

CPSC 421/501

Sept 13, 2024

Today!

- An unrecognizable

language in a toy

language Duck

NEW!

Somewhere
in section

4

- An unrecognizable

language in a version

of Python

We work with

Σ_{ASCII} = the usual 128
symbol ASCII
character set

and its subalphabet

$\Sigma_{\text{Digits}} = \{0, 1, \dots, 9\}$

We built a map

$\Sigma_{\text{ASCII}}^* \rightarrow \text{Power}(\Sigma_{\text{ASCII}}^*)$

Specify a programming language called Duck.

- A valid Duck statement

$$S_1 S_2 = S_1 \circ S_2 \quad \left. \vphantom{S_1 S_2} \right\} \leftarrow \begin{array}{l} \text{string} \\ \text{concatenation} \end{array}$$

$$S_1 = \text{quack} \in \sum_{\text{ASCII}}^S$$

$$S_2 \in \sum_{\text{Digits}}^*$$

Valid Duck statements!

quack421, quack3, quack♡7

quack 0, ...

Invalid Duck statement!

Quack 7, CPSC421, ...

quack 07q

This gives

VALID-DUCK-STATEMENT

$$C \sum_{\text{ASCII}}^*$$

Rem:

quack i.e. quack = ϵ

is OK

- A valid duck program

is an element of

(VALID-DUCK-STATEMENT)

where for any set, S ,

$$S^* = S^0 \cup S^1 \cup S^2 \cup \dots$$

↑

ϵ

So; really, for any Σ ,

any $L \subset \Sigma^*$, L^* refers to

~~*~~

see below

thanks to
management

$$L^* = L^0 \cup L^1 \cup L^2 \cup \dots$$

where

$$L^2 = \left\{ s_1 s_2 = s_1 \circ s_2 \mid s_1, s_2 \in L \right\}$$

- OR: Equivalently, a valid duck program is a string $s_1 s_2 \dots s_k$ for

some k , where

$$s_1, s_2, \dots, s_k \in$$

VALID

DUCK

STATEMENT

e.g.

quack quack 3 quack 002024

let's say, after consulting with
management,

(VALID-DUCK-STATEMENT)⁺

where

$$L^+ = L \cup L^2 \cup L^3 \cup \dots$$

L^k means in the sense of
concatenation

so:

$\epsilon \notin \text{VALID-DUCK-PROGRAM}$

\equiv

If $p \in \text{VALID-DUCK-PROGRAM}$

and $i \in \sum_{\text{ASCII}}^*$ (i think of)
input

we say

" p accepts i "

if $|i|$ is a number

described by p

for example

$$p = \text{quack}^3 \text{quack} \text{quack}^{\text{13}}$$

then

p accepts i iff

$$|i| = 3, 6, 13$$

In other words, for this p ,

p accepts i iff

$$i \in \sum_{\text{ASCII}}^0 \cup \sum_{\text{ASCII}}^3 \cup \sum_{\text{ASCII}}^{13}$$

Question: does

quack quack Σ

i.e.

quack $\circ \varepsilon \circ$ quack $\circ \Sigma$

\Rightarrow that

p accepts $\varepsilon \in \sum_{\text{ASCII}}^{\circ}$

Management met and

p does not accept ε

The language accepted by
 P (if $p \in \text{VALID-DUCK-PROGRAMS}$)

is

$$\{ i \mid P \text{ accepts } i \}$$

$$\{ i \in \Sigma_{\text{ASCII}}^* \mid P \text{ accepts } i \}$$

denote this set

Language Rec By (P)

If $p \notin \text{VALID-DUCK-PROGRAM}$

then

$$\text{Language RecBy}(p) = \emptyset,$$

i.e.

p does not accept ~~any~~ any

string $w \in \Sigma_{\text{ASCII}}^*$

Hence:

$$\text{Language RecBy} : \Sigma_{\text{ASCII}}^* \rightarrow \text{Power}(\Sigma_{\text{ASCII}}^*)$$

Hence

$$T = \left\{ s \in \Sigma_{\{A, B, C, D\}}^* \mid s \notin \text{LanguageRecBy}(S) \right\}$$

is not in the image of

$$\text{LangRecBy} : \Sigma_{\{A, B, C, D\}}^* \rightarrow \text{Power}(\Sigma_{\{A, B, C, D\}}^*)$$

Rem: $L = \{a\}$ is not in the image of LangRecBy

A language L is
recognizable if

(1) it is in the image of LangRecBy ,
equiv

(2) for some p , $L = \text{LangRecBy}(p)$

(3) for some p ,

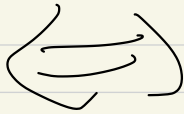
p accepts i iff $i \in \text{LangRecBy}(p)$

(4) $i \in \text{LangRecBy}(p) \Rightarrow p$ accepts i
 $i \notin \text{LangRecBy}(p) \Rightarrow p$ does not accept i

Rem:

$$L \subset \sum_{k \in \mathbb{N}} \text{ASCII}^k \text{ is in}$$

LangRecBy Duck™



$$L = \sum^{k_1} \cup \sum^{k_2} \cup \dots \cup \sum^{k_m}$$

for $k_1, \dots, k_m \in \mathbb{Z}_{\geq 0} = \{0, 1, 2, \dots\}$

quack3quack13 ∈
LangRecBy (quack3quack13)

quack 5 \notin LangRecBy (quack 5)

$$|quack 5| = \text{length}(quack 5) = 6$$

LangRecBy (quack 3 quack 3)

$$= \sum_{ASCI}^3 \cup \sum_{ASCI}^{13}$$