# CPSC 421/501, Homework 3, Part (2) solutions

(4) 9.2.26 (e, g, h, m)

(e) If $L_1, L_2$ are decidable, then to determine if $s \in \Sigma^*_{\mathrm{ASCII}}$ is in $L_1 \cup L_2$ we can run an $L_1$ decider on $s$ (which always terminates and returns either "yes" or "no"), and then similarly run an $L_2$ decider. We have $s \in L_1 \cup L_2$ iff at least one decider returns "yes";

hence we return "yes" ($s \in L_1 \cup L_2$) if either decider returns "yes," and otherwise we return "no" ($s \notin L_1 \cup L_2$). This combined algorithm decides $L_1 \cup L_2$

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(g) Yes! Similarly to (e), we answer the question "does $s \in L_1 \cup L_2$?" by running both $L_1$ and $L_2$ recognizers in parallel on $s$ (e.g. run $L_1$ and $L_2$ each for one step, then each for 2 steps, etc.) and accept $s$ (as lying in $L_1 \cup L_2$) if either $L_1$ or $L_2$ returns "yes".

[Note! We cannot run an $L_1$ recognizer on $s$ and then run an $L_2$ recognizer, since the $L_1$ recognizer may not terminate.]

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(h) $L_1$ = NON-ACCEPTANCE and $L_2$ = NON-REJECTION are both unrecognizable, but

$(L_1 \cup L_2)^{comp}$ = NOT-PYTH-INP

is decidable; hence so is $L_1 \cup L_2$

So (h) does not generally hold

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(m) $L_1 = \Sigma^*_{ASCII}$ and $L_2$ = ACCEPTANCE are both recognizable, but

$$L_1 \setminus L_2 = \Sigma_{ASCII}^{*} \setminus ACCEPTANCE$$

$$= ACCEPTANCE^{Comp}$$

is unrecognizable.

(5) 9.2.27, parts (a, d, f)

(a) We first detect if $p$ is a valid Python program. If so, we can simulate $p$ on $i$ for 10 steps in a finite amount of time; we can then if $p$ has reached return("yes") ~~be~~ by then or not. If not, then we return("no"), thereby deciding the language $\{p \sigma_o i \mid p$ accepts $i$ after running for 10 steps$\}$.

(d) This is just the language NON-REJECTION. One can prove this ~~is~~ unrecognizable by either:

(i) Showing that NON-REJECTION is unrecognizable by assuming it isn't and getting a recognizer for

$$T = \{ q \mid q \notin \mathrm{LangRecBy}(q) \}$$

adapting the proof in class that NON-ACCEPTANCE is unrecognizable, getting the table:

| Where is $q\sigma_0 q$ ? | ALG. 1 Feed $q\sigma_0 q$ Universal Py program | ALG 2 Feed $q\sigma_0 q$ into NON-REJ recognizer | Is $q \in T$ ? |
|---|---|---|---|
| NOT-PYTH-INP | | defer to Alg 1 | $q \in T$ |
| ACCEPTANCE | | | $q \notin T$ |
| REJECTION | | defer to Alg 1 | $q \in T$ |
| LOOPING | | | $q \in T$ |

Where = may not terminate

(but Alg 1 can't be fooled, and Alg 2 can always defer to Alg 1 if Alg 2 terminates without returning "yes"

(ii) You can assume NON-REJECTING is recognizable, and prove (contrary to what we know) that NON-ACCEPTANCE is recognizable: namely, given any string of the form $p\sigma_0 i$ with $p$ a valid Python program, we can form $\tilde{p}$ that returns $\begin{cases} \text{"yes" if } p \text{ returns "no"} \\ \text{"no" if } p \text{ returns "yes"} \end{cases}$

Then

$p\sigma_0 i \in$ NON-ACCEPTANCE iff

$\tilde{p}\sigma_0 i \in$ NON-REJECTION

Hence being able to recognize NON-REJECTION implies the same for NON-ACCEPTANCE, which is impossible.

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(f) Since $\Sigma^*_{ASCII}$ is countable, we can list all possible inputs to $p$ as a list

$$\{ i_1, i_2, i_3, \ldots \} = \Sigma^*_{ASCII}$$

Now we can run $p$ on various inputs as follows (after checking that $p$ is a valid prog)

Phase 1: Run $p$ for 1 step on $i_1$.

Phase 2: Run $p$ for 2 steps on $i_1$ and $i_2$.

$\vdots$

Phase $k$: Run $p$ for $k$ steps on each of

$\bar{i}_1, \bar{i}_2, \ldots, \bar{i}_k$

Phase $k+1$:

$\vdots$

We stop at Phase $k$ if $p$ accepts two of $i_1, \ldots, i_k$, and (if this ever happens) we declare

$$p \in L = \{ p \mid p \text{ accepts at least 2 of its inputs} \}$$

This recognizes $L$.

We claim that $L$ is undecidable; if not, we claim that we could decide ACCEPTANCE. Indeed, given a string $q \sigma_0 j$, we can decide if $q \sigma_0 j \in$ ACCEPTANCE (i.e. $q$ accepts $j$) by creating a program $\hat{q}$ that ignores its input, and runs $q$ with input $j$

(replace the statement

$$i = \text{input}(\text{"Your input: "})$$

with

$$i = j$$ ).

Hence

$$q\,\sigma_0\,j \in \text{ACCEPTANCE}$$

iff

$$\tilde{q} \in L.$$

Hence $L$ is decidable $\Longrightarrow$

ACCEPTANCE is decidable,

which is impossible.