

GROUP HOMEWORK 9, CPSC 421/501, FALL 2024

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Please note:

- (1) You must justify all answers; no credit is given for a correct answer without justification.
- (2) Proofs should be written out formally.
- (3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
- (4) You may work together on homework in groups of up to four, but **you must submit a single homework as a group submission under Gradescope.**

- (1) Who are your group members? Please print if writing by hand.
- (2) Let $f: \{F, T\}^3 \rightarrow \{F, T\}$ be the Boolean function such that $f(\mathbf{x}) = T$ if $\mathbf{x} = (T, F, T), (F, F, F)$, and otherwise $f(\mathbf{x}) = F$.
 - (a) Write a Boolean formula, ϕ , representing f , such that ϕ is a DNF formula, specifically the disjunction (i.e., OR, i.e., \vee) of two clauses, each clause being the conjunction (i.e., AND, i.e., \wedge) of three literals, in the same manner as done in class on November 18.
 - (b) Write a Boolean formula representing $\neg f$ (the negation of f), based on part (a), that is a 3CNF with two clauses.
- (3) Show that there is a Boolean function $\{F, T\}^4 \rightarrow \{F, T\}$ that cannot be expressed as a 3CNF Boolean formula. [Hint: show that a disjunction of three literals is either never false (e.g., $x_1 \vee x_1 \vee \neg x_1$) or false on at least $1/8$ of the $16 = 2^4$ possible assignments of x_1, x_2, x_3, x_4 to $\{F, T\}$.]
- (4) Let $n \geq 4$, and let $a_1, \dots, a_n \in \{F, T\}$. Show that

$$a_1 \vee a_2 \vee \dots \vee a_n = T$$

iff the formula

$$\phi(z_1, \dots, z_{n-3}) = (a_1 \vee a_2 \vee z_1) \wedge (\neg z_1 \vee a_3 \vee z_2) \wedge \dots \wedge (\neg z_{n-4} \vee a_{n-2} \vee z_{n-3}) \wedge (\neg z_{n-3} \vee a_{n-1} \vee a_n)$$

is satisfiable.

- (5) Reduce 3SAT to 3COLOUR: that is, given a 3CNF Boolean formula ϕ in variables x_1, \dots, x_n , produce a graph, G such that $\phi \in \text{SAT}$ (and therefore $\phi \in \text{3SAT}$ iff $G \in \text{3COLOUR}$, with the additional property that the number of edges and vertices in the graph, G is no more than a constant times the number of clauses in ϕ . Do this by following the hints in Problem 7.29 of [Sip].

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