Homework 8, Solutions, Nov 2024

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(2) There are many possible solutions.

Note: If we see $\boxed{0\ \vert\quad\quad\quad}$ in the

initial state, q_0, we can simply write

a 1 there and we are done (after, moving
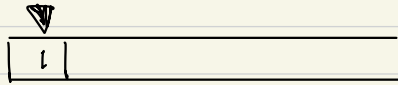
right and left). (Since we don't allow leading 0's.)

So we q_0 the initial state, we set

$$\delta(q_0, 0) = (q_{\text{saw leading zero}}, 1, R)$$

$$\delta(q_{\text{saw leading zero}}, \sqcup) = (q_{\text{accept}}, \sqcup, L)$$

we brings us to $\boxed{1\ \vert\ 1\ \vert \sqcup\vert\sqcup\vert\ \cdots}$

If initially we see

we can mark this cell as the first 1, and
and move to the end of the input. Adding
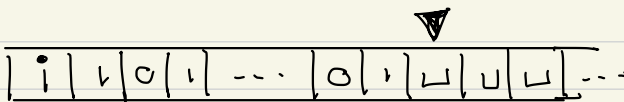a symbol $\dot{1}$ to Γ, we write

$$\delta(q_0, 1) = (q_{\text{move to end}}, \dot{1}, R)$$

and

$$\delta\left(q_{\text{move to end}}, 1\right) = \left(q_{\text{move to end}}, 1, R\right)$$

$$\delta\left(q_{\text{move to end}}, 0\right) = \left(q_{\text{move to end}}, 0, R\right)$$

We wind up at something like

When we see a ⊔ in state $q_{\substack{move\ to\\end}}$,

we move left to the first O

$$\underline{\begin{array}{|c|c|c|c|c|c|c|}\hline & O & 1 & 1 & 1 & ⊔ & ⊔ \\ \hline\end{array}}\text{---}$$
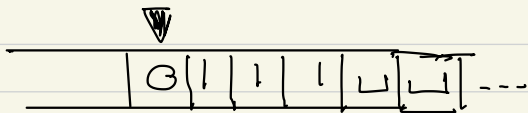
change the O to a 1, and change
all 1's to the right to O's. Hence

$$\delta\left(q_{\substack{move\ to\\end}}, ⊔\right) = \left(q_{\substack{move\\left}}, ⊔, L\right)$$

$$\delta\left(q_{\substack{move\\left}}, 1\right) = \left(q_{\substack{move\\left}}, 1, L\right)$$

$$\delta\left(q_{\substack{move\\left}}, O\right) = \left(q_{\substack{swap\ 1's\\to\ O's}}, 1, R\right)$$

$$\delta\left(q_{\substack{swap\ 1's\\to\ O's}}, 1\right) = \left(q_{\substack{swap\ 1's\\to\ O's}}, O, R\right)$$

$$\delta\left(q_{\substack{swap\ 1's\\to\ O's}}, ⊔\right) = \left(q_{\substack{almost\\done}}, ⊔, L\right)$$

In the "almost done" state we have
added one to the counter and have
taken a first step to the right

| | i | i | 0 | i | ... | i | 0 | i | i | ⊔ | ⊔ | ... |
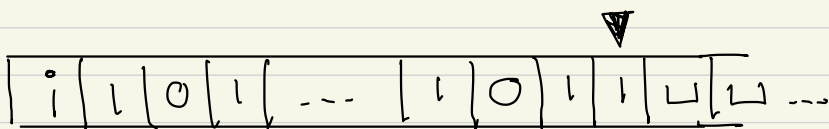
Now we just move to the left and
change the i to a 1 :

$$\delta(q_{\text{almost done}}, 0) = (q_{\text{almost done}}, 0, L)$$

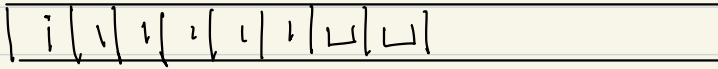$$\delta(q_{\text{almost done}}, 1) = (q_{\text{almost done}}, 1, L)$$

$$\delta(q_{\text{almost done}}, i) = (q_{\text{saw } i}, 1, R)$$

$$\delta(q_{\text{saw } i}, x) = (q_{\text{accept}}, x, L)$$

for $x = 0, 1$

The only exception is something like

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        ▽
┌─┬─┬─┬─┬─┬─┬─┬─┐
│ i│ 1│ 1│ 1│ 1│ 1│⊔│⊔│
└─┴─┴─┴─┴─┴─┴─┴─┘
```

where there are no 0's in the

counter. In this case we are in

state $q_{move \, left}$ (see page 3), but

we see a i symbol. In this

case we enter $q_{only \, 1's}$, via

$$\delta \left(q_{move \, left}, 1 \right) = \left(q_{only \, 1's}, \bar{i}, R \right)$$

(leaving i, i.e. the marked 1, on the

first tape cell, and change

all the 1's to 0's, and add

an extra 0 to the end. Hence
we set

$$\delta\left(q_{\text{only 1's}}, 1 \right) = \left(q_{\text{only 1's}}, 0, R \right)$$

when we see a $\sqcup$, we make this a
0, and then move left to $\overset{q}{1}$ and
replace it by a 1, as before. Hence
we set

$$\delta\left(q_{\text{only 1's}}, \sqcup \right) = \left(q_{\text{almost done}}, 0, L \right)$$