THE UNIVERSITY OF BRITISH COLUMBIA
CPSC 421/501: MIDTERM EXAMINATION – November 1, 2024

Last Name: ______________________ First Name: ______________________

Signature: ______________________ UBC Student #: |_|_|_|_|_|_|_|_|

Important notes about this examination

1. You have 45 minutes to write this examination.
2. You may use a pencil to write your solutions, although a very light pencil might be harder to read after scanning.
3. No textbooks or electronic devices are permitted. We permit a "cheat-sheet" consisting of one page of handwritten or typed notes, on double-sided 8.5x11" paper.
4. Answer all the questions in the exam.
5. Good luck!

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0. Identification

Please make sure that the following is your 8-character Student ID:

`Student ID:`

Your answer to each problem should be written on its page; if needed, you can use the back side of the page as well.

1. Question 1. (10 points, 2 points per correct T/F Answer — No Penalty for Incorrect Responses)

Circle either T for true, or F for false, for each of the statements below. In these questions, $\Sigma = \Sigma_{\text{ASCII}}$ denotes the ASCII alphabet, $L_1, L_2 \subset \Sigma^*$, $L_1 \setminus L_2$ denotes the set difference of L_1 and L_2.

Let $f \colon A \to B$ and $g \colon B \to C$. If f is not injective, then $gf \colon A \to C$ is not injective. T F

See Homework 2, Problem 2

If L_1 and $\Sigma^* \setminus L_1$ are both recognizable, then L_1 is decidable. T F

By running recognizers for L_1 and $\Sigma^* \setminus L_1$ "in parallel"

Power(Σ^*) is uncountable. T F

Σ^* is in bijection with $\mathbb{N}$, Power($\mathbb{N}$) is uncountable by Cantor's theorem.

The set of functions $\Sigma^* \to \{1, 2\}$ is countably infinite. T F

These functions are in bijection with Power(Σ^*) (via f maps to $\{s \in \Sigma^* \mid f(s) = 2\}$).

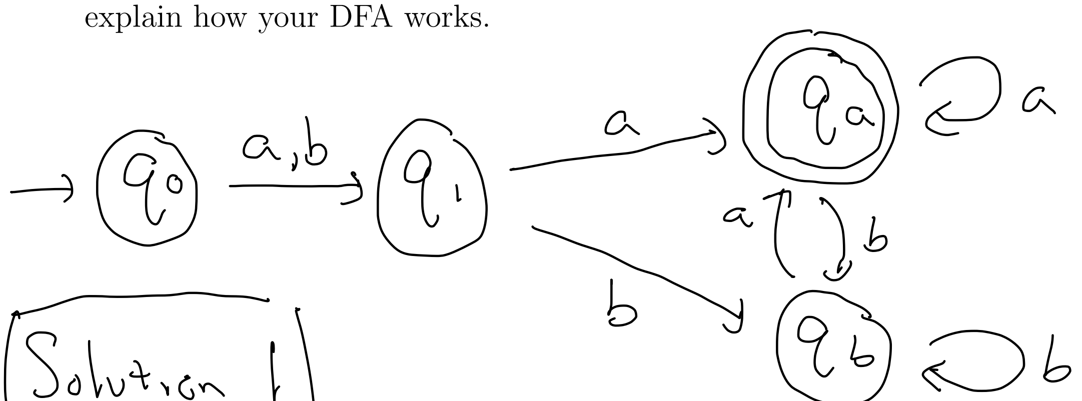
Let L be the set of strings of digits that represent base 10 integers that are divisible by 7 (where we allow leading 0's but we don't allow the empty string, i.e., $\epsilon \notin L$). Then L can be recognized with a DFA with at ~~most than 12~~ states. T F

at most 12 (correction given during exam)

Group Homework 5, Problem 5

2. Question 2 (5 points)

Let $\Sigma = \{a, b\}$, and let $L \subset \Sigma^*$ be the set of strings ending in a whose length is at least two. Hence $L = \{aa, ba, aaa, aba, baa, bba, \ldots\}$. Build a DFA that recognizes L and briefly explain how your DFA works.



