

THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 421/501: MIDTERM EXAMINATION – November 1, 2024

Last Name: _____

First Name: _____

Signature: _____

UBC Student #: _____

Important notes about this examination

1. You have 45 minutes to write this examination.
2. You may use a pencil to write your solutions, although a very light pencil might be harder to read after scanning.
3. No textbooks or electronic devices are permitted. We permit a “cheat-sheet” consisting of one page of handwritten or typed notes, on double-sided 8.5x11” paper.
4. Answer all the questions in the exam.
5. Good luck!

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - i. speaking or communicating with other examination candidates, unless otherwise authorized;
 - ii. purposely exposing written papers to the view of other examination candidates or imaging devices;
 - iii. purposely viewing the written papers of other examination candidates;
 - iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)—(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).



0. IDENTIFICATION

Please make sure that the following is your 8-character Student ID:

Student ID:

Your answer to each problem should be written on its page; if needed, you can use the back side of the page as well.

1. QUESTION 1. (10 POINTS, 2 POINTS PER CORRECT T/F ANSWER — NO PENALTY FOR INCORRECT RESPONSES)

Circle either T for true, or F for false, for each of the statements below. In these questions, $\Sigma = \Sigma_{\text{ASCII}}$ denotes the ASCII alphabet, $L_1, L_2 \subset \Sigma^*$, $L_1 \setminus L_2$ denotes the set difference of L_1 and L_2 .

Let $f: A \rightarrow B$ and $g: B \rightarrow C$. If f is not injective, then $gf: A \rightarrow C$ is not injective.

See Homework 2, Problem 2

T F

If L_1 and $\Sigma^* \setminus L_1$ are both recognizable, then L_1 is decidable.

By running recognizers for L_1 and $\Sigma^* \setminus L_1$ "in parallel"

T F

$\text{Power}(\Sigma^*)$ is uncountable.

Σ^* is in bijection with \mathbb{N} , $\text{Power}(\mathbb{N})$ is uncountable by Cantor's theorem.

T F

The set of functions $\Sigma^* \rightarrow \{1, 2\}$ is countably infinite.

These functions are in bijection with $\text{Power}(\Sigma^*)$ (via f maps to $\{s \in \Sigma^* \mid f(s) = 2\}$).

T F

Let L be the set of strings of digits that represent base 10 integers that are divisible by 7 (where we allow leading 0's but we don't allow the empty string, i.e., $\epsilon \notin L$). Then L can be recognized with a DFA with ~~at most 12~~ states.

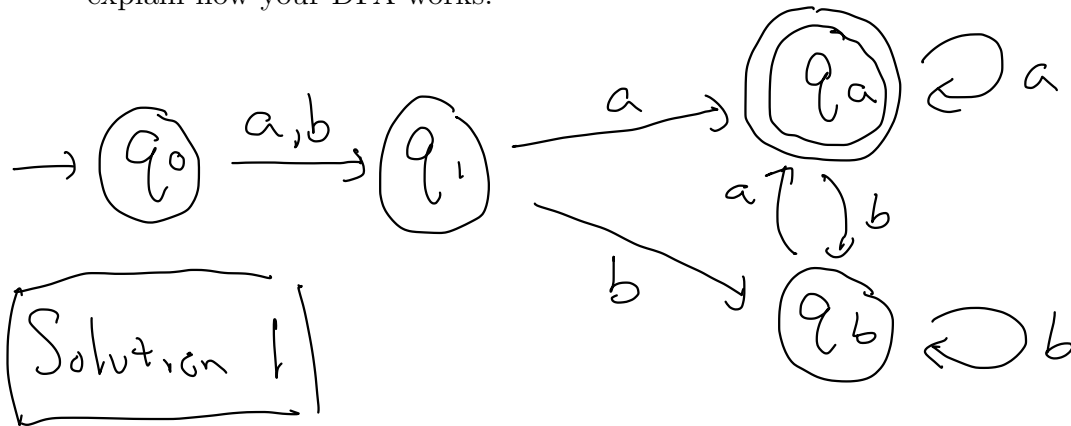
T F

Group Homework 5,
Problem 5

at most 12 (correction given during exam)

2. QUESTION 2 (5 POINTS)

Let $\Sigma = \{a, b\}$, and let $L \subset \Sigma^*$ be the set of strings ending in a whose length is at least two. Hence $L = \{aa, ba, aaa, aba, baa, bba, \dots\}$. Build a DFA that recognizes L and briefly explain how your DFA works.



