Some CPSC 259 Sample Exam Questions on Graph Theory (Part 6)
Sample Solutions

<u>DON'T LOOK AT THESE SOLUTIONS UNTIL YOU'VE MADE AN HONEST ATTEMPT AT ANSWERING THE QUESTIONS YOURSELF.</u>

1. {3 marks} Can a simple graph have 5 vertices and 12 edges? If so, draw it; if not, explain why it is not possible to have such a graph.

   ANSWER:

   In a simple graph, no pair of vertices can have more than one edge between them. In other words, there are no parallel edges.

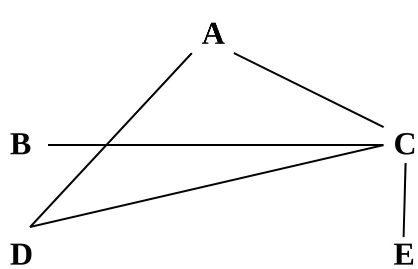
   For a simple graph, the "densest" graph we can get is one in which every vertex is connected to every other vertex. This is called a *complete graph*. The maximum number of edges in the complete graph containing 5 vertices is given by $\mathbf{K_5}$: which is C(5, 2) edges = "5 choose 2" edges = 10 edges. Since 12 > 10, it is not possible to have a simple graph with more than 10 edges.

2. {6 marks} Suppose that in a group of 5 people: A, B, C, D, and E, the following pairs of people are acquainted with each other.
   - A and C
   - A and D
   - B and C
   - C and D
   - C and E

   a) Draw a graph G to represent this situation.
   b) List the vertex set, and the edge set, using set notation. In other words, show sets V and E for the vertices and edges, respectively, in G = {V, E}.
   c) Draw an adjacency matrix for G.

   ANSWER:

   a) One such graph for G is:



b) For sets V and E, any order to the elements is fine. Furthermore, in edge set E, you can specify (A, C) or (C, A); they mean the same thing.

V = {A, B, C, D, E}
E = {(A, C), (A, D), (B, C), (C, D), (C, E)}

c) Adjacency matrix (0 = no edge; 1 = edge):

	A	B	C	D	E
A	0	0	1	1	0
B	0	0	1	0	0
C	1	1	0	1	1
D	1	0	1	0	0
E	0	0	1	0	0

3. {3 marks} How many *more* edges are there in the complete graph $\mathbf{K_7}$ than in the complete graph $\mathbf{K_5}$?

ANSWER:

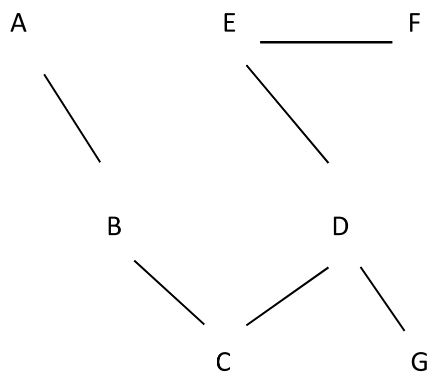
C(7, 2) – C(5, 2) = 21 – 10 = 11

4. {4 marks} Given a graph for a tree (with no designated root), briefly describe how a root can be chosen so that the tree has *maximum* height. Similarly, describe how a root can be chosen so that the tree has *minimum* height. (Note that path length is described as the number of edges that need to be traversed between two vertices.)

ANSWER:

For the maximum height, choose either end of the longest path as the root. For the minimum height, choose the vertex at the half-way point of the path.

5. {6 marks} Perform a *breadth-first search* of the following graph, where E is the starting node. In other words, show the output if we issue the call `BFS(E)`. Provide two cases: (a) Use a counterclockwise ordering from the top (12 o'clock position); and (b) Use a clockwise ordering from the top.



ANSWER:

(a) When we visit adjacent nodes in a counterclockwise order from the top, the order in which we visit the nodes is:

E, D, F, C, G, B, A

(b) When we visit adjacent nodes in a clockwise order from the top, the order in which we visit the nodes is:

E, F, D, G, C, B, A

6. {6 marks} Perform a *depth-first search* of the same graph as in Question 5, but use D as the starting node. In other words, show the output if we issue the call `DFS(D)`. Provide two cases: (a) Use a counterclockwise ordering from the top (12 o'clock position); and (b) Use a clockwise ordering from the top.

ANSWER:

(a) When we visit adjacent nodes in a counterclockwise order from the top, the order in which we visit the nodes is:

D, E, F, C, B, A, G

(b) When we visit adjacent nodes in a clockwise order from the top, the order in which we visit the nodes is:

D, G, C, B, A, E, F