GROUP HOMEWORK 7, CPSC 421/501, FALL 2024

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

(1) Who are your group members? Please print if writing by hand.

(2) For each infinite increasing sequence $n_1 < n_2 < n_3 < \cdots$ of non-negative numbers, let

$$L = L_{n_1,n_2,\ldots} = \{a^{n_1}, a^{n_2}, \ldots\} \subset \{a\}^*.$$

Show that if L is regular, then for some $p \in \mathbb{N}$ we have for all $i \in \mathbb{N}$, $n_{i+1} \le n_i + p$.

(3) (a) Show that there are uncountably many different sequences $n_1 < n_2 < n_3 < \cdots$ such that for all $i \in \mathbb{N}$, $n_{i+1} \le n_i + 2$. [Hint: We know that Power($\mathbb{N}$) is uncountable, so it suffices to give an injection from Power($\mathbb{N}$) to the set of such sequences. In class we discussed one way of doing this, which involved setting $n_1 = 3$, $n_3 = 6$, $n_5 = 9$, etc. There are other ways to produce such an injection.]

(b) Show that there are is a sequence of non-negative integers $n_1 < n_2 < n_3 < \cdots$ such that for all $i \in \mathbb{N}$, $n_{i+1} \le n_i + 2$, and that

$$L = L_{n_1,n_2,\ldots} = \{a^{n_1}, a^{n_2}, \ldots\}$$

is not regular.

(4) Find the eventual period of

$$L = \{a^{3n} \mid n \in \mathbb{N}\} \cup \{a^{4n} \mid n \in \mathbb{N}\}.$$

This means that you must find a $p \in \mathbb{N}$ such that L is p-eventually periodic, and you must prove that if $1 \le p' < p$, then L is not p'-eventually periodic. (You can use results from the last homework.)

(5) Let $L = \{a^3\}$ (so L contains only one string). How many different values are there for $\mathrm{AccFut}_L(s)$ as s varies over all strings over $\{a\}$? Draw the minimum state DFA that recognizes L, and show how each state corresponds to one of these values of $\mathrm{AccFut}_L(s)$.

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