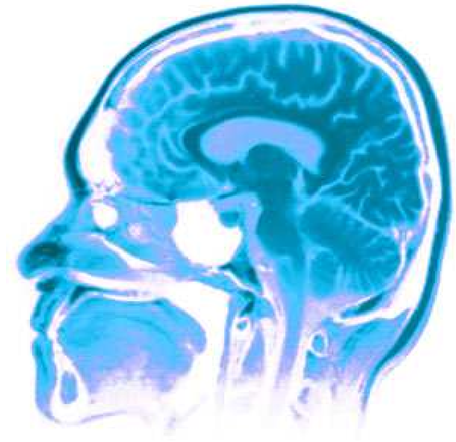
CPSC540



Decision trees

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February, 2013

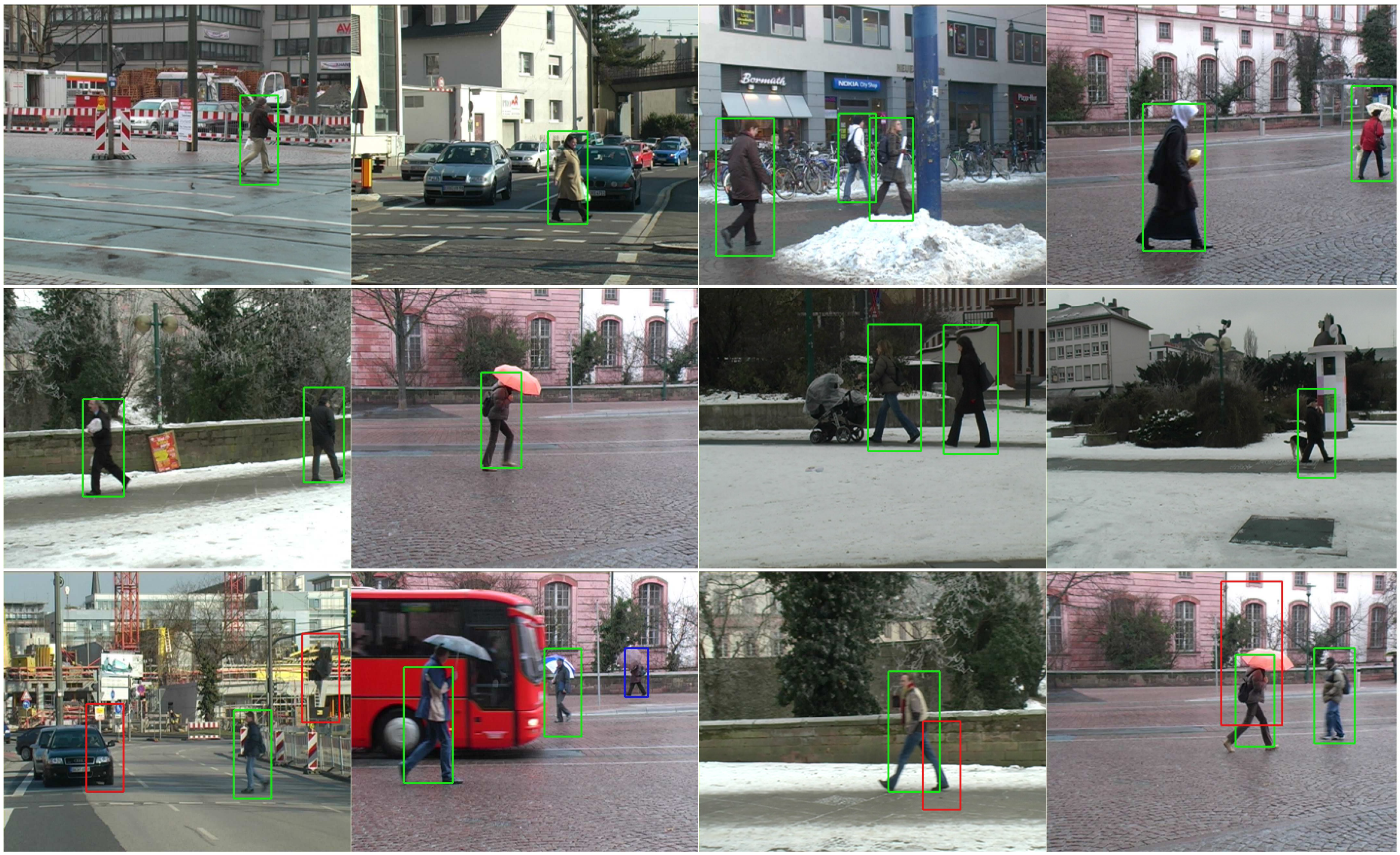
University of British Columbia

Outline of the lecture

This lecture provides an introduction to decision trees. It discusses:

