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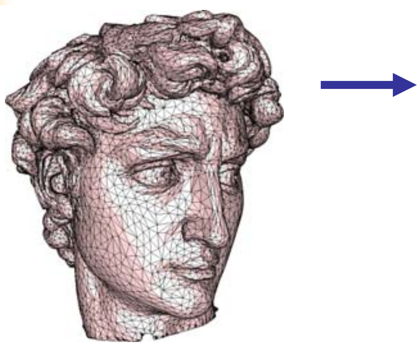
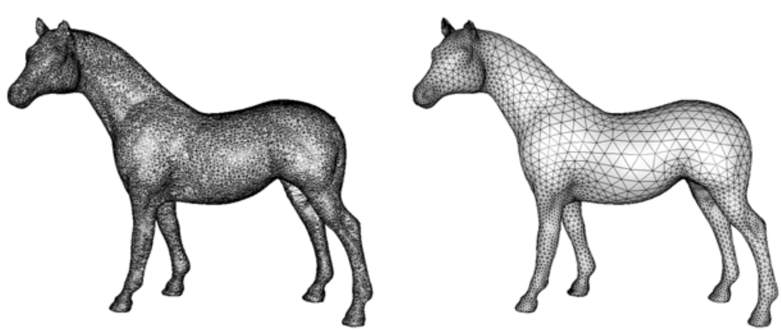
Re-Meshing Surfaces



Re-Meshing Surfaces

- Given input mesh generate new mesh which is “better”
  - Element sizing
  - Element shape
- BUT is near (geometrically) to original surface



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Hausdorff Metric

- Given two sets (surfaces) P and Q

  $H_P(Q)=\max_p \min_q ||p,q||$

  $H(P,Q)=\max(H_q(P), H_P(Q))$

  - point to point
- On mesh approximate by
  - measuring vertex to surface distance
  - measuring vertex to vertex distance
- Expensive to compute
  - Public Domain Code: Metro

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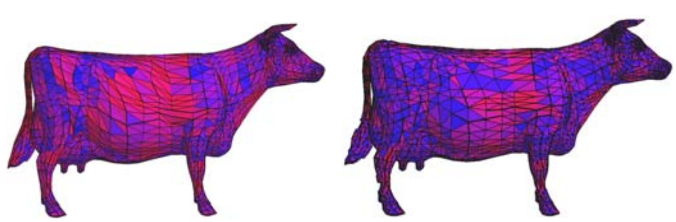
How to (re)mesh surfaces?

- Can we apply Delaunay triangulation?
  - What is Delaunay criterion on surface?
    - Option 1: Use sphere instead of circle
      - Works for volumetric meshes (tets)
    - Option 2: Use pairwise test only
    - Theoretical Delaunay properties do not hold
  - Boundary recovery = Approximation quality



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Approaches

- Mesh adaptation/Local Remeshing
  - Modify existing mesh using sequence of local operations
    - Evaluate approximation quality at every step
- Reduction to 2D/Global Remeshing
  - Segment surface into parameterizable pieces
  - Parameterize in 2D
  - Mesh in 2D (Delaunay)
  - Project back

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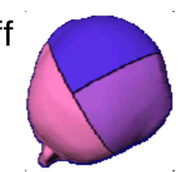
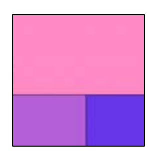
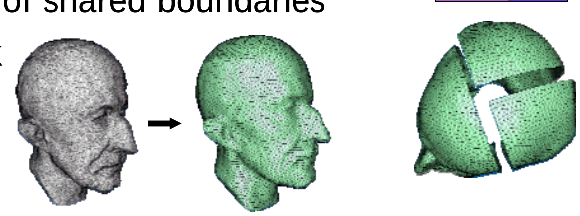
Reduction to 2D/Global Remeshing

- Segment surface into parameterizable charts
  - distortion/chart size (count) trade-off
- Parameterize in 2D
  - Distortion affects 3D mesh quality
- Mesh charts in 2D (Delaunay)
  - Take parametric distortion into account (sizing)
  - Take care of shared boundaries
- Project back







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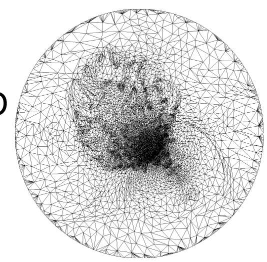
Parameterization

