CPSC 421/501 Sept 11, 2024

Last time:

$|\mathbb{Q}^+| = |\Sigma^*| = |\mathbb{N}|$

$\mathbb{Q}^+, \Sigma^*, \mathbb{N}$ are countably infinite

$\text{Power}(\mathbb{Q}^+), \text{Power}(\Sigma^*)$ are uncountable

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Today: Generalized Cantor's Theorem:

- an example
- a proof

Some "decision problems" are "unsolvable"

Admin stuff!

Office hour locations announced on piazza: there is an issue with ICCS having limited access after 5pm ...

Gradescope / Piazza:

If you need access to Piazza, or aren't on Gradescope, email: To: jf@cs.ubc.ca Subject: CPSC 421, SID

Generalized Cantor's Theorem

example:

(1) Ursula Le Guin has written:

The Dispossessed
The Left Hand of Darkness
The Lathe of Heaven

(2) Daniel Abraham and Ty Frank have cowritten

Leviathen Wakes

$S = \{ \text{Ursula}, \text{Daniel}, \text{Ty} \}$

a smaller set than

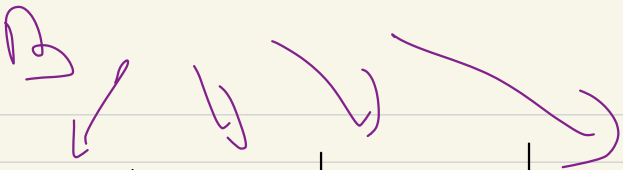
$B = \{ \text{Dispossessed}, \text{Hand}, \text{Lathe}, \text{Wakes} \}$

$\text{Cowritten} : S \to \text{Power}(B)$

given by

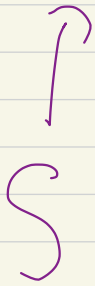
$\text{Cowitten}(\text{Ursula}) = \{ \text{Disp.}, \text{Hand}, \text{Lathe} \}$

$\text{Cowritten}(\text{Dan}) = \text{Cowritten}(\text{Ty}) = \{ \text{Wakes} \}$



Did s cowrite b?

	b = Disp.	b = Left	b = Lathe	b = Wakes
s = Ursula	yes	yes	yes	no
s = Dan	no	no	no	yes
s = Ty	no	no	no	yes



Partial info: Injection $S \to B$

True Partial Info

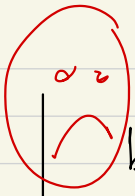
	b = Disp.	b = Left	b = Lathe	b = Wakes
s = Ursula		yes		
s = Dan	no			
s = $\overline{\text{Ty}}$				yes

↓ yes ↓ nd ↓ no

$T = \{ \text{Disp}, \quad , \quad \ddots \quad , \quad \}$

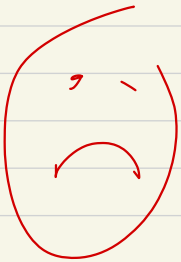
$T = \{ \{\text{Disp}\}, \{\text{Disp}, \text{Lathe}\} \}$

Not real, fictive



	b = Disp.	b = Left	b = Lathe	b = Wakes
s = Ursula		yes		
s = Dan	no			
s = Ty	yes			~~yes~~

yes no no



$S \to B$

not an injection

Not really how things happen

	b = Disp.	b = Left	b = Lathe	b = Wakes
s = Ursula		yes		
s = Dan	no			
s = Ty	no			~~yes~~ no

$S \to B$ not an injection.

Homework

Generalized Cantor's Theorem:

Injective form:

Let $h : S \to B$ be an injection, and

$$f : S \to \text{Power}(B).$$

Then

$$T = \{ h(s) \mid s \in S \text{ and } h(s) \notin f(s) \}$$

is not in the image of f.

Σ alphabet = finite, nonempty set

$$\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \cdots$$

A language over Σ is any subset of Σ^*

Decision problem over Σ is also a subset of Σ^*

$$\Sigma_{digits} = \{0, 1, \ldots, 9\}$$

PRIMES

$$= \{2, 3, 5, 7, 11, 13, \ldots\}$$

$$\subset \Sigma^*_{digits}$$

EVEN-PRIMES $= \{2\}$

$$\subset \Sigma^*_{digits}$$

DIV-BY-3

$= \{0, 3, 6, 9, 12, \ldots\}$

$\subset \Sigma^*_{digits}$

036 is div by 3?