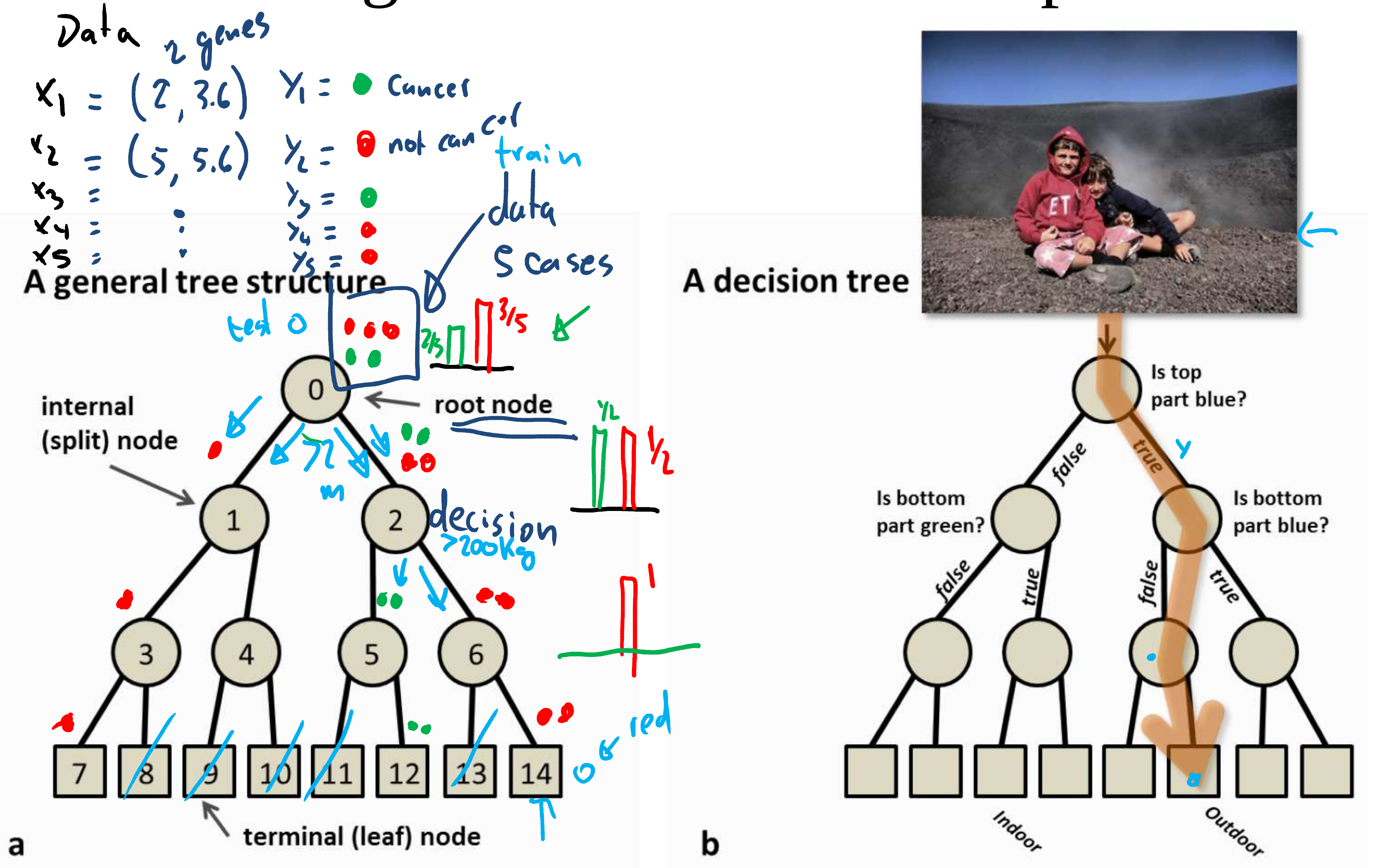
- ❑ Decision trees
- ❑ Using reduction in entropy as a criterion for constructing decision trees.
- ❑ The application of decision trees to classification

Motivation example 1: object detection



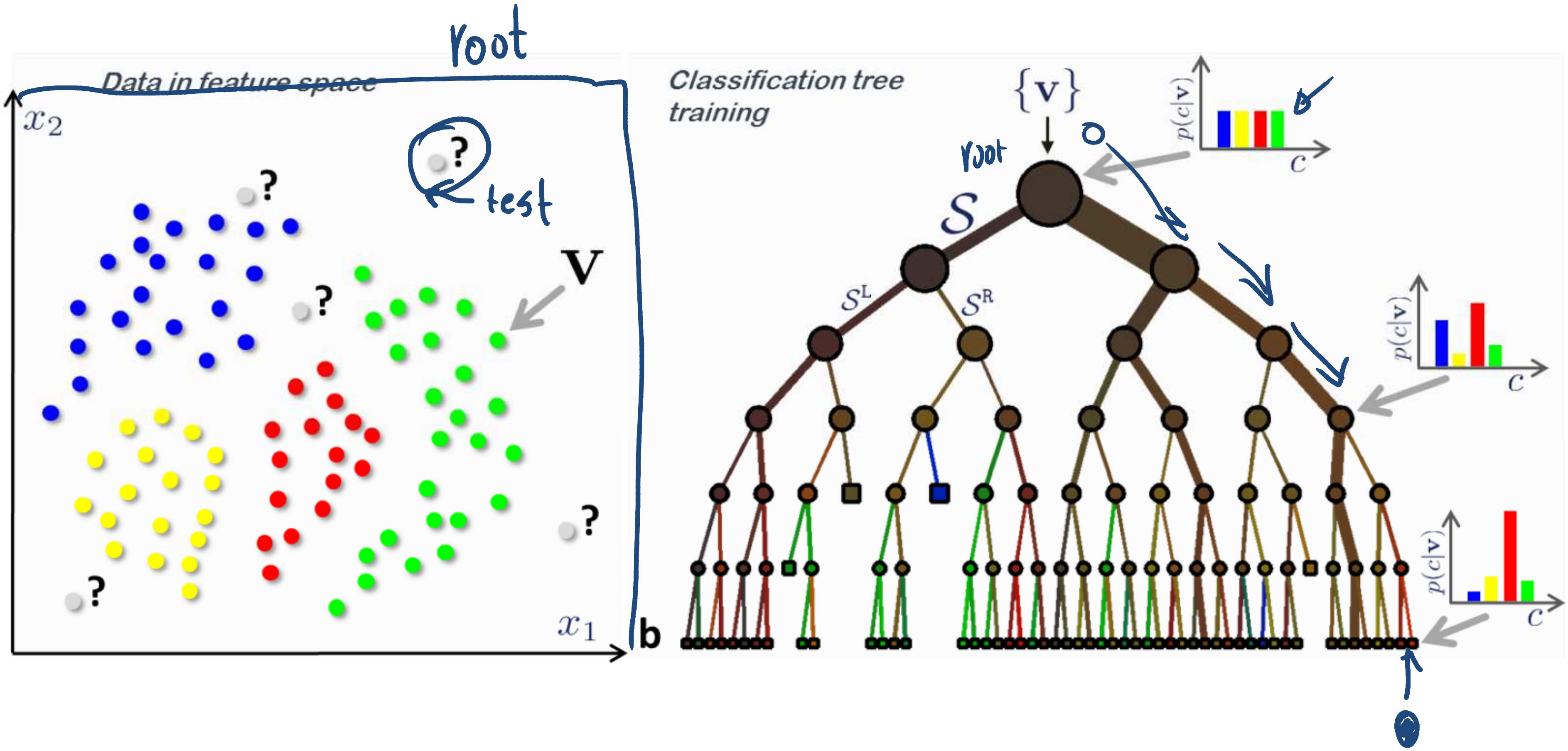
Motivation example 2: Kinect

Image classification example



[MSR Tutorial on decision forests by Criminisi et al, 2011]