Solution 1

q_1 is reached after one letter (i.e. symbol)

q_a, q_b are reached after at least 2 letters,

q_a means the string currently ends in a

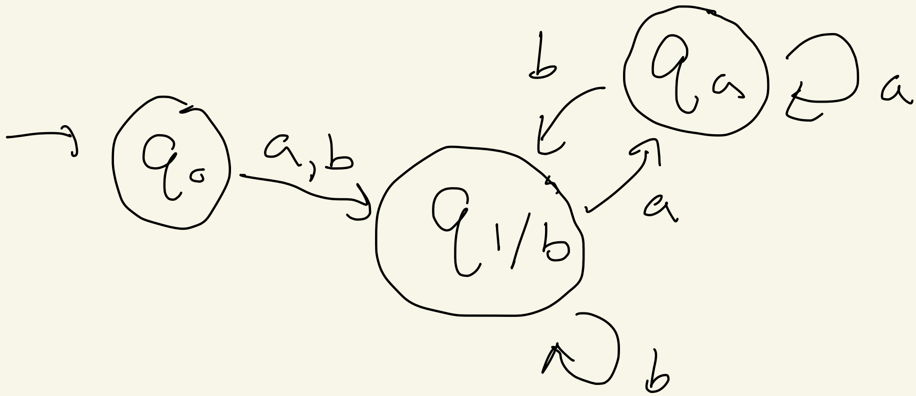
q_b " " " " " " b

The state q_a, q_b is determined by the last current symbol, and hence when in q_1, q_a, q_b, we transition to q_a on reading an "a," and to q_b on reading a b. We therefore accept a string on q_a, and reject the others.

Solution 2: We can merge q_1 and q_b, since they have the same accepting futures, i.e.

from q_1 or q_b we: do not accept, move to q_a on reading an "a", and move to q_b on reading a "b".

Hence:



also works.

3. Question 3 (8 points)

Let $\Sigma = \{a, b\}$, and let $L \subset \Sigma^*$ be the set of strings ending in a whose length is at least two. Hence $L = \{aa, ba, aaa, aba, baa, bba, \ldots\}$. (This is the same language as in Question 2.) Use the Myhill-Nerode theorem to prove that any DFA recognizing L must have at least ~~four~~ three states. You must therefore determine $\mathrm{AccFut}_L(s)$ for four values of s, and explain why these sets are all different.

(correction given during exam)

Note that $ss' \in L$ whenever $s' \in L$, since then ss' has at least 2 letters (symbols) and ends in "a." Similarly $ss' \notin L$ if s' has at least 2 symbols and ends in "b." Hence for <u>any s</u>,

$$\mathrm{AccFut}_L(s) = L \cup \{\text{all } s' \text{ of length } 0 \text{ or } 1 \text{ such that } ss' \in L\}.$$

Checking the s' of length 0 or 1 above, we see that:

$$\mathrm{AccFut}_L(\varepsilon) = L = \{aa, ba, \ldots\}$$

$$\mathrm{AccFut}_L(a) = \{s' \mid as' \in L\} = L \cup \{a\}$$

$$\mathrm{AccFut}_L(aa) = \{s' \mid aas' \in L\} = L \cup \{\varepsilon, a\}$$

Since the strings of length 0 or 1 in the above values of AccFut_L are, respectively, $\emptyset$, $\{a\}$, $\{\varepsilon, a\}$,

which are all different, these

are 3 distinct sets.

4. Question 4 (5 points)

What is the eventual period of $L = \{a^4, a^5\}$? **Explain.** (You get no points for simply giving a number; you must explain.)

Since $a^n \notin L$ for $n \geq 6$, we have

$$n \geq 6 \Rightarrow (a^n \in L \text{ iff } a^{n+1} \in L),$$

and hence L is eventually 1-periodic.
(Since the eventual period of L is the smallest $p \in \mathbb{N}$ s.t. L is eventually p-periodic,)
the eventual period of L is 1.

5. Question 5 (5 points)

Let L_1, L_2 be Duck-recognizable. Is $L_1 \cup L_2$ Duck-recognizable? **Explain why or why not.** (You get no points for simply answering "yes" or "no"; you must explain.)

L is Duck-recognizable iff L is a finite union of sets of the form Σ^k with $k \geq 0$.

Since a union of finite unions (of such sets) is again a finite union (of such sets) if L_1, L_2 are Duck-recognizable, then so is $L_1 \cup L_2$.