q_1 is reached after one letter (i.e. symbol)

q_a, q_b are reached after at least 2 letters,

q_a means the string currently ends in a

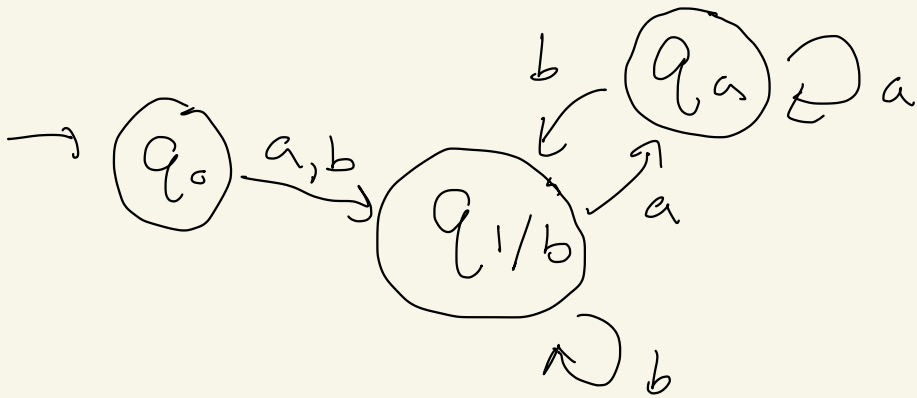
q_b " " " " " " " " b

The state q_a, q_b is determined by the last current symbol, and hence when in q_1, q_a, q_b , we transition to q_a on reading an "a", and to q_b on reading a b . We therefore accept a string on q_a , and reject the others.

Solution 2: We can merge q_1 and q_b , since they have the same accepting futures, i.e.

from q_1 or q_b we do not accept,
move to q_a on reading an "a", and
move to q_b on reading a "b."

Hence:



also works.

3. QUESTION 3 (8 POINTS)

Let $\Sigma = \{a, b\}$, and let $L \subset \Sigma^*$ be the set of strings ending in a whose length is at least two. Hence $L = \{aa, ba, aaa, aba, baa, bba, \dots\}$. (This is the same language as in Question 2.) Use the Myhill-Nerode theorem to prove that any DFA recognizing L must have at least ~~four~~ ^{three} states. You must therefore determine $\text{AccFut}_L(s)$ for four values of s , and explain why these sets are all different.

Note that $ss' \in L$ whenever $s' \in L$, since then ss' has at least 2 letters (symbols) and ends in "a." Similarly $ss' \notin L$ if s' has at least 2 symbols and ends in "b." Hence for any s ,

$$\text{AccFut}_L(s) = L \cup \left\{ \text{all } s' \text{ of length 0 or 1 such that } ss' \in L \right\}.$$

Checking the s' of length 0 or 1 above, we see that:

$$\text{AccFut}_L(\varepsilon) = L = \{aa, ba, \dots\}$$

$$\text{AccFut}_L(a) = \{s' \mid as' \in L\} = L \cup \{a\}$$

$$\text{AccFut}_L(aa) = \{s' \mid aas' \in L\} = L \cup \{\varepsilon, a\}$$

Since the strings of length 0 or 1 in the above values of AccFut_L are,

respectively, \emptyset , $\{a\}$, $\{\varepsilon, a\}$,
which are all different, these
are 3 distinct sets.

4. QUESTION 4 (5 POINTS)

What is the eventual period of $L = \{a^4, a^5\}$? **Explain.** (You get no points for simply giving a number; you must explain.)

Since $a^n \notin L$ for $n \geq 6$, we have

$$n \geq 6 \Rightarrow (a^n \in L \text{ iff } a^{n+1} \in L),$$

and hence L is eventually 1-periodic.

(Since the eventual period of L is the smallest $p \in \mathbb{N}$ s.t. L is eventually p -periodic,) the eventual period of L is 1.

5. QUESTION 5 (5 POINTS)

Let L_1, L_2 be Duck-recognizable. Is $L_1 \cup L_2$ Duck-recognizable? **Explain why or why not.** (You get no points for simply answering "yes" or "no"; you must explain.)

L is Duck-recognizable iff L is a finite union of sets of the form \sum^k with $k \geq 0$.

Since a union of finite unions (of such sets) is again a finite union (of such sets) if L_1, L_2 are Duck-recognizable, then so is $L_1 \cup L_2$.