GROUP HOMEWORK 5, CPSC 421/501, FALL 2024

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Disclaimer: The material may sketchy and/or contain errors, which I will elaborate upon and/or correct in class. For those not in CPSC 421/501: use this material at your own risk...

Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

(1) Who are your group members? Please print if writing by hand.

(2) Exercise 9.2.43 on the handout "Uncomputability in CPSC421/501."

(3) Exercise 9.3.1 on the handout "Uncomputability in CPSC421/501."

(4) Exercise 9.3.2, part (b) on the handout "Uncomputability in CPSC421/501."

(5) Let L be the set of strings of digits, i.e., strings over the alphabet $\Sigma_{\text{digits}} = \{0, 1, \ldots, 9\}$, that represent integers in base 10 that are divisible by 7, where we allow leading 0's but we don't consider the empty string, ϵ, to be part of L. Hence

$$L = \{0, 7, 00, 07, 14, 21, 28, \ldots, 91, 98, 000, 007, 014, \ldots, 098, 105, 112, \ldots\}.$$

Recall that if $n \in \mathbb{Z}$, the expression $n \bmod 7$ refers to the unique integer, $a \in \{0, 1, \ldots, 6\}$ such that $n = 7p + a$ for some $p \in \mathbb{Z}$, and that if n, n' are integers, then $n - n'$ is divisible by 7 iff $n \bmod 7 = n' \bmod 7$.

(a) Show that for any integers $m, n \in \mathbb{Z}$, we have

$$(10m + n) \bmod 7 = \Big(3(m \bmod 7) + (n \bmod 7)\Big) \bmod 7.$$

(b) Use the previous part to design an 8-state DFA, M, that recognizes L. [Hint: it is easiest to name states in some convenient way so that you can describe the values of $\delta(q, \sigma)$ by a simple formula. You probably don't want to draw a graph that would need to depict $8 \cdot 10 = 80$ transition arrows...]

(c) Say that $L' = L \cup \{\epsilon\}$, so that L' is the language of strings representing integers divisible by 7, where we allow leading 0's and we do allow consider the empty string ϵ, to be part of L'.

  (i) Describe a simple modification of M that yields another 8-state DFA, M, that recognizes L'.

  (ii) Can you describe a 7-state DFA that recognizes L' ?

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