Classification tree

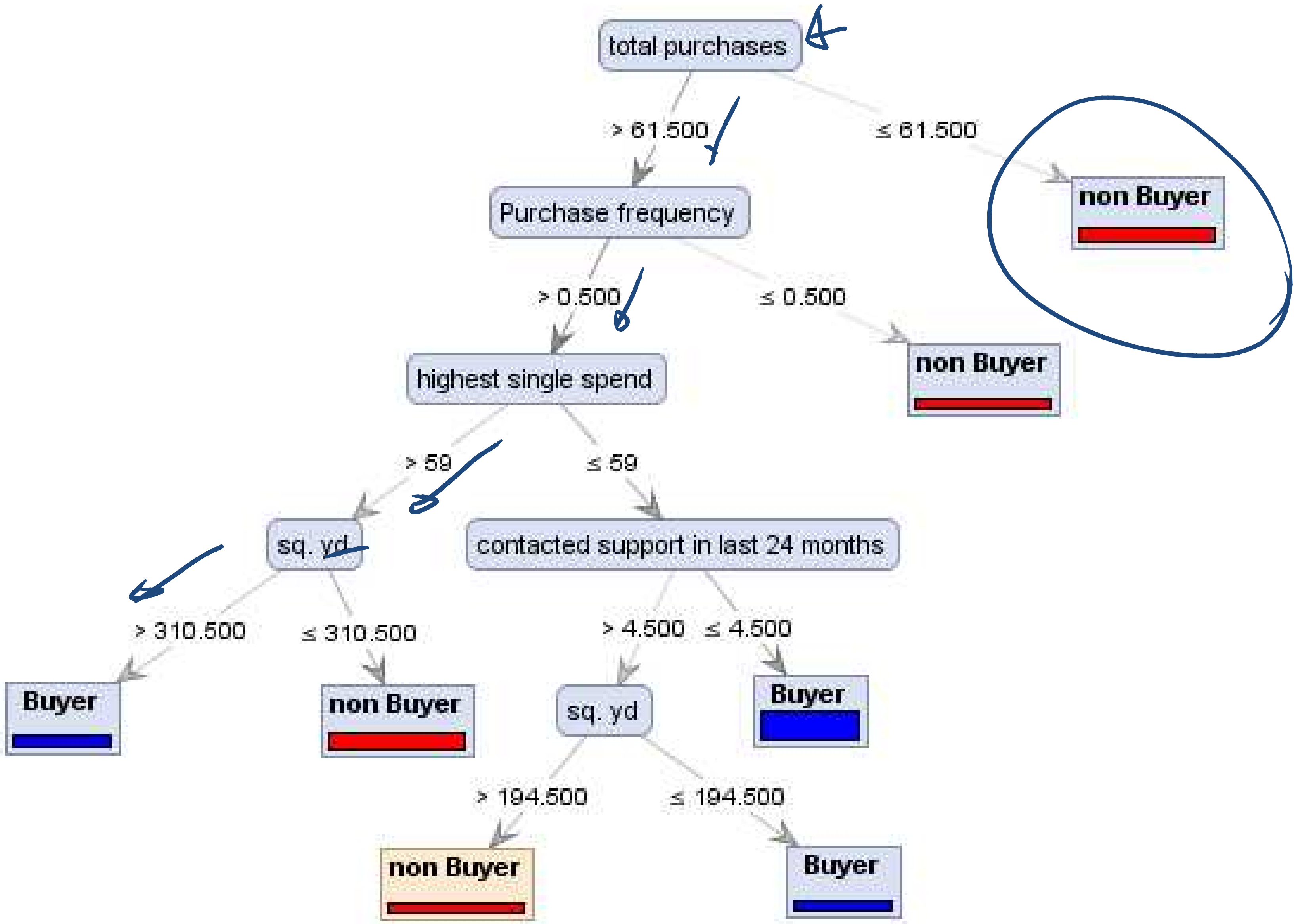


A generic data point is denoted by a vector $\mathbf{v} = (x_1, x_2, \cdots, x_d)$

$$\mathcal{S}_j = \mathcal{S}_j^{\text{L}} \cup \mathcal{S}_j^{\text{R}}$$

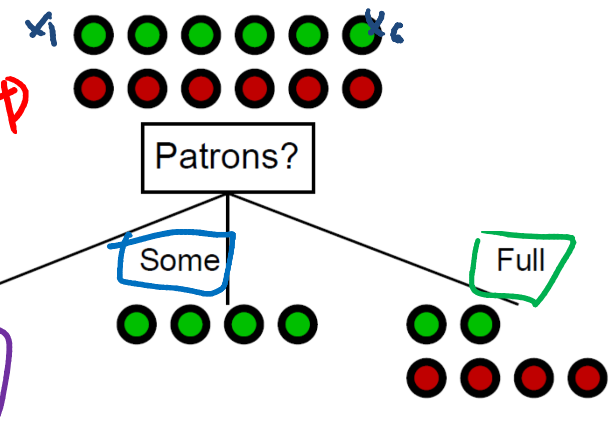
[Criminisi et al, 2011]

Another commerce example



Simafore.com

From a spreadsheet to a decision node



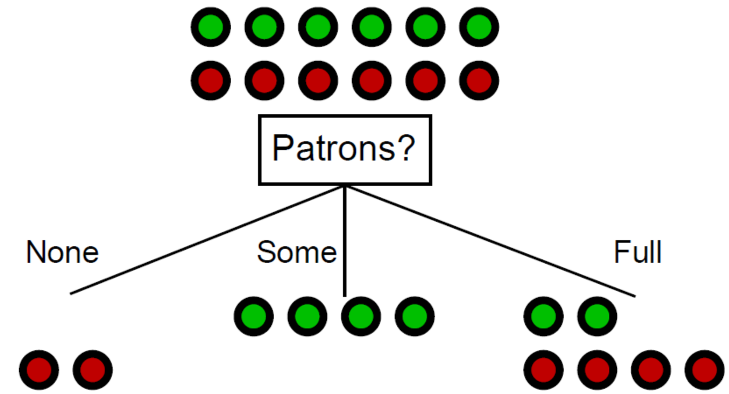
Examples described by attribute values (Boolean, discrete, continuous, etc.)
E.g., situations where I will/won't wait for a table:

