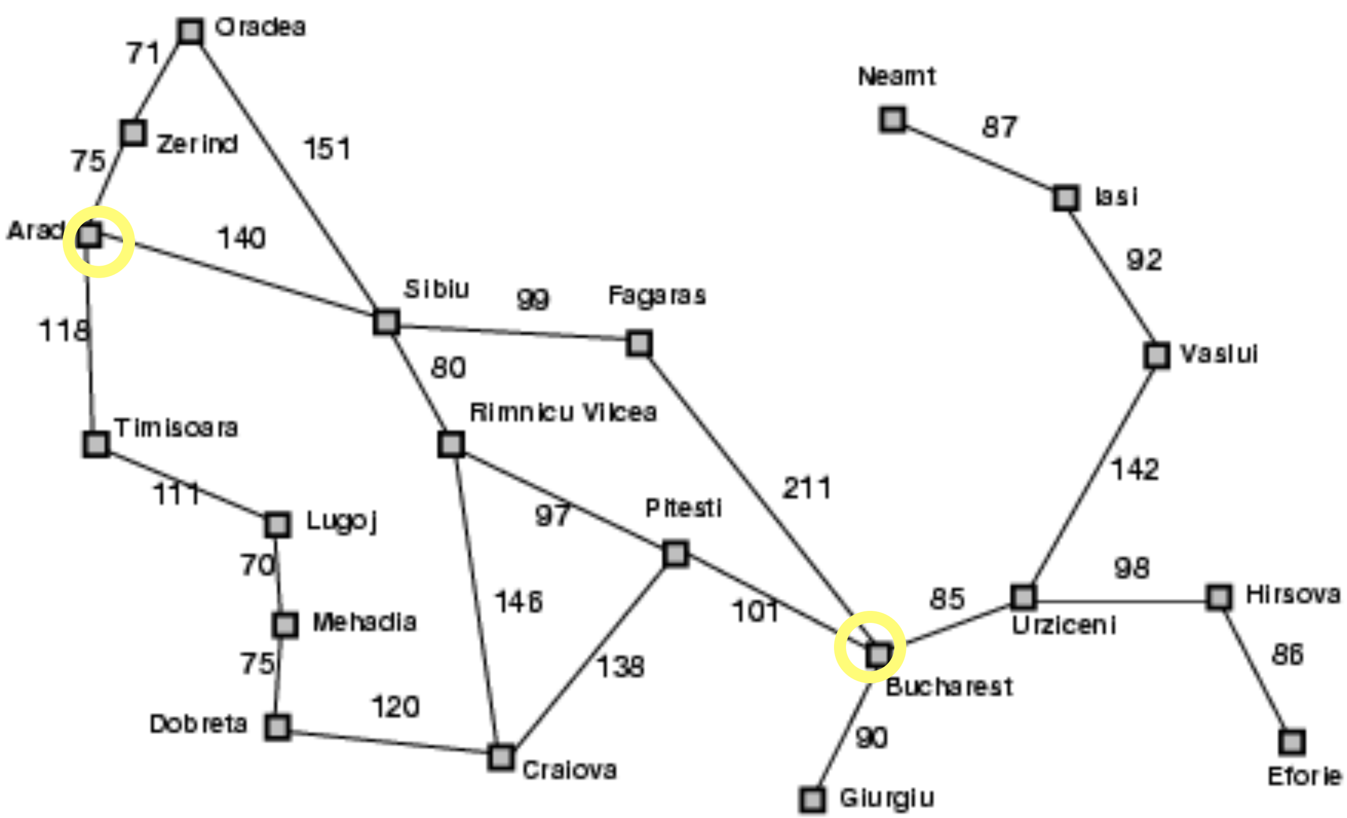
Example: finding routes

- What could we use as h(n)?



