

**SUPPLEMENTAL FINAL EXAM PRACTICE, CPSC 421/501,  
FALL 2023**

JOEL FRIEDMAN

**DOCUMENT UNDER CONSTRUCTION AND MAY BE INCOMPLETE**

**Copyright:** Copyright Joel Friedman 2023. Not to be copied, used, or revised without explicit written permission from the copyright owner.

- (1) Which of the following are true? Explain: explain why they are (always) true, or give a counterexample and explain why this is a counterexample.
  - (a) If the Boolean formulas associated to an NP-complete language over the alphabet  $\Sigma = \{T, F\}$  don't have polynomial size circuits, it follows that  $P \neq NP$ .
  - (b) If the Boolean formulas associated to an NP-complete language over the alphabet  $\Sigma = \{T, F\}$  don't have polynomial size formulas, it follows that  $P \neq NP$ .
  - (c) As of November 2023, we know that PARTITION is NP-complete.
  - (d) As of November 2023, it is possible that  $a\{a, b\}^*b$  is NP-complete.
  - (e)  $\text{Threshold}_{2,n}$  can be expressed by formulas of size  $O(n \log_2 n)$ .
  - (f)  $\text{Threshold}_{2,n}$  can be expressed by circuits of size  $O(n \log_2 n)$ .
  - (g)  $\text{Parity}_n$  can be expressed by formulas of size  $O(n \log_2 n)$ .
  - (h)  $\text{Parity}_n$  can be expressed by formulas of size  $O(n^2)$ .

- (2) Write a 3CNF formula that is satisfiable for all values of  $x_1, \dots, x_5 = T, F$  iff

$$(x_1) \wedge (x_2 \vee x_3 \vee x_4 \vee \neg x_5) = T;$$

(you may add additional variables).

- (3) Is there a 3CNF formula in  $x_1, \dots, x_5$  that is equivalent to  $x_2 \vee x_3 \vee x_4 \vee \neg x_5$ ? Explain.

- (4) MORE PROBLEMS MAY BE ADDED LATER.

DEPARTMENT OF COMPUTER SCIENCE, UNIVERSITY OF BRITISH COLUMBIA, VANCOUVER, BC V6T 1Z4, CANADA.

*E-mail address:* [jf@cs.ubc.ca](mailto:jf@cs.ubc.ca)

*URL:* <http://www.cs.ubc.ca/~jf>

---

Research supported in part by an NSERC grant.