

CPSC 421/501

Sept 6, 2024

Today!

- Cantor's Theorem
- Injections, surjections, bijections
- What does $|S| \leq |B|$ mean?
for (infinite) sets
- Generalized Cantor's Theorem
- Start:
 - Alphabets
 - Strings
 - Languages
 - Complexity Classes
are ... ?

First 2 weeks:

Most of Sections 1-7

of

Uncomputability in CPSC 421/501

Last time:

Cantor's Theorem: Let

$f: S \rightarrow \text{Power}(S)$. Then

$$T = \{ s \in S \mid s \notin f(s) \}$$

is not the image.

→ hopefully today
Piazza & Office

will start next week



LaTeX example will

appear

Remark on Handout:

Uncomputability in CPSC 42/501

The exercises are undergoing

many changes, including

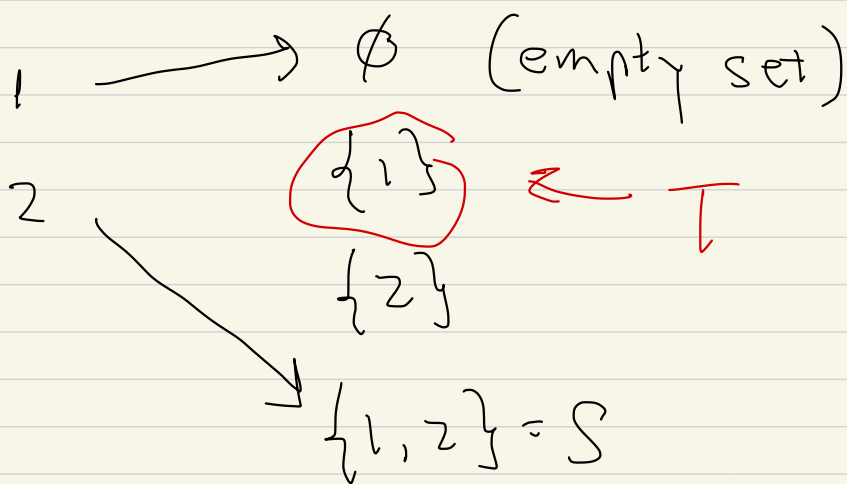
exercise numbers. But

assigned homework numbers

will not change.

Example 1: $S = \{1, 2\}$

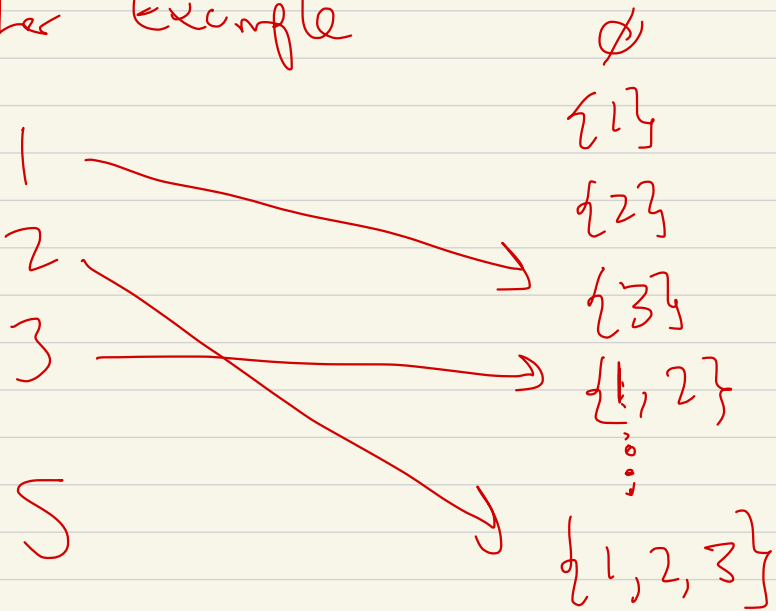
say that f given by



S $\text{Power}(S)$

$$\begin{array}{l} f(1) = \emptyset \\ f(2) = \{1, 2\} \end{array} \quad |$$

Another Example



$\mathcal{P}(S)$

$$T = \{s \in S \mid s \notin f(s)\}$$

In Example 1:

