Midterm Solutions

CPSC 421/501, Fall 2021

Problem 1 (True/False)

(a) True: Follows from a discussion of NFA's, e.g. Section 1.2 [Sip]

(b) True: essentially because there are only countably many standardized DFA's.

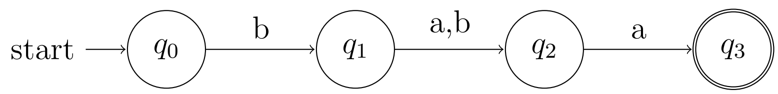
(c) False: The set of languages is in 1-1 correspondence with the set of subsets of words over Σ, and this set of words is (countably) infinite.

(d) False: This can seen numerous ways: for one, the gaps between numbers of the form n^4 is too large to make this set of integers eventually periodic.

(e) False This set of maps is 1-1 correspondence with the set of subsets of natural numbers, and this set of numbers is (countably) infinite.

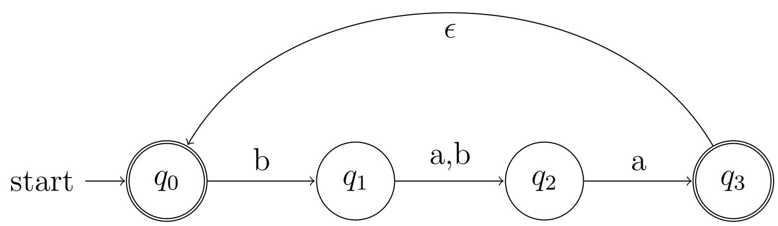
Problem 2

(a)

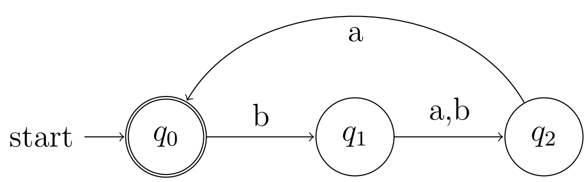


(b)

We need only make a ϵ connection from the accepting state in part (a) to the initial state and make the initial state the accepting state.



or as state q_3 can be reduced.



Problem 3

(a) We have that the shortest word in $\mathrm{AccFut}(a^n)$ for $n = 0, 1, 2, 3$ is a^{3-n}, and these shortest words are all distinct for $n = 0, 1, 2, 3$. Hence $\mathrm{AccFut}(a^n)$ are distinct for $n = 0, 1, 2, 3$. However, we easily see that for $n \geq 2$ and even,

$$\mathrm{AccFut}_L(a^n) = a(a^2)^*, \quad \mathrm{AccFut}_L(a^{n+1}) = (a^2)^*.$$

Hence

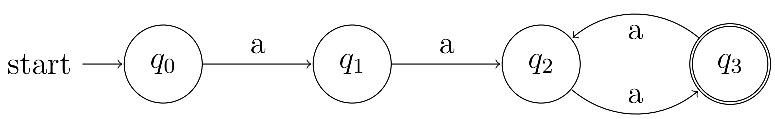
$$\mathrm{AccFut}_L(a^2) = \mathrm{AccFut}_L(a^4) = \mathrm{AccFut}_L(a^6) = \cdots$$

and similarly

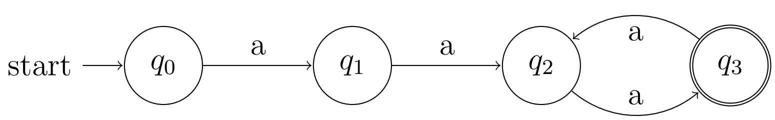
$$\mathrm{AccFut}_L(a^3) = \mathrm{AccFut}_L(a^5) = \mathrm{AccFut}_L(a^7) = \cdots$$

Hence there are exactly four different possible "accepting futures," and hence the minimum number of states of DFA accepting L is 4.

(b)



(c)



Our NFA has no arrows labelled b, since any such letter forces the input to be rejected; the NFA has a path from the initial state to a^3, which is the first word in L, and then cycles on a cycle of length two, since L has eventual period 2 at a^3. The NFA pictured transitions from a^3 to the a^2 state, since this is an equivalent way to cycle to a^3 (one could have built a cycle from a^3 to one new state and then cycle back to a^3).

Problem 4
Since $f(c) = \{b, c\}$, we have $c \in f(c)$, and therefore $c \notin T$. Since c lies in $f(c)$ and not in T, the sets $f(c)$ and T must be distinct.