GROUP HOMEWORK 9, CPSC 421/501, FALL 2024

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Please note:

(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

(1) Who are your group members? Please print if writing by hand.

(2) Let $f\colon \{F,T\}^3 \to \{F,T\}$ be the Boolean function such that $f(\mathbf{x}) = T$ if $\mathbf{x} = (T,F,T),(F,F,F)$, and otherwise $f(\mathbf{x}) = F$.
   (a) Write a Boolean formula, ϕ, representing f, such that ϕ is a DNF formula, specifically the disjuction (i.e., OR, i.e., $\vee$) of two clauses, each clause being the conjunction (i.e., AND, i.e., $\wedge$) of three literals, in the same manner as done in class on November 18.
   (b) Write a Boolean formula representing $\neg f$ (the negation of f), based on part (a), that is a 3CNF with two clauses.

(3) Show that there is a Boolean function $\{F,T\}^4 \to \{F,T\}$ that cannot be expressed as a 3CNF Boolean formula. [Hint: show that a disjunction of three literals is either never false (e.g., $x_1 \vee x_1 \vee \neg x_1$) or false on at least 1/8 of the $16 = 2^4$ possible assignments of x_1, x_2, x_3, x_4 to $\{F,T\}$.]

(4) Let $n \geq 4$, and let $a_1, \ldots, a_n \in \{F,T\}$. Show that

$$a_1 \vee a_2 \vee \ldots \vee a_n = T$$

iff the formula

$$\phi(z_1,\ldots,z_{n-3}) = \bigl(a_1\vee a_2\vee z_1\bigr)\wedge\bigl(\neg z_1\vee a_3\vee z_2\bigr)\wedge\ldots\wedge\bigl(\neg z_{n-4}\vee a_{n-2}\vee z_{n-3}\bigr)\wedge\bigl(\neg z_{n-3}\vee a_{n-1}\vee a_n\bigr)$$

is satisfiable.

(5) Reduce 3SAT to 3COLOUR: that is, given a 3CNF Boolean formula ϕ in variables $x_1, \ldots, x_n$, produce a graph, G such that $\phi \in$ SAT (and therefore $\phi \in$ 3SAT iff $G \in$ 3COLOUR, with the additional property that the number of edges and vertices in the graph, G is no more than a constant times the number of clauses in ϕ. Do this by following the hints in Problem 7.29 of [Sip].

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