SUPPLEMENTAL MIDTERM PRACTICE, CPSC 421/501, FALL 2021

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In all the exercises below, for any $k \in \mathbb{N}$, let C_k be, as usual,

$$C_k = \bigl\{ w \in \Sigma^* \mid \textbf{the } k\textbf{-th last symbol of } w \textbf{ is } a \bigr\},$$

where $\Sigma = \{a, b\}$.

(1) (a) Find a word, w, of length 4 for which

$$\mathrm{AccFut}_{C_4}(aaa) = \mathrm{AccFut}_{C_4}(w)$$

and briefly justify your answer.

(b) Briefly explain why for any $\sigma_1, \ldots, \sigma_6 \in \Sigma$ we have

$$\mathrm{AccFut}_{C_4}(\sigma_1 a \sigma_2 \sigma_3) \neq \mathrm{AccFut}_{C_4}(\sigma_4 b \sigma_5 \sigma_6).$$

(c) Does

$$\mathrm{AccFut}_{C_4}(\sigma_1 a \sigma_2 \sigma_3) \neq \mathrm{AccFut}_{C_4}(b \sigma_4 \sigma_5 \sigma_6)$$

for all $\sigma_1, \ldots, \sigma_6 \in \Sigma$? Briefly explain.

(d) Does

$$\mathrm{AccFut}_{C_5}(b \sigma_1 a \sigma_2 \sigma_3) \neq \mathrm{AccFut}_{C_4}(b \sigma_4 b \sigma_5 \sigma_6)$$

for all $\sigma_1, \ldots, \sigma_6 \in \Sigma$? Briefly explain.

(2) (a) Find a word, w, of length three over Σ such that

$$\mathrm{AccFut}_{C_5}(wab) = \mathrm{AccFut}_{C_5}(ab)$$

and briefly justify your answer.

(b) Find a word, w, of length two over Σ such that

$$\mathrm{AccFut}_{C_2}(w) = \mathrm{AccFut}_{C_2}(a).$$

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(3) What is the minimum number of states of a DFA needed to recoginze the language $L = \{a^3, a^7\}$ over the alphabet $\Sigma = \{a\}$? Brielfy explain. Would your answer change over the alphabet $\Sigma = \{a, b\}$? Brielfy explain.

(4) Let $L = \{a^{4n+2} \mid n \in \mathbb{N}\}$. What is the minimum number of states in a DFA needed to recognize L? **Explain this as briefly as possible.** Give such a DFA.

(5) Let $L \subset \{a\}^*$ be an infinite, regular language over the alphabet $\Sigma = \{a\}$, such that $a^{20}, a^{50} \in L$, but $a^{51}, a^{52} \notin L$. Determine the minimum number of states that a DFA recognizing any such L must have. You may use any formula given on the homework (but make sure that it really applies).

(6) Let $L \subset \{a\}^*$ be an infinite, regular language over the alphabet $\Sigma = \{a\}$, such that $a^{145}, a^{150} \in L$, but $a^{151}, a^{152}, \ldots, a^{160} \notin L$. Determine the minimum number of states that a DFA recognizing any such L must have. You may use any formula given on the homework (but make sure that it really applies).

(7) John feeds those who don't feed themselves. Does John feed themself? Explain.

(8) In a set of five humans, Batiste loves everyone. Let S consist of each of the humans who does not love themself. Can S equal the set of humans whom Batiste loves? Explain.

(9) Say that each of 50 profs reside in one of 13 bird sanctuaries. How many bird sanctuaries must be a residence for at least 4 profs?

(10) MORE PROBLEMS MAY BE ADDED LATER.

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