421/501 Homework 1 Solutions, 2024

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(1) 9.2.2. (a) Since $T = \{s \mid s \notin f(s)\}$ and $1 \in f(1)$, $1 \notin T$. Since $1 \in \{1,2\}$, $T \neq \{1,2\}$.

(b) No! (there are many possible examples to show that both $2 \in T$ and $2 \notin T$ are possible)

if $f(2) = \emptyset$, then $2 \notin f(2)$ so $2 \in T$;

if $f(2) = S$, then $2 \in f(2)$ so $2 \notin T$;

hence both $2 \in T$ and $2 \notin T$ are possible

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(2) 9.2.3

(a) $a \in S$ but $a \notin f(a)$; hence

$S \neq f(a)$

(b) Similarly $b \notin f(b)$ but $b \in S$

so $S \neq f(b)$, and $c \notin f(c)$ but $c \in S$

so $S \neq f(c)$.

(c) Since $s \notin f(s)$ for $s = a, b, c$,

$T = \{ s \in S \mid s \notin f(s) \} = \{a, b, c\}$.

(3) 9.2.4 (this is similar to 9.2.3)

(a) $b \in f(b)$ but $b \notin \{a, c\}$.

Hence $f(b) \neq \{a, c\}$.

(b) $a \notin f(a)$ but $a \in \{a, c\}$. And

$c \notin f(c)$ but $c \in \{a, c\}$.

Hence $f(a), f(c) \neq \{a, c\}$.

(c) Since:

$s \notin f(s)$ for $s = a, c$, $\quad a, c \in T$

$s \in f(s)$ for $s = b$ $\quad b \notin T$

hence $T = \{a, c\}$.

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(4) 9.2.5

[There are many possible examples here...]

Let $f(a) = f(b) = f(c) = S = \{a, b, c\}$.

Then

$$\{s \mid s \in f(s)\} = \{a, b, c\} = S$$

which is in the image of f

(since $f(a) = S$).