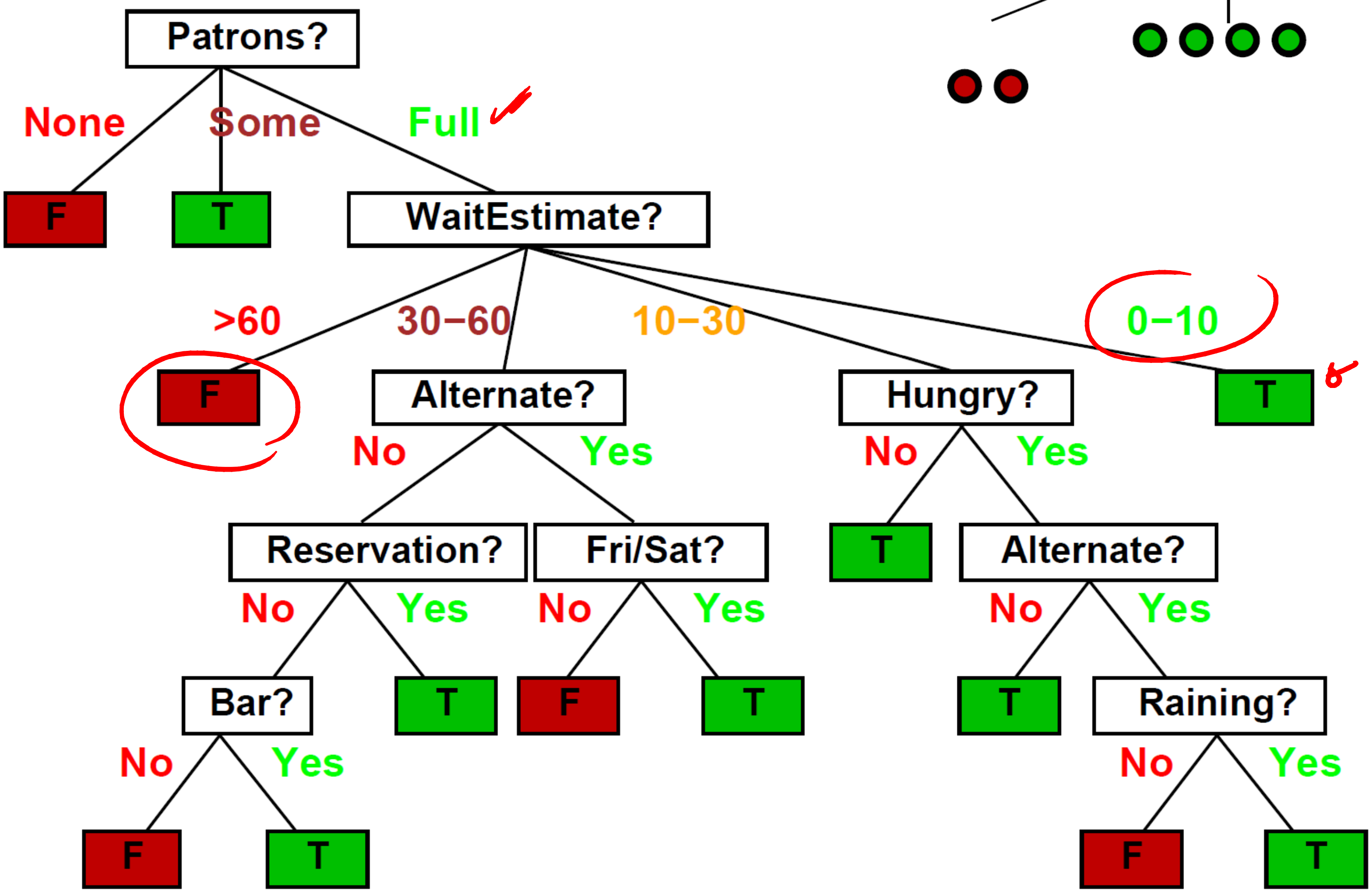
Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	T	F	F	T	Some	$$$	F	T	French	0–10	T
X_2	T	F	F	T	Full	$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	$$$	F	T	French	>60	F
X_6	F	T	F	T	Some	$$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	$$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	$$$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	$	F	F	Burger	30–60	T

Classification of examples is positive (T) or negative (F)

[AI book of Stuart Russell and Peter Norvig]

A learned decision tree

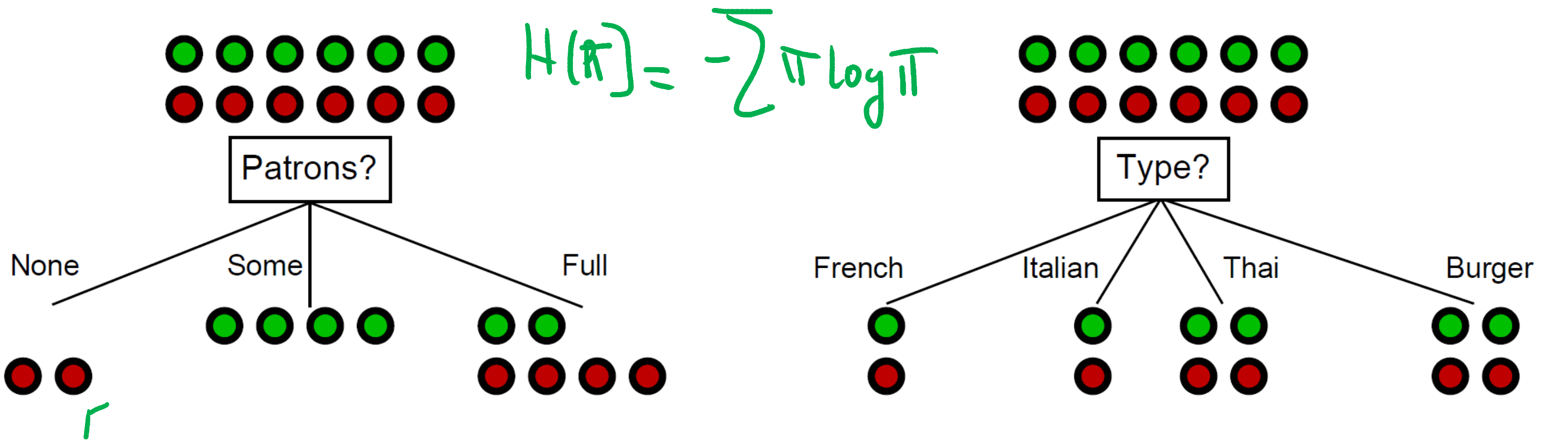




[AI book of Stuart Russell and Peter Norvig]

How do we construct the tree ?
i.e., how to pick attribute (nodes)?

Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"



Patrons? is a better choice—gives **information** about the classification

For a training set containing p positive examples and n negative examples, we have:

$$H\left(\frac{p}{p+n}, \frac{n}{p+n}\right) = -\frac{p}{p+n}\log_2\frac{p}{p+n} - \frac{n}{p+n}\log_2\frac{n}{p+n}$$

How to pick nodes?

- A chosen attribute ***A***, with ***K*** distinct values, divides the training set ***E*** into subsets ***E_1, … , E_K***.
- The **Expected Entropy (EH)** **remaining** after trying attribute ***A*** (with branches *i=1,2,…,**K***) is

$$EH(A)=\sum_{i=1}^{K}\frac{p_i+n_i}{p+n}H\left(\frac{p_i}{p_i+n_i},\frac{n_i}{p_i+n_i}\right)$$

points in child i

- **Information gain (I)** or **reduction in entropy** for this attribute is:

$$I(A)=H\left(\frac{p}{p+n},\frac{n}{p+n}\right)-EH(A)$$

- Choose the attribute with the largest I

[Hwee Tou Ng & Stuart Russell]

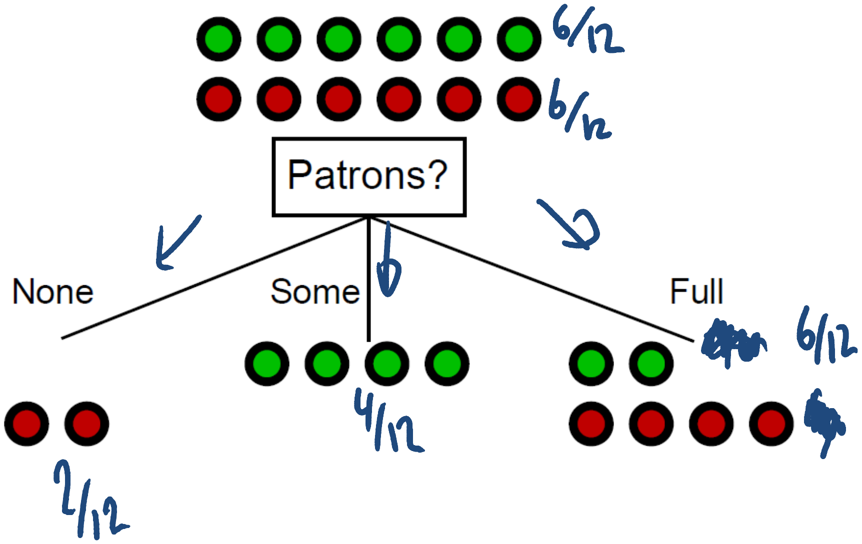
Example

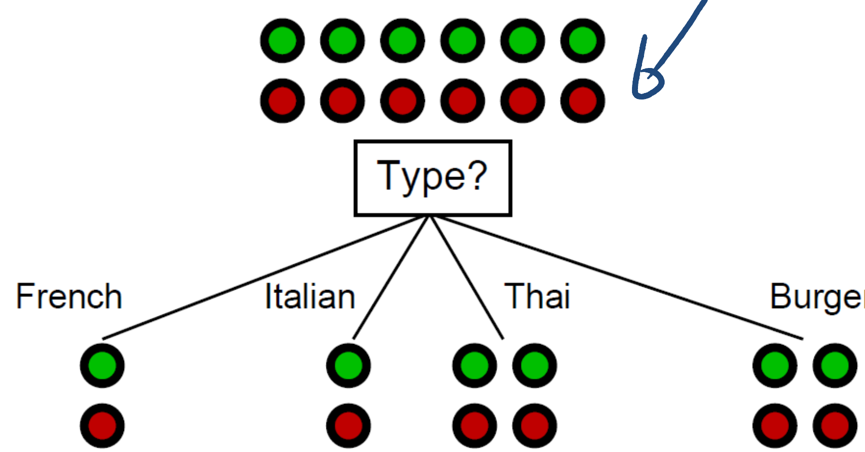
- **Convention**: For the training set, $p = n = 6$, $H(6/12, 6/12) = 1$ bit

- Consider the attributes *Patrons* and *Type* (and others too):

$$I(Patrons) = 1 - [\frac{2}{12}H(0,1) + \frac{4}{12}H(1,0) + \frac{6}{12}H(\frac{2}{6},\frac{4}{6})] = .0541 \text{ bits}$$