$$1, f(1) = \emptyset$$

$$1 \notin \emptyset$$

so

$$\boxed{\begin{array}{l} 1 \notin f(1) \\ 1 \in T \end{array}}$$

(if $u \in T$ but $u \notin T'$)
are different

$$f(1) \neq T$$

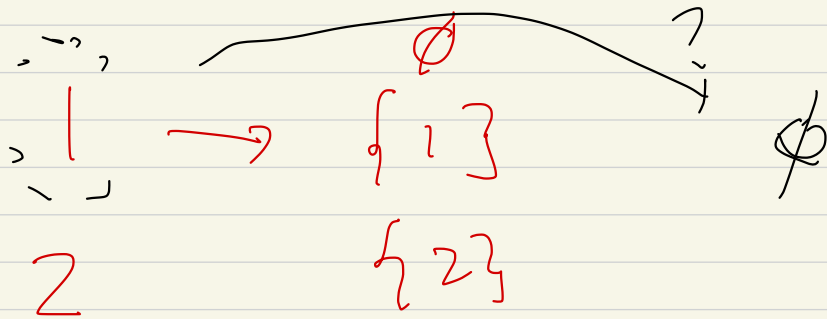
Similarly

$$2 \in f(2) = \{1, 2\}$$

$$\text{so } 2 \in f(2), 2 \notin T$$

$$f(2) \neq T$$

Example 3 (K.T.'s example)



$$\rightarrow \{1, 2\}$$

$$T = \emptyset$$

$$T = \{ s \in S \mid s \notin f(s) \}$$

$$1 \in f(1) \Rightarrow 1 \notin T$$

$$2 \in f(2) \Rightarrow 2 \notin T$$

$$T = \emptyset$$

Example 4: (D.3. example)

$$2 \mapsto \{2\}$$

$$1 \mapsto \{1\}$$

$$1 \in f(1), 2 \in f(2) \Rightarrow T = \emptyset$$

Looking ahead --- ultimately

set $S = \sum^*$

maybe $\sum^* \text{ASCII}$

Proof of Cantor's Theorem:

$$f: S \rightarrow \text{Power}(S),$$

$$T = \{ s \in S \mid s \notin f(s) \}$$

Say that $T \in \text{Image}(f)$

$$\text{then } T = f(t)$$

Either $t \in f(t) = T$

$t \notin f(t) = T$

We'll show both cases are impossible.

$$\boxed{t \in f(t)} = T = \{s \mid s \notin f(s)\}$$

so $t \in T$ then $t = s$, $s \notin f(s)$

impossible

$$\boxed{t \notin f(t)}$$

Similarly, if

$$t \in f(t) = T = \{s \mid s \in f(s)\}$$

$$t \in \{s \mid s \in f(s)\}$$



this isn't true

when $t = s$

Alt proof

(A.S.)

$$s \in f(s)$$

when $s = t$

$$t \in f(t) = T$$

$$t \in T$$

by id

$\in T$

$$t \in f(t)$$

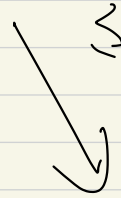
$$t \in f(t) = T$$

$$S \rightarrow \text{Power}(S)$$

$$\text{Say } |S| \leq |B|$$

$$f: S \rightarrow \text{Power}(S)$$

all f not surjective



$$\text{Power}(B)$$

B

Are you ≥ 3 metres tall

Have you met one of your grandchildren

Have you seen the TV series Dark

S
↓

Student 1

no

Student 2

no

Student 3

no

Fictive Student

yes

yes

yes

$$S \xrightarrow{f} \text{Power}(B)$$

$$f(\text{student 1})$$

$$= \{na, na, na\} = \emptyset \text{ in } B$$

$$f(\text{student 2}) = \{na, na, na\} = \emptyset \text{ in } B$$

$$f(\text{student 3}) = \emptyset \in B$$

Answers

student 1 { no, no, no }

student 2 { yes, no, no }

student 3 { yes, no, yes }

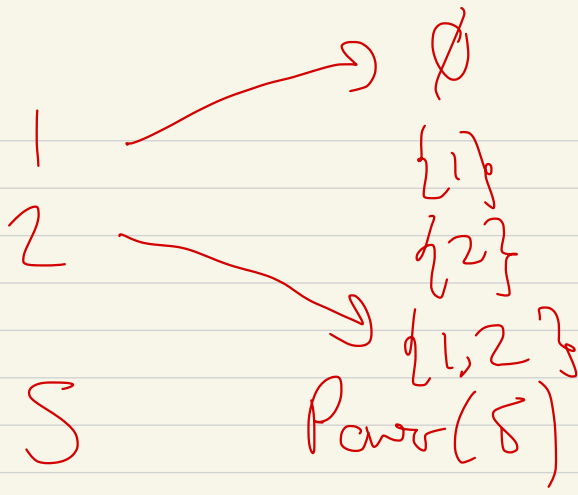
⋮

⋮

⋮

Map $B \rightarrow \{ \text{no}, \text{yes} \}$

view as a subset of B



	$f(1) = \emptyset$	$f(2) = S$
$\downarrow_S x \in f(y)$	$y=1$	$y=2$
$x=1$	no	
$x=2$		yes
	\downarrow	\downarrow
	yes	no

$T = \{1\}$