Example: finding routes

- What could we use as h(n)? E.g., the straight-line (Euclidean) distance between source and goal node



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	10
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

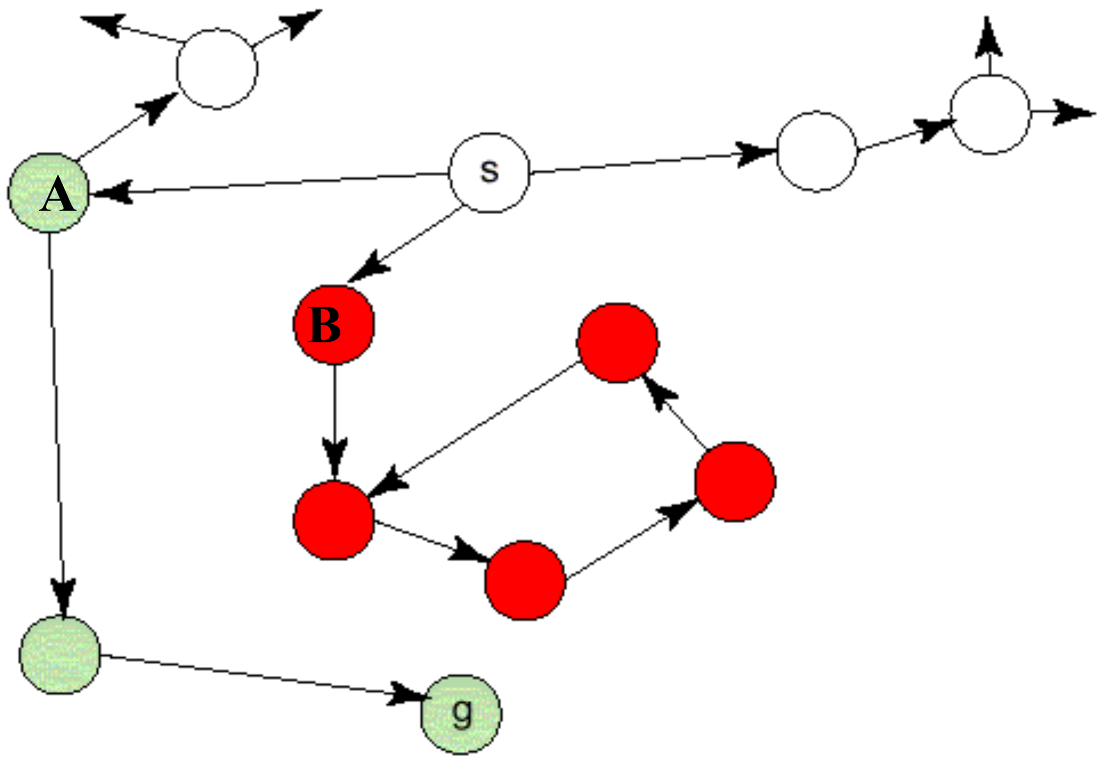
Best First Search (BestFS)

- Idea: always choose the path on the frontier with the smallest h value.
- BestFS treats the frontier as a priority queue ordered by h.
- Greedy approach: expand path whose last node seems closest to the goal

Let's look at this in action: AIspace

Optimal? AIspace example, load from URL
http://www.cs.ubc.ca/~mack/CS322/ex-best-first-search.txt

Best-first Search: Illustrative Graph



- A low heuristic value can mean that a cycle gets followed forever -> not complete

Analysis of BestFS

- Complete? No, see the example last slide
- Optimal? No, see the AIspace example from above: http://www.cs.ubc.ca/~mack/CS322/ex-best-first-search.txt
- Time Complexity $\tilde{O}(b^m)$ $O(m^b)$ $O(bm)$ $O(b+m)$

  - Worst case: has to explore all nodes

- Space Complexity $O(b^m)$ $O(m^b)$ $O(bm)$ $O(b+m)$

  - Heuristic could be chosen to emulate BFS:
    E.g. h(n) = distance of n from start

Learning Goals for today's class

- Select the most appropriate algorithms for specific problems.
  - Depth-First Search vs. Breadth-First Search vs. Least-Cost-First Search vs. Best-First Search
- Define/read/write/trace/debug different search algorithms
- Construct heuristic functions for specific search problems (just started, more on this next time)