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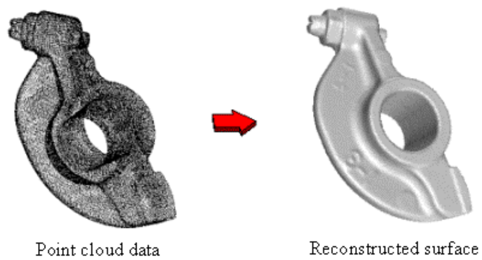
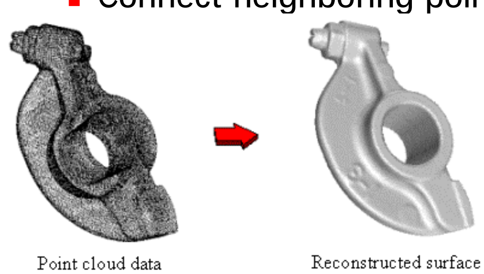
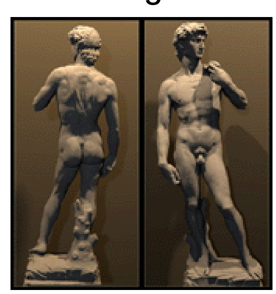
Meshes from Point Clouds



Triangulating Point Clouds

- General Idea
  - Connect neighboring points into triangles





- Issues
  - Who are the neighbors?
    - connectivity/topology
  - Efficiency – (David 32GIGA)

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Basic Reconstruction - Zippering

- Use range scanner properties for reconstruction
  - Single scan from given direction produces regular lattice of points in X and Y with changing depth (Z)
- Register/Combine multiple scans to create complete model
  - Find “optimal” registration
- Variations of this setup used by commercial scanners

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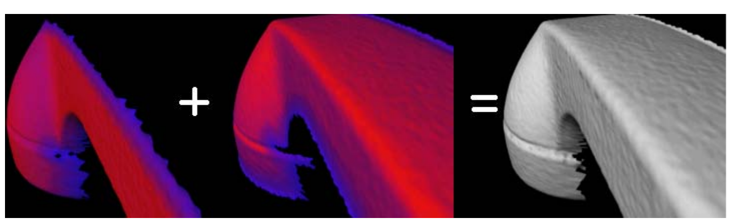
Algorithm steps

- Generate separate mesh from each scan
  - Use X & Y adjacency info
- Combine
  - Register positions
  - Merge meshes



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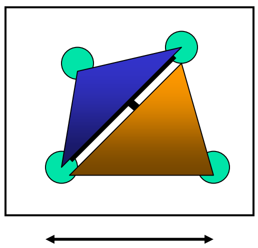
Single Mesh from Range Image

- Find quadruples of lattice points
- Form triangles
  - Find shortest diagonal
  - Form two triangles (test depth)



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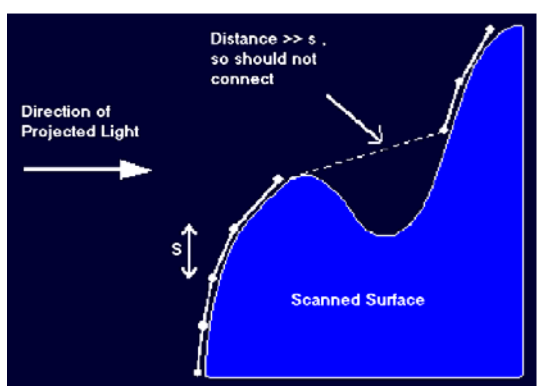
Single Mesh from Range Image

- Avoid connecting depth discontinuities:
  - Test 3D distance between points when generating triangles
  - Do not generate if depth >> S



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Registration of Range Images

- Align corresponding portions of different range images
- Variation of ***Iterated closest-point*** (ICP) algorithm
- Initial alignment from camera positions (user)

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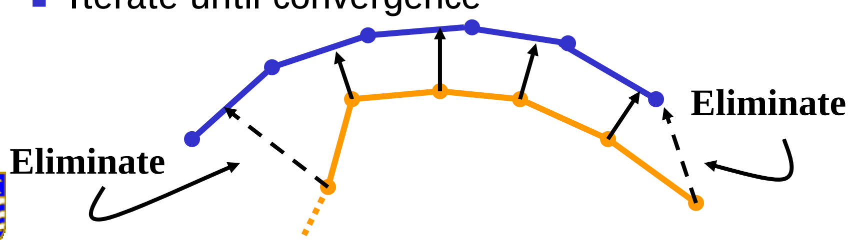
Alignment (ICP)