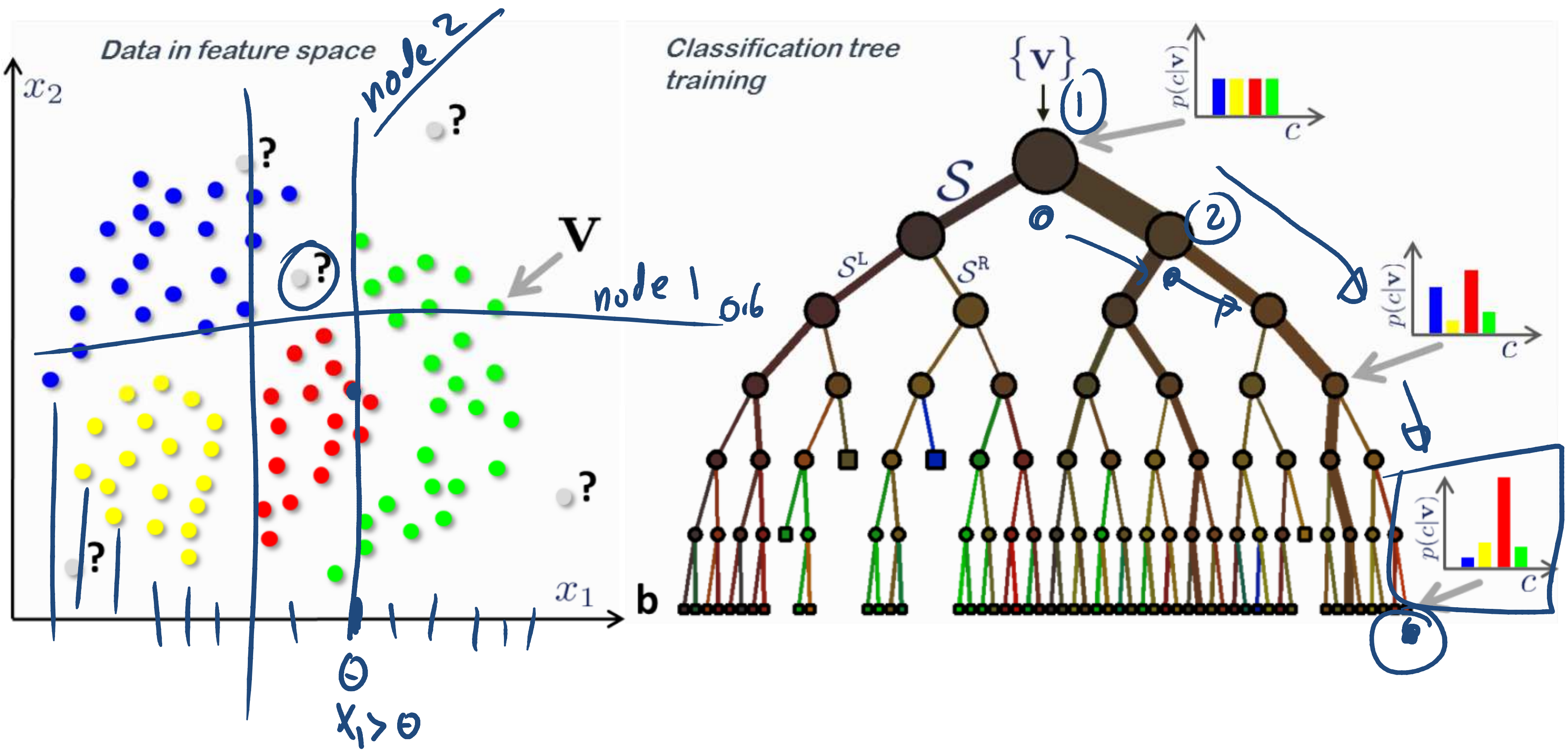
$$I(Type) = 1 - [\frac{2}{12}H(\frac{1}{2},\frac{1}{2}) + \frac{2}{12}H(\frac{1}{2},\frac{1}{2}) + \frac{4}{12}H(\frac{2}{4},\frac{2}{4}) + \frac{4}{12}H(\frac{2}{4},\frac{2}{4})] = 0 \text{ bits}$$





[Hwee Tou Ng & Stuart Russell]

