Structural Differences Between Two Graphs through Hierarchies

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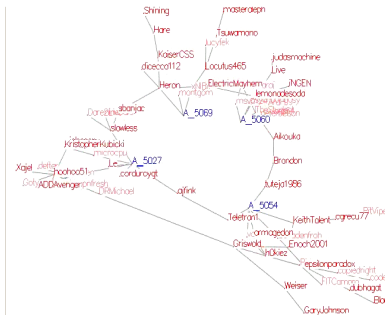
The Wine Difference Problem

Wikimedia Project, Creative Commons Licence, André Karwath

- Carefully studying this important problem...

The Graph Difference Problem

- Structural evolution of dynamic graphs
    - how does a dynamic graph evolve over time
- Typical technique in graph drawing community
    - show graph evolution through animation

Video

**Y. Frishman and A. Tal. Online Dynamic Graph Drawing. EuroVis 2007**

Difference Map and Small Multiples Approach

$\Delta t = 10$ and 11 $\qquad$ $\Delta t = 11$ and 12 $\qquad$ $\Delta t = 12$ and 13

- Is this an improvement?
  - Need an experiment to support this argument
- Contributes new algorithms to generate coarsened difference map

Animation as Dynamic Graph Evolution

Krists Boitmanis, Ulrik Brandes, and Christian Pich. Visualizing Internet Evolution on the Autonomous Systems Level.

Symp. Graph Drawing (GD '07)

- By far, most common method for dynamic graph visualization
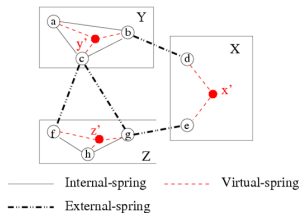- Changes faded in and out, node movement interpolated

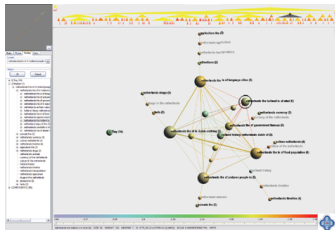Small Multiples and Dynamic Attributes
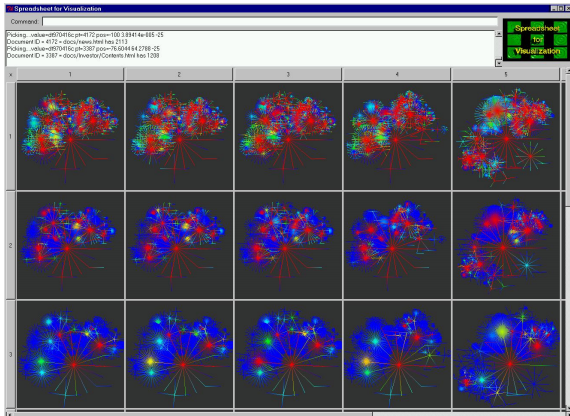
E. H. Chi and S. Card. Sensemaking of Evolving Web Sites using Visualization Spreadsheets (InfoVis '99)

- Not frequently used to depict structural evolution of graphs
- Experiments suggest may be better for dynamic data
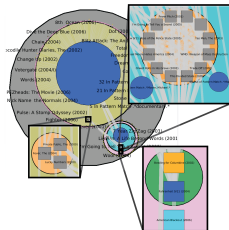
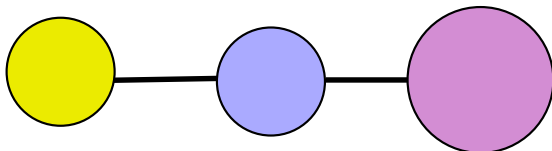# Hierarchy-Based Graph Visualization

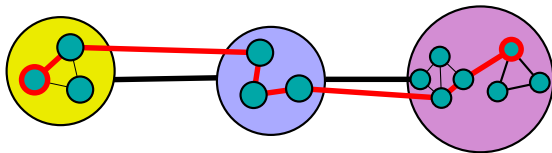(a) DA-TU, 2000

(b) ASK-GraphView, 2006

(c) GrouseFlocks, 2008

- Abstract away parts of the graph where details not required
  - used to abstract away structural difference
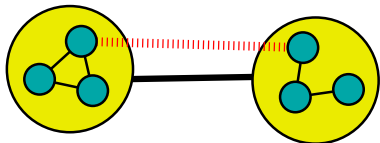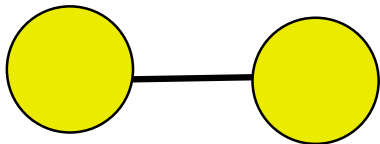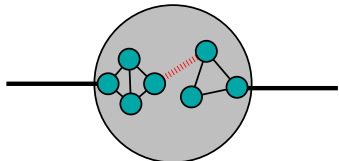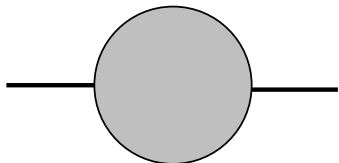
Path-Preserving Hierarchy

(a)

