GROUP HOMEWORK 8, CPSC 421/501, FALL 2024

JOEL FRIEDMAN

Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

(1) Who are your group members? Please print if writing by hand.

(2) The point of this exercise is to write a "code snippet" for a Turing machine, as described in class; we will call this Turing machine a *counter*. It is a useful subroutine for a larger Turing machine project. Here is a formal description.

Let $\Sigma = \{0, 1\}$. If $n \in \{0, 1, 2, 3, \ldots\}$ is a non-negative integer, let $\langle n \rangle \in \Sigma^*$ denote the usual base 2 (binary) representation of i without leading 0's; e.g.,

$$\langle 0 \rangle = 0,\ \langle 1 \rangle = 1,\ \langle 6 \rangle = 110,\ \langle 15 \rangle = 1111.$$

Let $\mathcal{I} \subset \Sigma^*$ be the subset

$$\mathcal{I} = \bigl\{ \langle i \rangle \mid i = 0, 1, 2, \ldots \bigr\} = \{0, 1, 10, 11, 100, 101, 110, 111, \ldots\}.$$

Give a formal description of a 1-tape Turing machine, M, with input alphabet $\Sigma = \{0, 1\}$, such that on input $i \in \Sigma^*$ we have

(a) if $i \in \mathcal{I}$, i.e., if $i = \langle n \rangle$ for some non-negative integer n, M accepts i, and when M halts it leaves the tape head over cell 1 and leaves the string $\langle n+1 \rangle$ on the tape (meaning this starts at cell 1, and every tape cell to the right of $\langle n+1 \rangle$ needs to have the blank symbol on it).
(b) if $i \notin \mathcal{I}$, we don't care what happens when M is run on input i.

[Recall from class: most likely you will want to "place a mark" over the first tape cell, i.e., a special tape symbol like $0'$ for 0 and $1'$ for 1, on your first step: our definition of Turing machines doesn't give you a way of figuring out when you are at cell 1, the leftmost tape cell.]

(3) Explain **in 20 to 60 words** how the "counter" Turing machine in Question (2) can be used in a multi-tape Turing machine algorithm to recognize $\mathrm{ACCEPTANCE}_{\mathrm{Duck}}$, the acceptance problem for Duck, as was described in class.

Department of Computer Science, University of British Columbia, Vancouver, BC V6T 1Z4, CANADA.
Email address: `jf@cs.ubc.ca`
URL: `http://www.cs.ubc.ca/~jf`