THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 421/501: MIDTERM EXAMINATION – November 3, 2023

Last Name: _____   First Name: _____

Signature: _____   UBC Student #: └─┴─┴─┴─┴─┴─┴─┴─┘

Important notes about this examination

1. You have 45 minutes to write this examination.
2. You may use a pencil to write your solutions, although a very light pencil might be harder to read after scanning.
3. No textbooks or electronic devices are permitted. We permit a "cheat-sheet" consisting of one page of handwritten or typed notes, on double-sided 8.5x11" paper.
4. Answer all the questions in the exam.
5. Good luck!

1

0. IDENTIFICATION

Please make sure that the following is your 8-character Student ID:

Student ID:

Your answer to each problem should be written on its page; if needed, you can use the back side of the page as well.

1. QUESTION 1. (10 POINTS, 2 POINTS PER CORRECT T/F ANSWER — NO PENALTY FOR INCORRECT RESPONSES)

Circle either T for true, or F for false, for each of the statements below. In these questions, $\Sigma = \Sigma_{\text{ASCII}}$ denotes the ASCII alphabet, $L_1, L_2 \subset \Sigma^*$, $L_1 \setminus L_2$ denotes the set difference of L_1 and L_2.

If L_1 and $\Sigma^* \setminus L_1$ are both recognizable, then L_1 is decidable.   T   F

If L_1, L_2 are both recognizable, then $L_1 \cup L_2$ is recognizable.   T   F

If L_1 is regular and $L_1 \cap L_2$ is regular, then L_2 is regular.   T   F

If L_1 is regular and $L_1 \cap L_2$ is non-regular, then L_2 is non-regular.   T   F

For all sets, S, T, if $g \colon S \to T$ and $f \colon T \to \text{Power}(S)$ are functions, then f, g cannot both be surjective.   T   F

2. Question 2 (5 points)

Let $S = \{a, b, c\}$, and let $f \colon S \to \mathrm{Power}(S)$ be any function with $f(a) = \{b, c\}$. Let $T = \{s \in S \mid s \notin f(s)\}$. Explain why it is impossible for $f(a)$ and T to be equal. (You have to argue this directly; you can't simply quote Cantor's Theorem.)

3. Question 3 (20 points)

Let
$$L = \left\{ s \in \{a, b\}^* \;\middle|\; s \text{ begins with } a \text{ and ends with } b \right\}$$
which begins
$$= \{ab, aab, abb, aaab, aabb, abab, abbb, aaaab, \ldots\}.$$

(1) (5 points) Build a DFA that recognizes L and briefly explain how your DFA works.

(2) (5 points) Build an NFA that recognizes L that has exactly 3 states, and briefly explain how your NFA works. [There is more than one possible answer.]

QUESTION 3, Continued

(3) (10 points) Use the Myhill-Nerode theorem to show that any DFA recognizing L has at least 4 states. (For full credit you must describe $\text{AccFut}_L(s)$ for four different values of s **and argue that these four sets are all different**.)