(b)

- Defined in GrouseFlocks work
- A path in the hierarchy means at least one path in the graph
- **Path-preserving hierarchies** respect this property

Path-Preserving Hierarchy

(a) Edge Conservation

(b) Connectivity Conservation

- Metaedge if and only if a pair of descendants connected
- Metanodes contain connected subgraphs
- If preserved, paths in cuts are also in underlying graph

Algorithm Overview

1. Construct difference map
2. Difference hierarchy construction
3. Degree One Coarsening
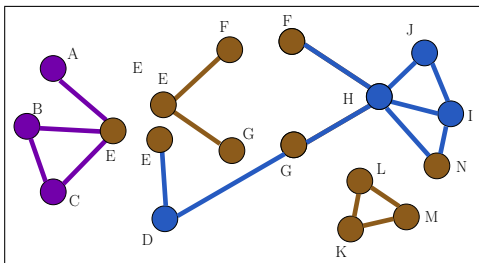4. Betweenness Centrality Coarsening

Difference Map Construction

- Each node in graph is guaranteed a unique labelling
- Can be done through a single scan of the node and edge list of each graph.
- $O(|N| + |E|)$ for the nodes in both graphs
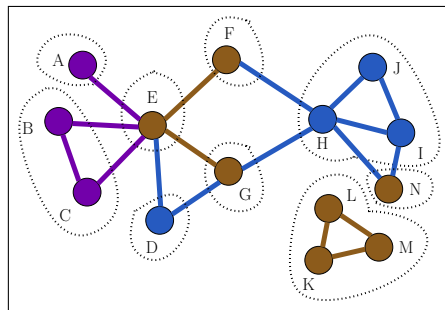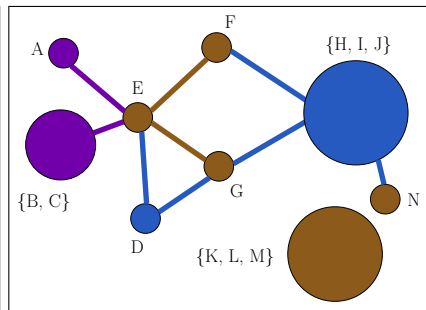
Difference Hierarchy Construction (1)
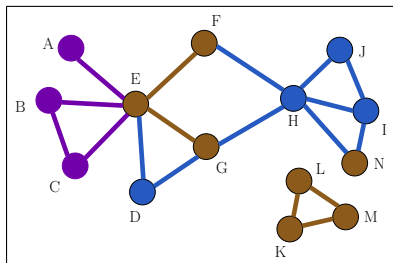
(a) Input   (b) Edge Decomposition

- Decompose the graph into connected components by edge difference
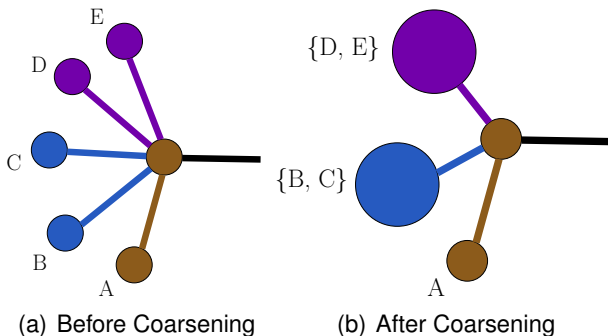
Difference Hierarchy Construction (2)

(a) Node Decomposition

(b) Graph Hierarchy

- Decompose by node difference
- Create a hierarchy based on node and edge difference
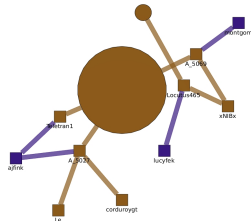
Degree One Coarsening
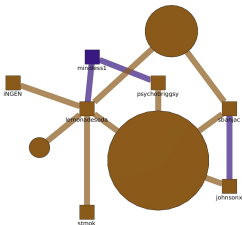

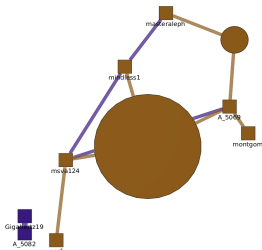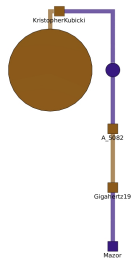(a) Before Coarsening   (b) After Coarsening

- Group degree one nodes attached same root together
- Does not need to be connected because paths begin/end here
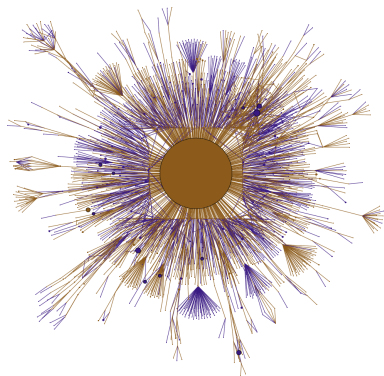
Betweenness Centrality Coarsening

- Coarsens away nodes with:
    - little change in betweenness centrality
    - large metanodes common to both graphs

