GROUP HOMEWORK 10.5, CPSC 421/501, FALL 2024

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Disclaimer: The material may sketchy and/or contain errors, which I will elaborate upon and/or correct in class. For those not in CPSC 421/501: use this material at your own risk. . .

Please note:
(1) You must justify all answers; no credit is given for a correct answer without justification.
(2) Proofs should be written out formally.
(3) You do not have to use LaTeX for homework, but **homework that is too difficult to read will not be graded.**
(4) You may work together on homework in groups of up to four, **but you must submit a single homework as a group submission under Gradescope.**

(1) Who are your group members? Please print if writing by hand.

(2) A 4-colouring of a graph $G = (V, E)$ is a maping of the vertices $V \to \{red, blue, green, black\}$ such that the endpoints of each edge has different colours. Show that 4COLOUR in NP-complete. You have to show that (1) 4COLOUR is in NP, and that (2) any language in NP can be reduced to 4COLOUR in polynomial time. [Hint: to prove (2), since any language in NP can be reduced to 3SAT in polynomial time, and 3SAT can be reduced to 3COLOUR in polynomial time, it suffices to show that 3COLOUR can be reduced to 4COLOUR in polynomial time.

(3) Let DOUBLE-3-SAT be the language of 3CNF formulas that have at least two satisfying assignments. Show that DOUBLE-3-SAT is NP-complete.

(4) Recall that 2COLOUR is the set of graphs that are 2-colourable.
   (a) Describe a graph, G_1, in 2COLOUR and another graph G_2, that is not in 2COLOUR.
   (b) Describe a PSPACE reduction from 3SAT to 2COLOUR.

(c) Is there a polynomial time reduction from 3SAT to 2COLOUR, that is, do we know if one exists as of November 2024?

(5) Describe a PSPACE algorithm for 3SAT.

(6) If L is in NPSPACE, is the complement of L necessarily in NSPACE? [Remark: as of November 2024, it is not known if the complement of a language in NP is also in NP: indeed, co-NP is defined as the class of languages whose complement lies in NP, and whether or not NP equals co-NP is wide open. If P equals NP, then co-NP also equals P, since the complement of any language in P is again in P.]

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