- Projection to/from 2D should not distort mesh
- Can handle some stretch
  - Measure & take into account during 2D meshing
    - Use as component of local sizing
- MUST be conformal
  - If care about quality
  - Want: Equilater (Delaunay) in 2D = equilateral (Delaunay) in 3D



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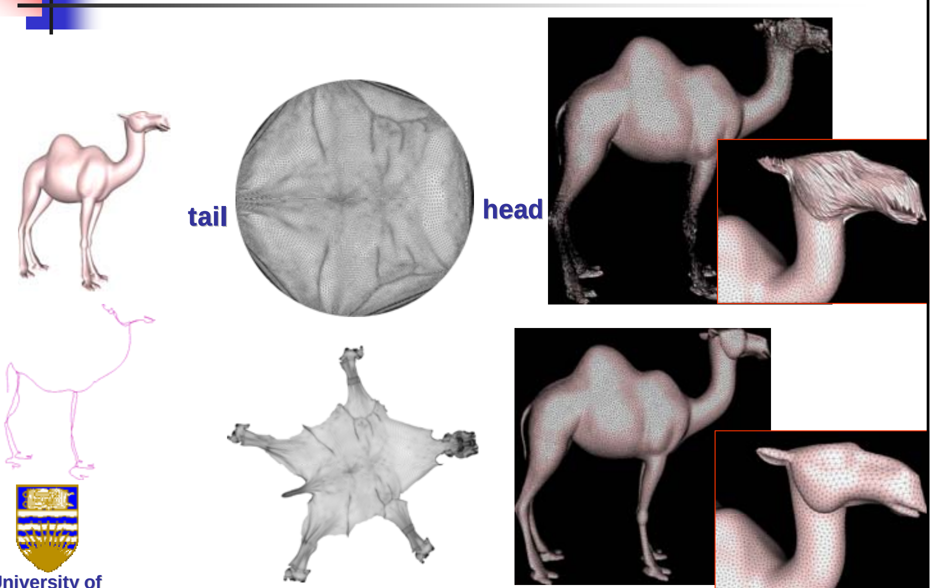
Impact of distortion



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Segmentation

- Chart Properties
  - parameterizable: open + genus 0
  - Low distortion
    - Ideal: Developable charts
- Approaches
  - Single chart
    - Generate (short) cuts to reduce genus
    - Cut through high curvature/distortion vertices
  - Multiple charts
    - More convex boundaries – easier to handle

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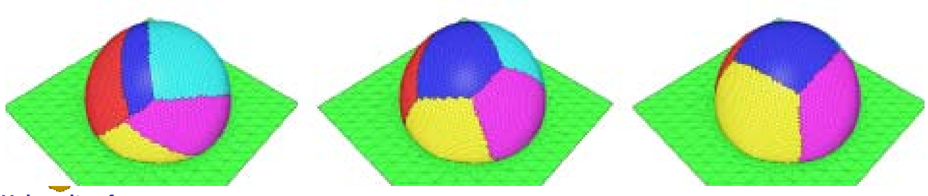
Lloyd Segmentation Framework

- Lloyd iterations:
  - Select random triangles to act as seeds
  - Grow charts around seeds using a greedy approach
  - Find new seed for each chart
    - Typically chart center
  - Repeat from step 2 until convergence



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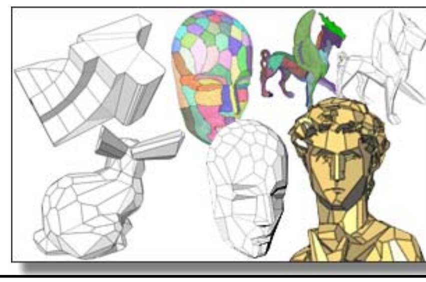
Proxies

- Charts represented by proxies – used for reseeding and growth
- Example I: Planar charts
  - Proxy: Normal to plane N_c
  - Compute: Average normal of chart triangles
  - Growth metric: Normal difference $F(C,t) = N_C \cdot n_t$



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Example II: D-Charts (Devlopable Charts

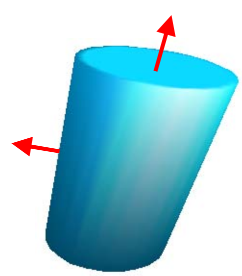
- Constant angle between surface normal and axis
  → Developable chart
- Proxy: <axis, angle> $\langle N_c, \theta_c \rangle$
- Compute:

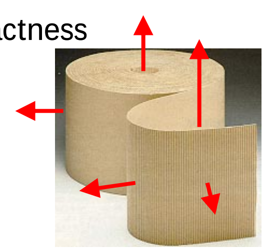
$$\min_{N_C,\theta_C} \frac{1}{A_C} \sum_{t \in C} A_t F(C,t) \text{ s.t. } \|N_C\| = 1$$

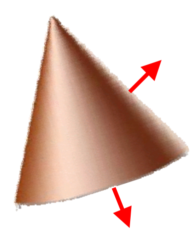
- Growth metric:

$$F(C,t) = (N_C \cdot n_t - \cos\theta_C)^2$$

  - Combine with compactness







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Examples

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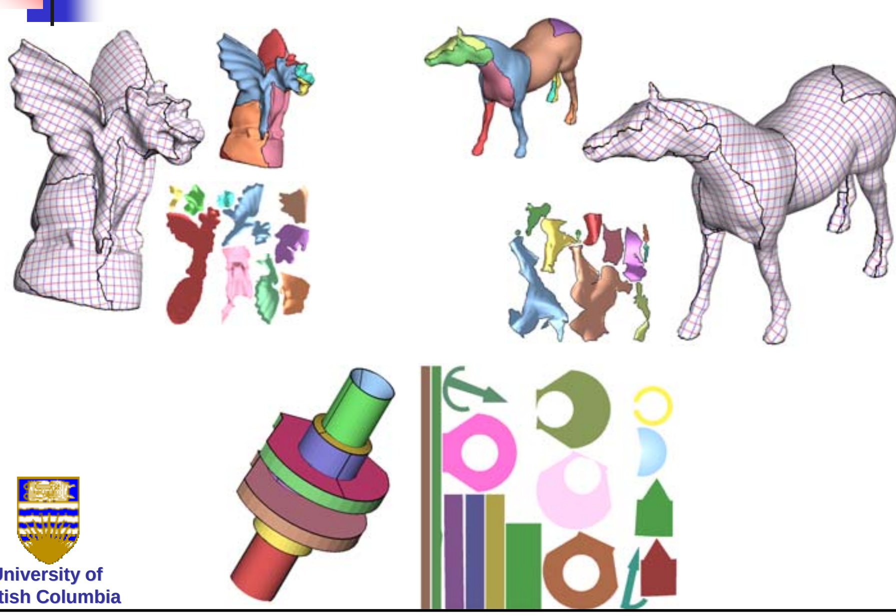
Example Results



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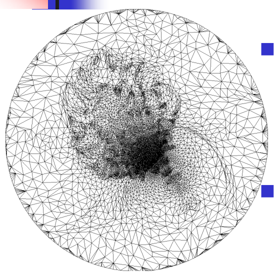
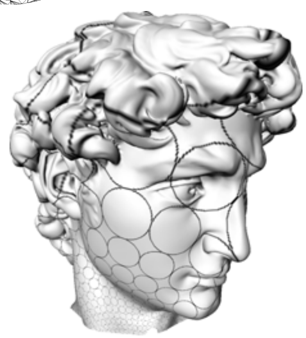
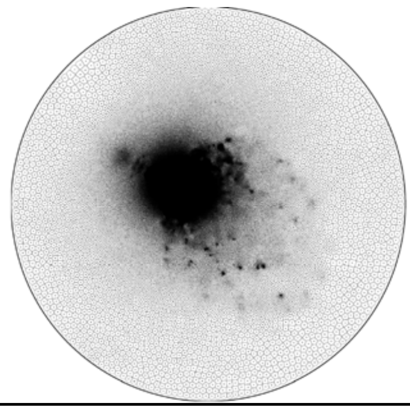
Meshing - sizing



- Measure parametric stretch (3D to 2D)
  - Measure stretch per edge $||e_{3D}||/||e_{2D}||$
  - Vertex stretch = average of edges
- Multiply sizing function (at vertices) by stretch