Classification tree

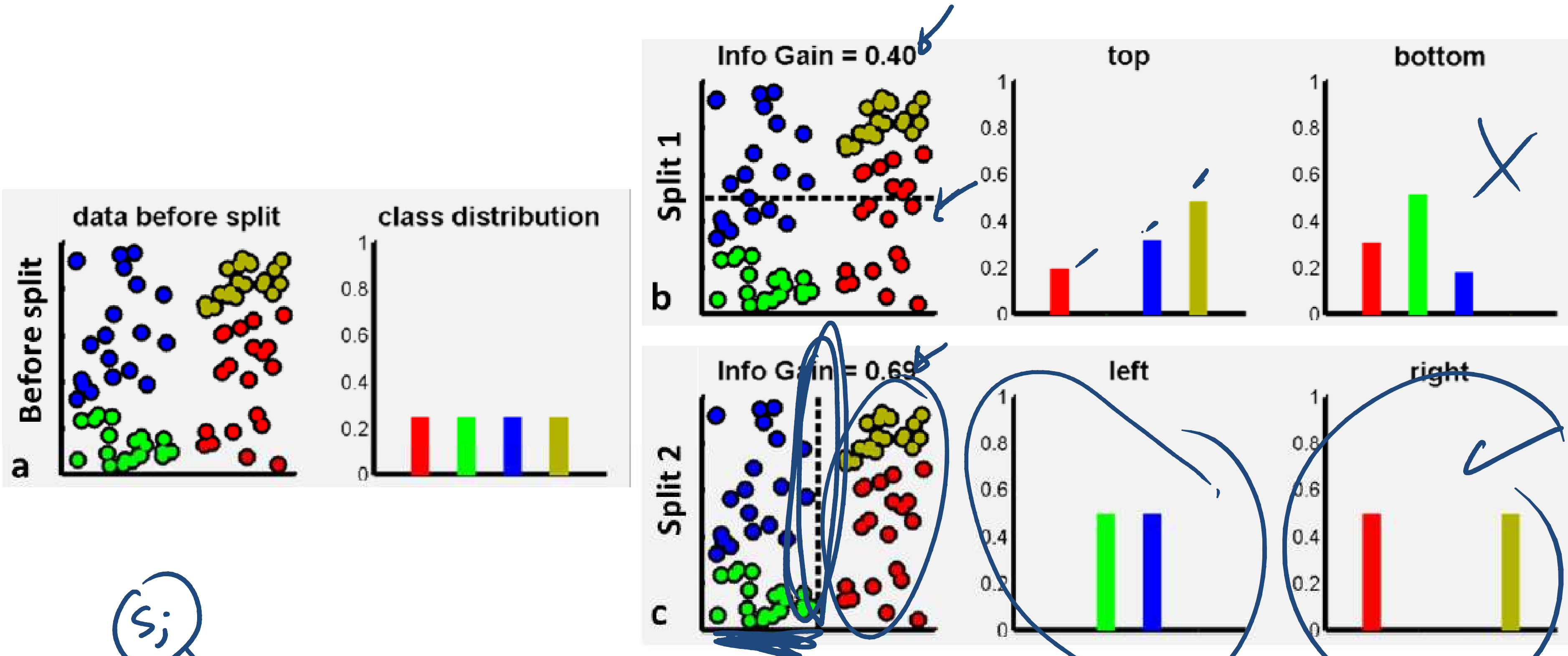


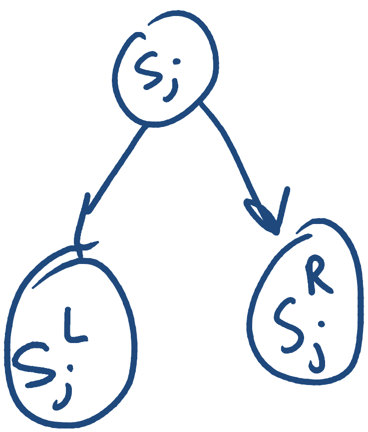
A generic data point is denoted by a vector $\mathbf{v} = (x_1, x_2, \cdots, x_d)$

$$\mathcal{S}_j = \mathcal{S}_j^{\text{L}} \cup \mathcal{S}_j^{\text{R}}$$

[Criminisi et al, 2011]

Use information gain to decide splits

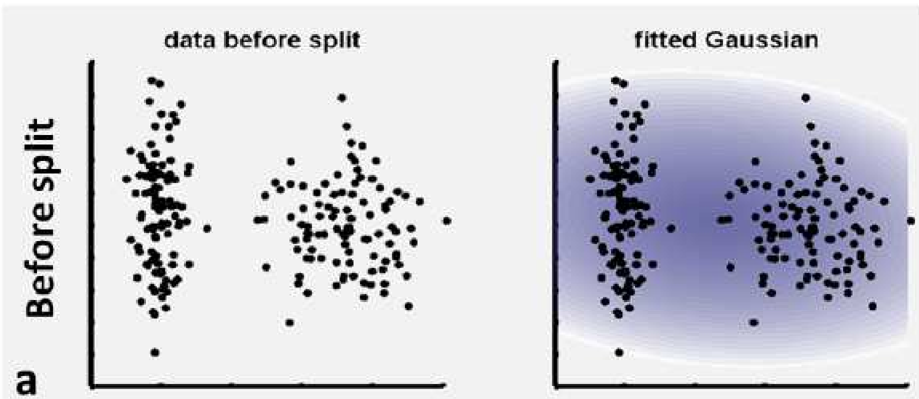


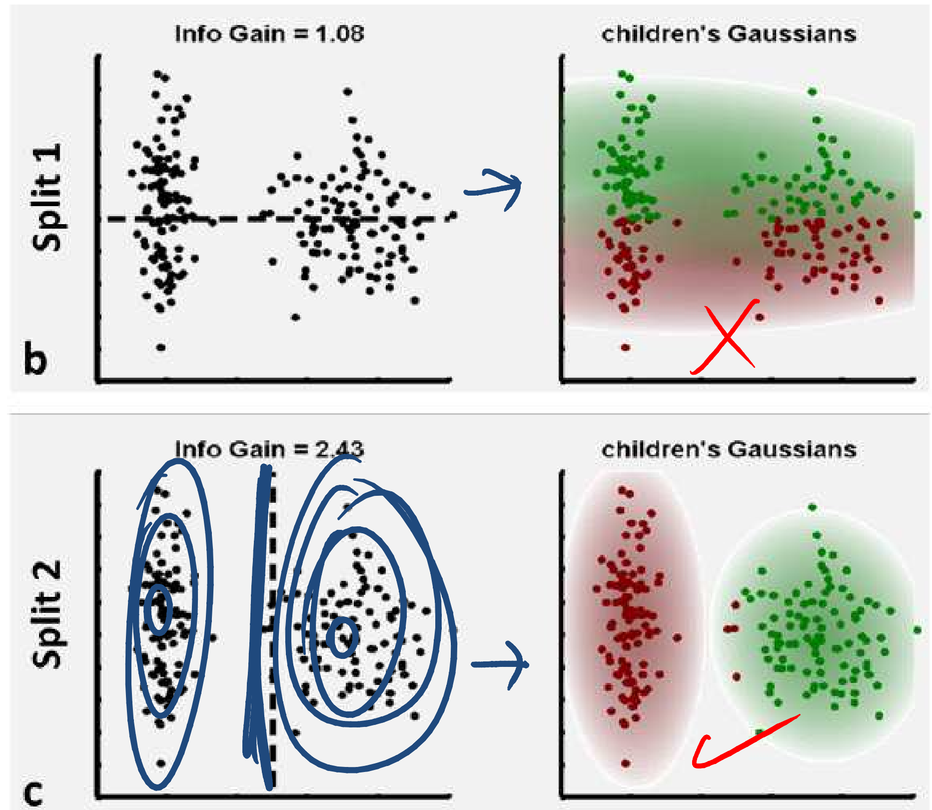


$$I_j = H(\mathcal{S}_j) - \sum_{i \in \{L,R\}} \frac{|\mathcal{S}_j^i|}{|\mathcal{S}_j|} H(\mathcal{S}_j^i)$$

[Criminisi et al, 2011]

