

THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 421/501: MIDTERM EXAMINATION – November 3, 2023

Last Name: _____

First Name: _____

Signature: _____

UBC Student #: _____

Important notes about this examination

1. You have 45 minutes to write this examination.
2. You may use a pencil to write your solutions, although a very light pencil might be harder to read after scanning.
3. No textbooks or electronic devices are permitted. We permit a “cheat-sheet” consisting of one page of handwritten or typed notes, on double-sided 8.5x11” paper.
4. Answer all the questions in the exam.
5. Good luck!

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - i. speaking or communicating with other examination candidates, unless otherwise authorized;
 - ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
 - iii. purposely viewing the written papers of other examination candidates;
 - iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).



0. IDENTIFICATION

Please make sure that the following is your 8-character Student ID:

Student ID:

Your answer to each problem should be written on its page; if needed, you can use the back side of the page as well.

1. QUESTION 1. (10 POINTS, 2 POINTS PER CORRECT T/F ANSWER — NO PENALTY FOR INCORRECT RESPONSES)

Circle either T for true, or F for false, for each of the statements below. In these questions, $\Sigma = \Sigma_{\text{ASCII}}$ denotes the ASCII alphabet, $L_1, L_2 \subset \Sigma^*$, $L_1 \setminus L_2$ denotes the set difference of L_1 and L_2 .

If L_1 and $\Sigma^* \setminus L_1$ are both recognizable, then L_1 is decidable.

This was explained in "Uncomputability ..." and in class. [This is how we proved that ACCEPTANCE is undecidable]

T F

If L_1, L_2 are both recognizable, then $L_1 \cup L_2$ is recognizable.

We can run algorithms for L_1 and for L_2 in parallel, as explained in class.

T F

If L_1 is regular and $L_1 \cap L_2$ is regular, then L_2 is regular.

Say $L_1 = \emptyset$ and L_2 is non-regular.

Then $L_1 \cap L_2 = \emptyset$ is regular, giving a counterexample

T F

If L_1 is regular and $L_1 \cap L_2$ is non-regular, then L_2 is non-regular.

If not, then L_2 is regular, so $L_1 \cap L_2$ is regular, which is impossible.

T F

For all sets, S, T , if $g: S \rightarrow T$ and $f: T \rightarrow \text{Power}(S)$ are functions, then f, g cannot both be surjective.

If f, g are surjective, then so

is $f \circ g: S \rightarrow \text{Power}(S)$, contradicting

Cantor's Theorem

T F

2. QUESTION 2 (5 POINTS)

Let $S = \{a, b, c\}$, and let $f: S \rightarrow \text{Power}(S)$ be any function with $f(a) = \{b, c\}$. Let $T = \{s \in S \mid s \notin f(s)\}$. Explain why it is impossible for $f(a)$ and T to be equal. (You have to argue this directly; you can't simply quote Cantor's Theorem.)

We have $a \notin \{b, c\} = f(a)$, hence

$a \in T$. But $a \notin f(a)$ and $a \in T$,

so $f(a) \neq T$.

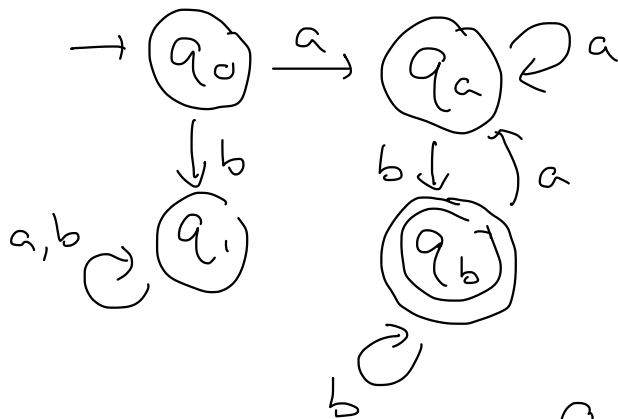
3. QUESTION 3 (20 POINTS)

