CPSC 421/501 Sept 13, 2024

Today:

- An unrecognizable language in a toy language Duck
- An unrecognizable language in a version of Python

NEW! Somewhere in section 4

We work with

$\Sigma_{ASCII}$ = the usual 128 symbol ASCII character set

and its subalphabet

$\Sigma_{Digits} = \{0, 1, \ldots, 9\}$

We built a map

$$\Sigma_{ASCII}^{*} \longrightarrow \mathrm{Power}\left(\Sigma_{ASCII}^{*}\right)$$

Specify a programming language called Duck.

- A valid Duck statement

$S_1 S_2 = S_1 \circ S_2$ ← string concatenation

$S_1 = quack \in \sum^{5}_{ASCII}$

$S_2 \in \sum^{*}_{Digits}$.

Valid Duck statements:

quack421, quack3, quack07

quack0, ...

Invalid Duck statement:

Quack7, CPSC421, ...

quack07q

This gives

VALID-DUCK-STATEMENT $\subset \Sigma^*_{ASCII}$

Rem:

quack i.e. quack$\circ\varepsilon$

is OK

~ A <u>valid duck program</u>

is an element of

$$(\text{VALID-DUCK-STATEMENT})^{+}$$

where for any set, $S$,

$$S^{*} = S^{0} \cup S^{1} \cup S^{2} \cup \cdots$$

$S^{0} = \{\varepsilon\}$

see below thanks to management

So, really, for any $\Sigma$, any $L \subset \Sigma^{*}$, $L^{*}$ refers to

$$L^* = L^0 \cup L^1 \cup L^2 \cup \cdots$$

where

$$L^2 = \{ s_1 s_2 = s_1 \circ s_2 \mid s_1, s_2 \in L \}$$

— OR: Equivalently, a valid duck program is a string $s_1 s_2 \cdots s_k$ for some $k$, where

$s_1, s_2, \ldots, s_k \in$ VALID DUCK STATEMENT

e.g.

quack quack 3 quack 02024

let's say, after consulting with management,

$$(\text{VALID-DUCK-STATEMENT})^+$$

where

$$L^+ = L \cup L^2 \cup L^3 \cup \cdots$$

$L^k$ means in the sense of concatenation

so:

$$\epsilon \notin \text{VALID-DUCK-PROGRAM}$$

=

If $p \in$ VALID-DUCK-PROGRAM

and $i \in \Sigma^*_{ASCII}$ (i think of input

we say

"p accepts i"

if $|i|$ is a number

described by p

for example

$p$ = quack 3 quack quack 13

then

$p$ accepts $i$ iff

$|i| = 3, 0, 13$ .

In other words, for this $p$,

$p$ accepts $i$ iff

$$i \in \sum_{\text{ASCII}}^{0} \cup \sum_{\text{ASCII}}^{3} \cup \sum_{\text{ASCII}}^{13}$$

Question: does

quack quack 5

i.e.

$quack \circ \varepsilon \circ quack \circ 5$

$\Rightarrow$ that

p accepts $\varepsilon \in \Sigma^{\circ}_{ASCII}$

Management met and

p does not accept $\varepsilon$

The language accepted by

p (if $p \in$ VALID-BUCK-PROGRAM)

is

$$\{ i \mid p \text{ accepts } i \}$$

$$\{ i \in \Sigma^*_{ASCII} \mid p \text{ accepts } i \}$$

denote this set

LanguageRecBy(p)

If $p \notin$ VALID-DUCK-PROGRAM

then

$LanguageRecBy(p) = \emptyset$,

i.e.

p does not accept any string i.e $\Sigma^*_{ASCII}$.

Hence:

$LanguageRecBy : \Sigma^*_{ASCII} \longrightarrow Power(\Sigma^*_{ASCII})$

Hence

$$T = \{ s \in \Sigma_{ASCII}^* \mid s \notin LanguageRecBy(s) \}$$

is not in the image of

$$LangRecBy : \Sigma_{ASCII}^* \to Power(\Sigma_{ASCII}^*)$$

---

Rem: $L = \{a\}$ is not in the image of $LangRecBy$

A language $L$ is

recognizable if

(1) it is in the image of $\text{LangRecBy}$,

equiv

(2) for some $p$, $L = \text{LangRecBy}(p)$

(3) for some $p$,

$p$ accepts $i$ iff $i \in \text{LangRecBy}(p)$

(4) $i \in \text{LangRecBy}(p) \Rightarrow p$ accepts $i$;

$i \notin \text{LangRecBy}(p) \Rightarrow p$ does not accept $i$

Rem:

$L \subseteq \Sigma_{ASCII}^*$ is in

$LangRecBy_{Duck^{TM}}$

$\Longleftrightarrow$

$L = \Sigma^{k_1} \cup \Sigma^{k_2} \cup \dots \cup \Sigma^{k_m}$

for $k_1, \dots, k_m \in \mathbb{Z}_{\geq 0} = \{0, 1, 2, \dots\}$

quack3quack13 $\in$
$LangRecBy(quack3quack13)$

$quack5 \notin LangRecBy(quack5)$

$|quack5| = length(quack5) = 6$

$LangRecBy(quack3\,quack13)$

$= \sum_{ASCII}^{3} \cup \sum_{ASCII}^{13}$