Results: `Threads`
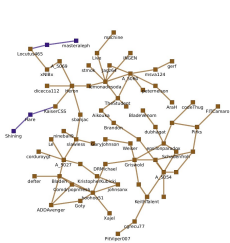
$\Delta t = 10$ and 11
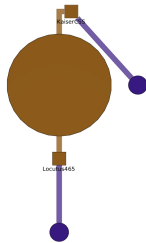
$\Delta t = 10$ and 11

$\Delta t = 11$ and 12

$\Delta t = 12$ and 13

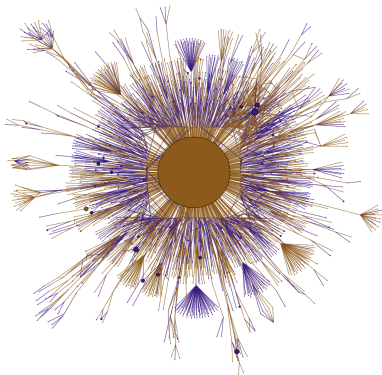$\Delta t = 13$ and 14

$\Delta t = 14$ and 15

Results: `Opte` (1)
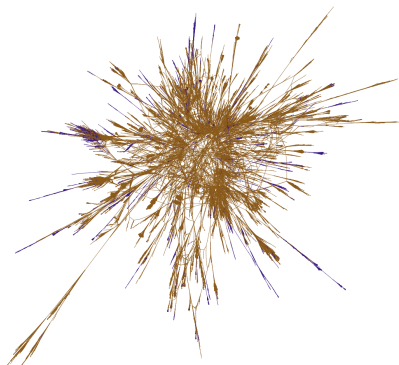
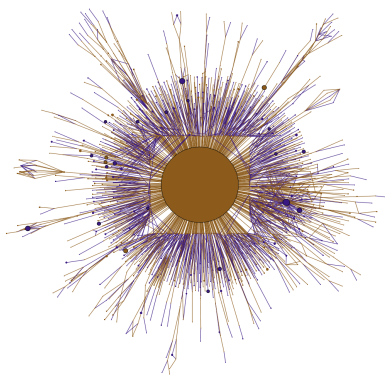(a) Difference Map

(b) Hierarchy

- Internet scan of about 40,000 nodes and 47,000 edges
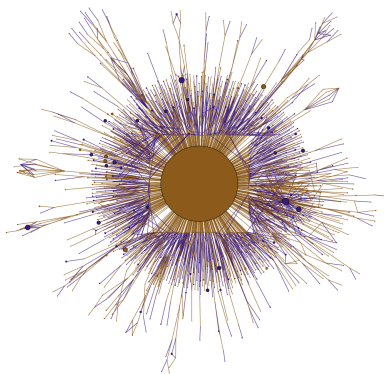
Results: `Opte` (2)

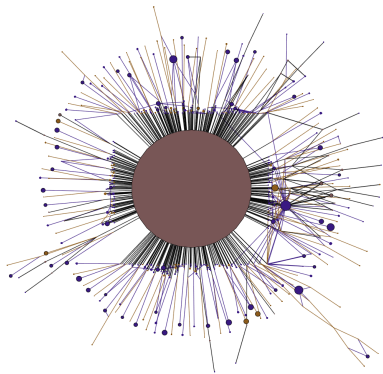(a) Hierarchy   (b) Degree One Coarsening

- Internet scan of about 40,000 nodes and 47,000 edges
- Degree one coarsening applied
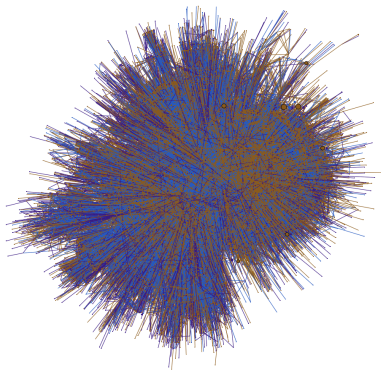
Results: `Opte` (3)

(a) Degree One Coarsening   (b) Betweenness Centrality Coarsening

- Internet scan of about 40,000 nodes and 47,000 edges
- Betweenness centrality coarsening applied

Results: `RouteViews`

- Internet scan of about 24,000 nodes and 58,000 edges
- Work still to be done
- Possible coarsening techniques to simplify hierarchy

Future Work

- Do difference maps help in understanding structural evolution?
  - currently designing an experiment to provide evidence
- Can people properly interpret hierarchies in a difference context?
- Coarsening techniques to scale to larger graphs

Conclusion

- Visualizing structural difference between two graphs
- Contributions
  - use hierarchy to coarsen away areas of similarity/difference
  - path-preserving coarsening technique
  - betweenness centrality coarsening technique
- Scales to graphs of tens of thousands of nodes

Acknowledgements

- Thank you Guy Melançon and David Auber for numerous discussions
- Thank you LaBRI for comments on the work
- Partially funded by the INRIA GRAVITÉ project
- Questions?