GROUP HOMEWORK 6, CPSC 421/501, FALL 2024

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Disclaimer: The material may sketchy and/or contain errors, which I will elaborate upon and/or correct in class. For those not in CPSC 421/501: use this material at your own risk...

Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

(1) Who are your group members? Please print if writing by hand.

(2) Let $\Sigma = \{a, b\}$.
  (a) Let L be the language of words over $\Sigma = \{a, b\}$ that end in an a; hence each string in L has length at least one, and the first few strings of L (in order of length, breaking ties with lexicographical order) are

$$L = \{a, aa, ba, aaa, aba, \ldots\}.$$

  Construct a two-state DFA that recognizes L, and explain why your DFA recognizes L. [Hint: Your DFA will have to "remember" the last symbol in the string that has been read up to that point. Hence the DFA needs at least two states. Are two states sufficient?]
  (b) Let L be the language of words over $\Sigma = \{a, b\}$ whose second to last letter is a; hence each string in L has length at least two, and the first few strings of L (in order of length, breaking ties with lexicographical order) are

$$L = \{aa, ab, aaa, aab, baa, bab, aaaa, \ldots\}.$$

  Construct a four-state DFA that recognizes L. [Hint: Your DFA will have to "remember" the last two symbols in the string that has been read up to that point. Hence the DFA needs at least four states. Are four states sufficient?]

(3) Exercise 6.1.2 of the handout "Non-regular languages..."

(4) Exercise 6.1.3 of the handout "Non-regular languages..."

(5) What is the fewest number of states in a DFA recognizing $L = \{a^5, a^7\}^*$? Answer this question in the following parts:
   (a) Show that $a^n \in L$ for $n = 24, 25, 26, 27, 28$.
   (b) Show that $a^n \in L$ for all $n \geq 24$.
   (c) Show that $a^{23} \notin L$. [Hint: say that $5x + 7y = 23$ for non-negative integers x, y. What is $5x$ modulo 7?]
   (d) What is the eventual period of L (i.e., the smallest p such that L is eventually p-periodic)?
   (e) What is the smallest n_0 such that for p in part (d) we have for all $n \geq n_0$, $x^n \in L$ iff $x^{n+p} \in L$?
   (f) What can you conclude about the smallest number of states in a DFA recognizing L? [You can use the result in Exercise 6.1.2 of the handout "Non-regular languages..."]

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