Advanced: Gaussian information gain to decide splits



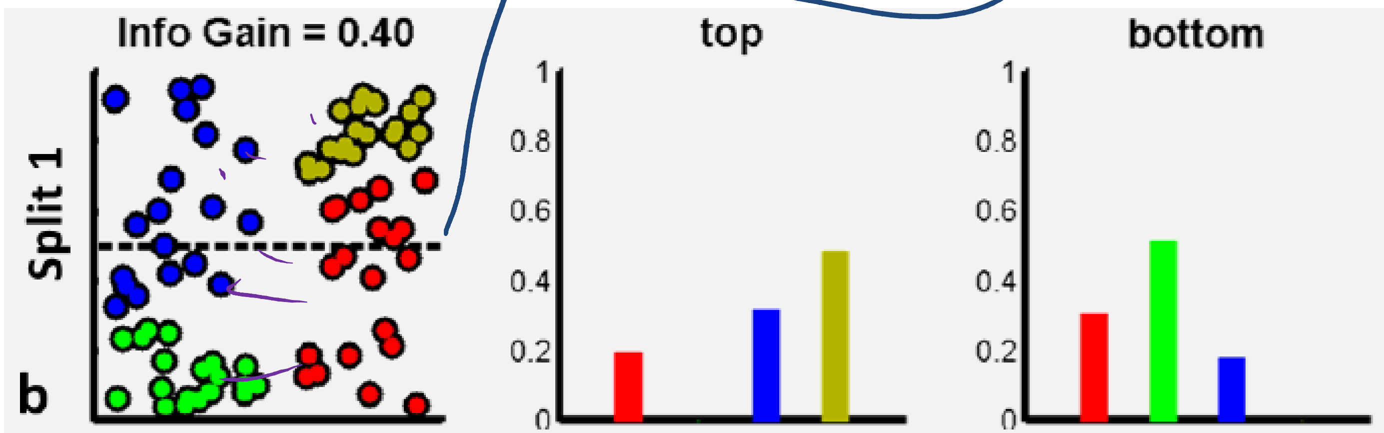


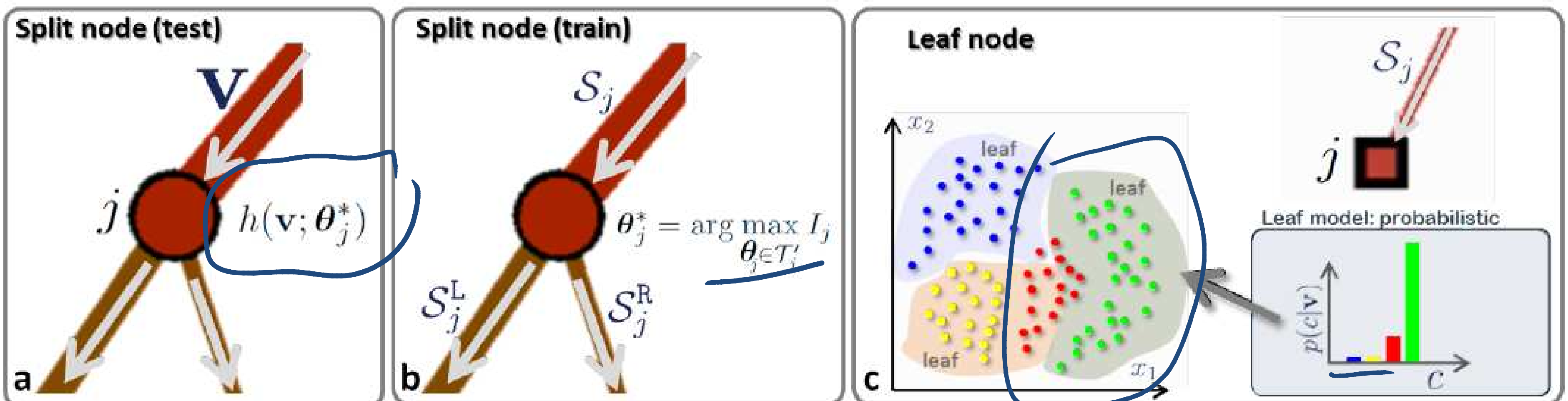
$$H(\mathcal{S}) = \frac{1}{2} \log \left((2\pi e)^d |\Lambda(\mathcal{S})| \right)$$

[Criminisi et al, 2011]

Each split node j is associated with a binary split function

$$h(\mathbf{v}, \boldsymbol{\theta}_j) \in \{0, 1\},$$

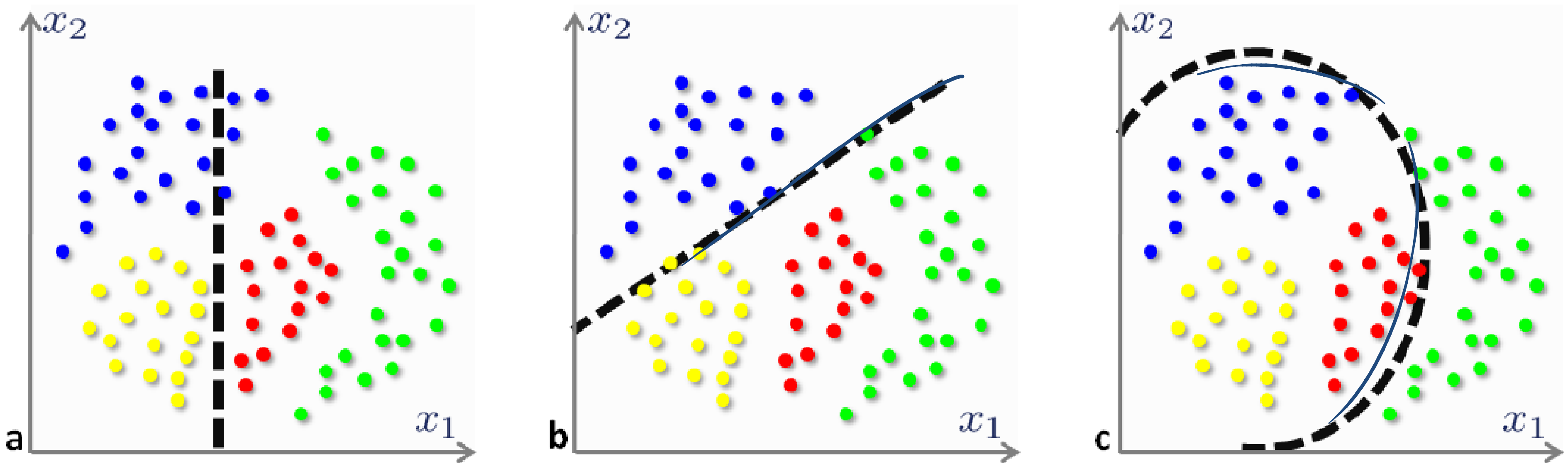




$$I_j = H(\mathcal{S}_j) - \sum_{i \in \{L,R\}} \frac{|\mathcal{S}_j^i|}{|\mathcal{S}_j|} H(\mathcal{S}_j^i)$$

[Criminisi et al, 2011]

Alternative node decisions



$$\mathbf{v} = (x_1 \ x_2) \in \mathbb{R}^2$$

[Criminisi et al, 2011]

Next lecture

The next lecture introduces random forests.