Let

$$L = \{s \in \{a, b\}^* \mid s \text{ begins with } a \text{ and ends with } b\}$$

which begins

$$= \{ab, aab, abb, aaab, aabb, abab, abbb, aaaab, \dots\}.$$

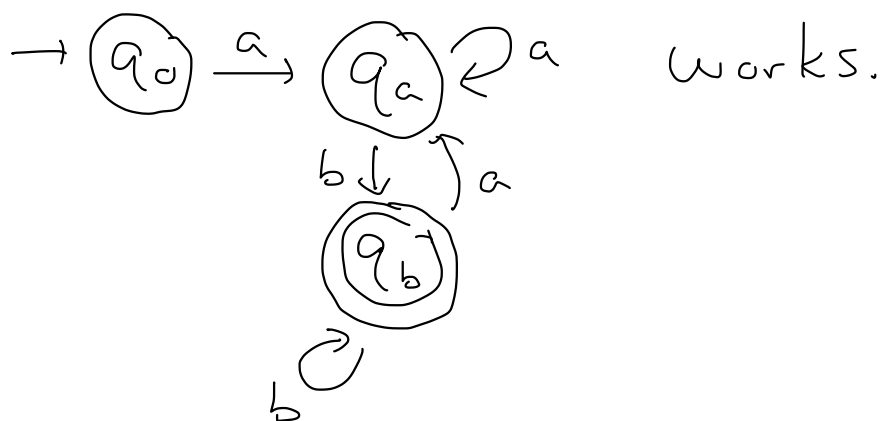
(1) (5 points) Build a DFA that recognizes L and briefly explain how your DFA works.

If first letter of s is b , we (transition to q_i which) rejects from that point on.

Otherwise we move to q_a , and use q_a and q_b to indicate the last symbol read. Hence q_b is accepting, and q_a is not.

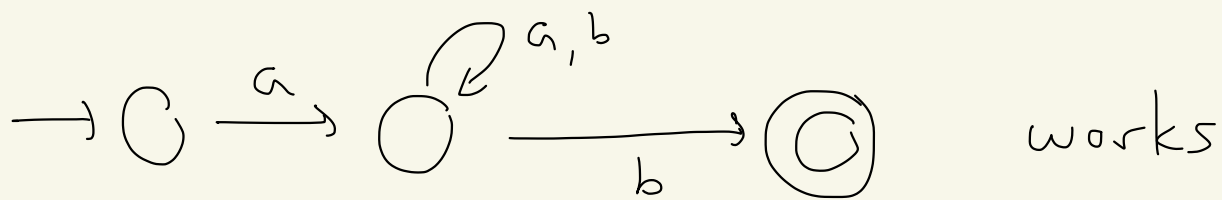
(2) (5 points) Build an NFA that recognizes L that has exactly 3 states, and briefly explain how your NFA works. [There is more than one possible answer.]

We can simply drop the arrow above, since without this arrow the NFA rejects. Hence



Alternative solution, starting from scratch:

We need to first read an a , then we can (non-deterministically) read any number of a 's and b 's, and then read a last b to accept. Hence



QUESTION 3, Continued

- (3) (10 points) Use the Myhill-Nerode theorem to show that any DFA recognizing L has at least 4 states. (For full credit you must describe $\text{AccFut}_L(s)$ for four different values of s and argue that these four sets are all different.)

(With the DFA of part (1) in mind, we)

$$\text{set } S_1 = \text{AccFut}_L(\varepsilon) = L$$

$$S_2 = \text{AccFut}_L(b) = \emptyset$$

$$S_3 = \text{AccFut}_L(a) = \Sigma^*b, \quad \Sigma = \{a, b\}$$

$$S_4 = \text{AccFut}_L(ab) = \varepsilon \cup \Sigma^*b$$

Proof that S_1, S_2, S_3, S_4 are all different:

$S_2 = \emptyset$, so it is different from S_1, S_3, S_4 .

The shortest strings in S_1, S_3, S_4 are, respectively, ab, b, ε , so S_1, S_3, S_4 are all different.

[Variant: S_4 is the only set containing ε ,
so $S_4 \neq S_1, S_3$. $b \in S_3$ and $b \notin S_1$ so $S_1 \neq S_3$.]