THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 421/501: MIDTERM EXAMINATION – November 3, 2023

Last Name: ______________________ First Name: _____________________

Signature: ______________________ UBC Student #:

Important notes about this examination

1. You have 45 minutes to write this examination.
2. You may use a pencil to write your solutions, although a very light pencil might be harder to read after scanning.
3. No textbooks or electronic devices are permitted. We permit a "cheat-sheet" consisting of one page of handwritten or typed notes, on double-sided 8.5x11" paper.
4. Answer all the questions in the exam.
5. Good luck!

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0. Identification

Please make sure that the following is your 8-character Student ID:

`Student ID:`

Your answer to each problem should be written on its page; if needed, you can use the back side of the page as well.

1. Question 1. (10 points, 2 points per correct T/F Answer — No Penalty for Incorrect Responses)

Circle either T for true, or F for false, for each of the statements below. In these questions, $\Sigma = \Sigma_{\mathrm{ASCII}}$ denotes the ASCII alphabet, $L_1, L_2 \subset \Sigma^*$, $L_1 \setminus L_2$ denotes the set difference of L_1 and L_2.

If L_1 and $\Sigma^* \setminus L_1$ are both recognizable, then L_1 is decidable. T F

This was explained in "Uncomputability ..." and in class. [This is how we proved that ACCEPTANCE is undecidable]

If L_1, L_2 are both recognizable, then $L_1 \cup L_2$ is recognizable. T F

We can run algorithms for L_1 and for L_2 in parallel, as explained in class.

If L_1 is regular and $L_1 \cap L_2$ is regular, then L_2 is regular. T F

Say $L_1 = \emptyset$ and L_2 is non-regular. Then $L_1 \cap L_2 = \emptyset$ is regular, giving a conterexample

If L_1 is regular and $L_1 \cap L_2$ is non-regular, then L_2 is non-regular. T F

If not, then L_2 is regular, so $L_1 \cap L_2$ is regular, which is impossible.

For all sets, S, T, if $g\colon S \to T$ and $f\colon T \to \mathrm{Power}(S)$ are functions, then f, g cannot both be surjective. T F

If f, g are surjective, then so is $fg: S \to \mathrm{Power}(S)$, contradicting Cantor's Theorem

2. Question 2 (5 points)

Let $S = \{a, b, c\}$, and let $f\colon S \to \text{Power}(S)$ be any function with $f(a) = \{b, c\}$. Let $T = \{s \in S \mid s \notin f(s)\}$. Explain why it is impossible for $f(a)$ and T to be equal. (You have to argue this directly; you can't simply quote Cantor's Theorem.)

We have $a \notin \{b,c\} = f(a)$, hence

$a \in T$. But $a \notin f(a)$ and $a \in T$,

so $f(a) \neq T$.

3. Question 3 (20 points)

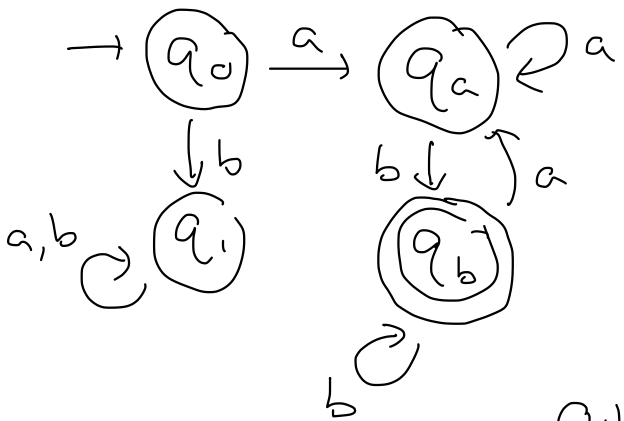
Let

$$L = \left\{ s \in \{a,b\}^* \;\middle|\; s \text{ begins with } a \text{ and ends with } b \right\}$$

which begins

$$= \{ab, aab, abb, aaab, aabb, abab, abbb, aaaab, \ldots\}.$$

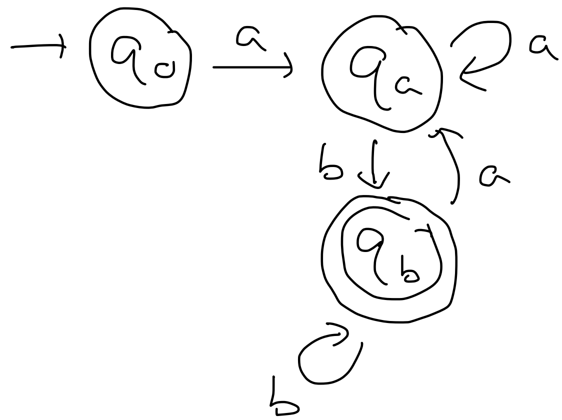
(1) (5 points) Build a DFA that recognizes L and briefly explain how your DFA works.



If first letter of s is b, we (transition to q_1 which) rejects from that point on. Otherwise we move to q_a, and use q_a and q_b to indicate the last symbol read. Hence q_b is accepting, and q_b is not.

(2) (5 points) Build an NFA that recognizes L that has exactly 3 states, and briefly explain how your NFA works. [There is more than one possible answer.]

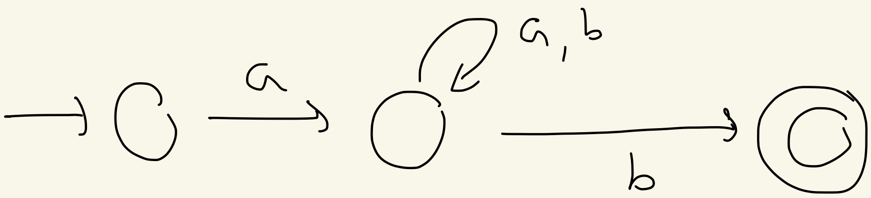
We can simply drop the arrow $q_0 \xrightarrow{b} q_1$ above, since without this arrow the NFA rejects. Hence



works.

Alternative solution, starting from scratch:

We need to first read an a, then we can (non-deterinistically) read any number of a's and b's, and then read a last b to accept. Hence



works

QUESTION 3, Continued

(3) (10 points) Use the Myhill-Nerode theorem to show that any DFA recognizing L has at least 4 states. (For full credit you must describe $\mathrm{AccFut}_L(s)$ for four different values of s **and argue that these four sets are all different**.)

(With the DFA of part (1) in mind, we)

set $S_1 = \mathrm{AccFut}_L(\varepsilon) = L$

$S_2 = \mathrm{AccFut}_L(b) = \emptyset$

$S_3 = \mathrm{AccFut}_L(a) = \Sigma^* b$, $\Sigma = \{a, b\}$

$S_4 = \mathrm{AccFut}_L(ab) = \varepsilon \cup \Sigma^* b$

Proof that S_1, S_2, S_3, S_4 are all different:

$S_2 = \emptyset$, so it is different from S_1, S_3, S_4.

The shortest strings in S_1, S_3, S_4 are, respectively, ab, b, ε, so S_1, S_3, S_4 are all different.

[Variant: S_4 is the only set containing ε, so $S_4 \neq S_1, S_3$. $b \in S_3$ and $b \notin S_1$ so $S_1 \neq S_3$.]