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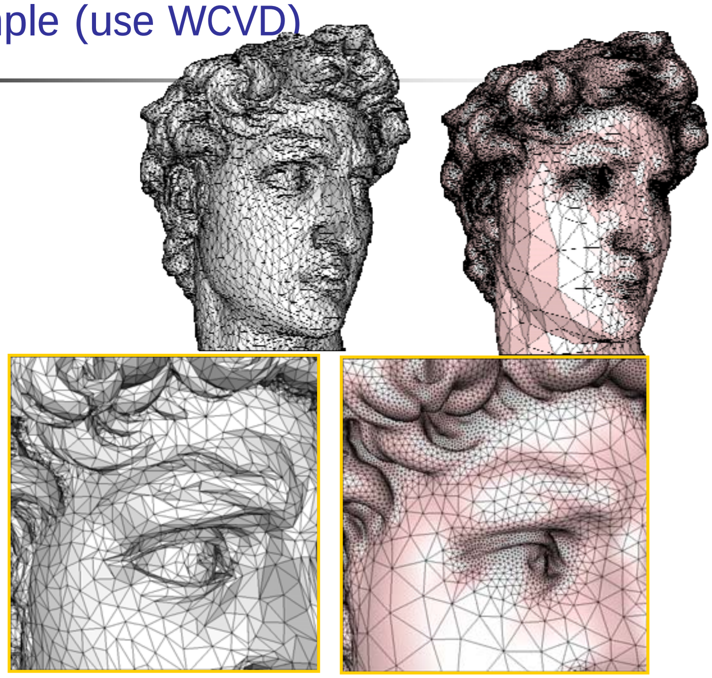
Example (use WCVD)



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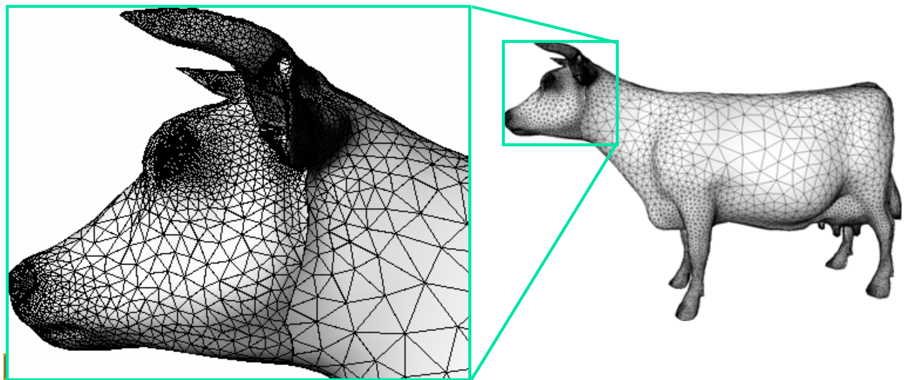
Boundary

- Need mesh consistency along boundaries
- Enforce shared boundary vertex positions



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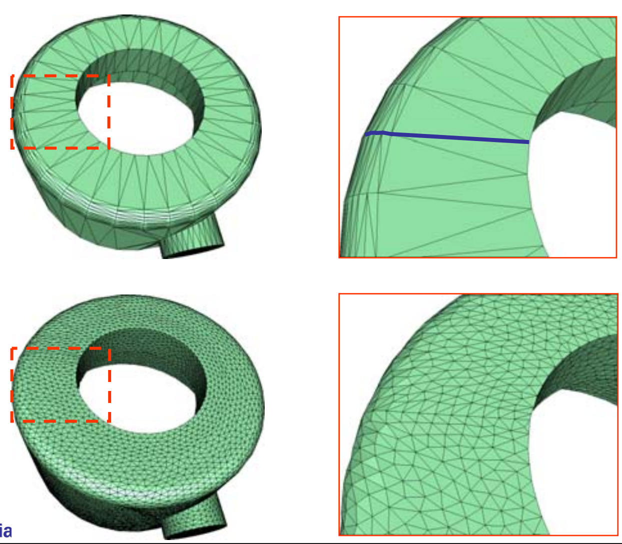
Boundaries

- Consistent but visible...



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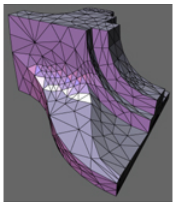
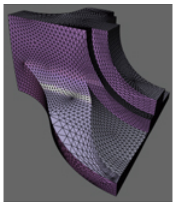
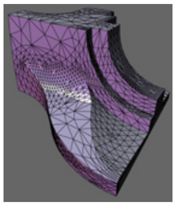
Features

- Preserving features – locate surface creases and prevent removing them
  - Special handling by segmentation and/or 2D meshing

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Global Methods - Properties

- Three major components:
  - Segment
  - Parameterize
  - Mesh in 2D
- Strongly depends on parameterization quality
  - In turn depends on segmentation
- More "formal"
- Typically more complex to implement from scratch

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