DIV-BY-3-ALLOWING-ZEROES

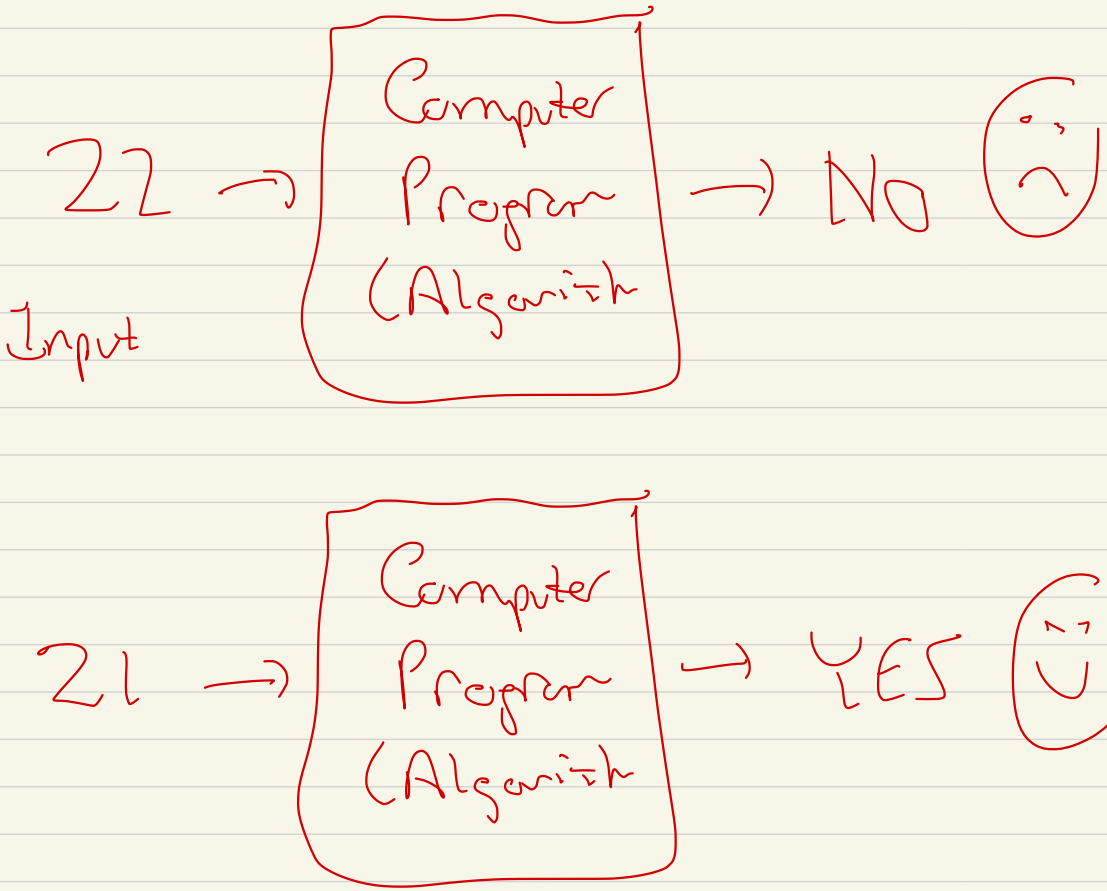
$= \{0, 3, 6, 9, 00, 03, 06, 09,$
$12, 15, \ldots, 99, 000,$
$003, 006, 009, \ldots\}$

BY-BY-3-ALLOWING-ZEROES
-AND-WE-ALLOW-THE
-EMPTY-STRING

$= \{ \varepsilon,$

$0, 3, 6, 9, 00, 03, 06, 09,$
$12, 15, \ldots, 99, 000,$
$003, 006, 009, \ldots \}$

DLV - By - 3


22
Computer Program (Algorith
NO
Input
21
Computer Program (Algorith
YES

A language over $\Sigma_{\text{digits}} = \{0, 1, \ldots, 9\}$

is a subset of Σ^*

$$\left\{ \begin{array}{l} \text{all languages} \\ \text{over } \Sigma^* \end{array} \right\}$$

$$= \text{Power}(\Sigma^*)$$

Decision problem over Σ

$$\Sigma^* \longrightarrow \{ \text{yes, no} \}$$

Given

$$f : \Sigma^* \longrightarrow \{yes, no\}$$

we associate the language

$$f^{-1}(yes)$$

$$= \{ s \in \Sigma^* \mid f(s) = yes \}$$

$$\subset \Sigma^*$$

$Power(\Sigma^*)$ is uncountable

Algorithm in C

$$C \subseteq \Sigma^*_{ASCII}$$

Alg in Python

$$C \subseteq \Sigma^*_{ASCII}$$

$$\{\text{Alg in your favourite language}\} \subseteq \Sigma^*_{ASCII}$$