- Find nearest position on mesh A to each vertex of mesh B
- Discard pairs of points that are too far apart
- Find rigid transformation that minimizes weighted least-squared distance
- Iterate until convergence



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Point Matching

- Input: 2 matching sets of 3D points (M, D)
- Find rigid transformation (rotation+translation) which minimizes the distance between M and D
- Use Least-Squares

$$\min_{R,T} \sum_i \left\| M_i - (RD_i + T) \right\|^2$$

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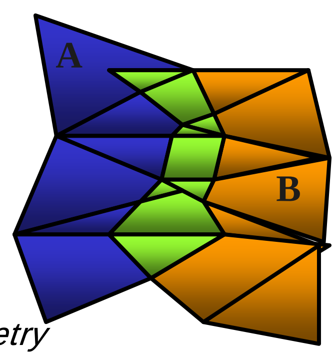
Integration: Mesh Zippering



- After registration have two overlapping meshes
  - Need to combine into single connectivity
- Zippering
  - Remove overlapping portion of the mesh
    - Use for *consensus geometry*
  - Clip one mesh against another
  - Remove triangles introduced during clipping

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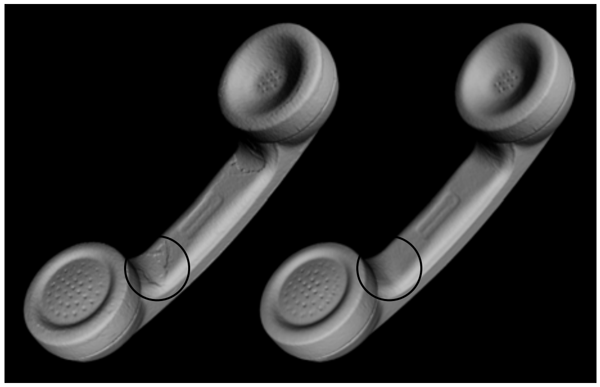
The Telephone Handset Example



Zippering results **consensus geometry**

Info : 10 range images, more than 160,000 triangles.

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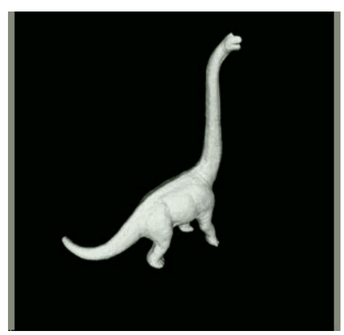
The Dinosaur Example



plastic dinosaur model **polygon mesh**

Info : 14 range images, more than 360,000 polygons.

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Some Related Algebra – Matrix Decomposition

Or how to solve for orthonormal transformation matrices...

Singular Value Decomposition

- Any m by n matrix $\boldsymbol{A}$ may be factored such that
$$\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^T$$
- $\boldsymbol{U}$: m by m, orthogonal, columns are the eigenvectors of $\boldsymbol{AA}^T$
- $\boldsymbol{V}$: n by n, orthogonal, columns are the eigenvectors of $\boldsymbol{A}^T\boldsymbol{A}$
- Σ: m by n, diagonal, r singular values are the square roots of the eigenvalues of both $\boldsymbol{AA}^T$ and $\boldsymbol{A}^T\boldsymbol{A}$

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Application: Pseudoinverse

- Given $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$, $\boldsymbol{x} = \boldsymbol{A}^{+}\mathbf{y}$
- For square $\boldsymbol{A}$, $\boldsymbol{A}^{+} = \boldsymbol{A}^{-1}$
- For any $\boldsymbol{A}$...

$$\boldsymbol{A}^{+} = \boldsymbol{V}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^{\mathrm{T}}$$

- $\boldsymbol{A}^{+}$ is called ***pseudoinverse*** of $\boldsymbol{A}$.
- $\boldsymbol{x} = \boldsymbol{A}^{+}\mathbf{y}$ - least-squares solution of $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$

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Polar Decomposition

$$\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^{T}$$
$$\boldsymbol{P} = \boldsymbol{V}\Sigma\boldsymbol{V}^{T}$$

- Polar or QR Decomposition

  $\boldsymbol{A}=\boldsymbol{Q}\boldsymbol{P}$

  $\boldsymbol{Q}=\boldsymbol{U}\boldsymbol{V}^{T}$
- Q is orthonormal

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