① Review universal Python

programs and proof that

NON-ACCEPTANCE and

NON-HALTING are

unrecognizable

② ACCEPTANCE and

HALTING are

recognizable and ==undecidable==

③ Reductions

Python program p, input i   LangRecBy
                              (₽)

p accepts i :   return ("yes")

p rejects i :   return ("no")

p loops on i :   anything else

        ⎧ - return ("quack")
        ⎪
        ⎪ - reaches last line without
        ⎨   returning anything
        ⎪
        ⎪ - doesn't terminate (e.g.
        ⎩   infinite loop)

LangRecBy(p) = { i | p accepts i }

New: A Python program is
a _decider_ if on any input,
it either:

    accepts (return ("yes"))
OR
    rejects (return ("no")).

L is _recognizable_ if for some p,

    L = LangRecBy (p)

L is _decidable_ if for some
decider p,   L = LangRecBy (p)

Catchy phrases!

"The **halting** problem is **undecidable**"

Less catchy:

"The **non-halting** problem is **unrecognizable**"

Decider ⊂ All valid Python prog.

— Σ^*_{ASCII} partitioned into

ACCEPTANCE $= \{ p\, \sigma_0\, i \mid p \text{ accepts } i \}$

REJECTION $= \{ p\, \sigma_0\, i \mid p \text{ rejects } i \}$

LOOPING $= \{ p\, \sigma_0\, i \mid p \text{ loops on } i \}$

(p above is a valid Python program)

NOT-PYTH-INP

$= \{ \text{everything else} \}$

$= \{ \text{strings that can't be written}$
$\text{as } p\, \sigma_0\, i \text{ where } p \text{ is a}$
$\text{valid Python program} \}$

- Each string in Σ_{ASCII}^{*} is in one of :

NOT-PYTH-INP = { not of the the form $p \sigma_o i$ with p a valid Python program

ACCEPTANCE = { $p \sigma_o i$ | p accepts i }

REJECTION = { $p \sigma_o i$ | p rejects i }

LOOPING = { $p \sigma_o i$ | p loops on i }

$\sigma_o = \langle FS \rangle$

Universal Program, u

Input: $s \in \Sigma_{ASCII}^{*}$:

① Figures out if $s = p \sigma_0 i$ with p
a valid Python program; if not,
u returns ==not valid==

② "==Simulates==" p on input i
  - if p accepts i, u says ==accepts==
  - if p rejects i, u says ==rejects==
  - if p terminates otherwise,
    u says ==loops==
  - p ==may never terminate== --- 🙂
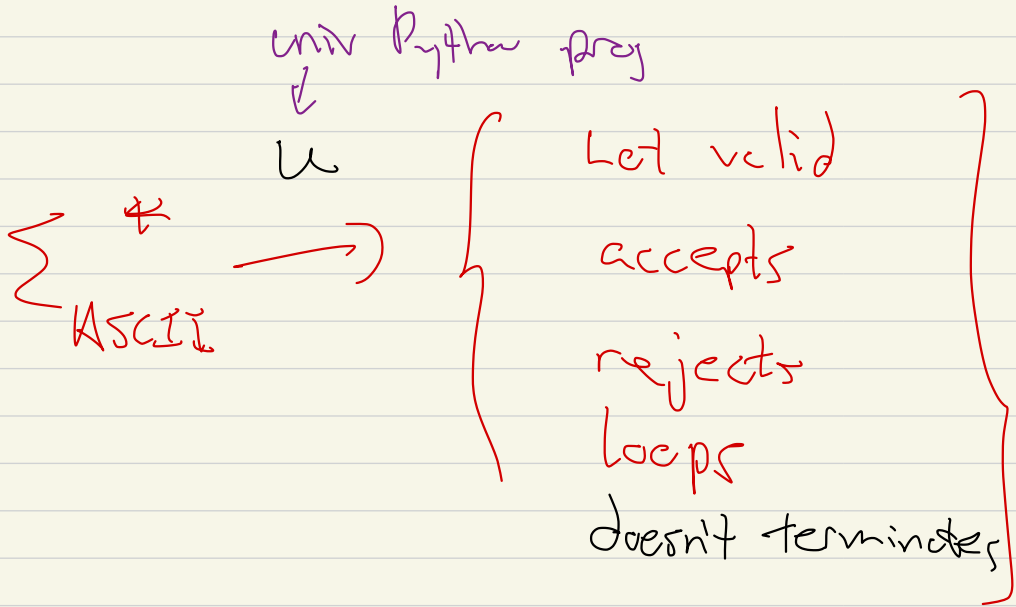
Note:

 = may not terminate,

but is never wrong

when it terminates

So u produces a map

$$\sum_{ASCII}^* \longrightarrow \left\{ \begin{array}{c} \text{``not valid''}, \text{``accepts''}, \\ \text{``rejects''} \text{``loops''}, \\ \text{doesn't terminate} \end{array} \right\}$$

u ⇒) ACCEPTANCE is recognizable

REJECTION "   "

HALTING "   "

univ Python prog

$$\sum_{ASCII}^{*} \xrightarrow{u} \begin{cases} \text{Let valid} \\ \text{accepts} \\ \text{rejects} \\ \text{loops} \\ \text{doesn't terminates} \end{cases}$$

$$\begin{cases} \text{Let valid} \\ \text{accepts} \\ \text{rejects} \\ \text{loops} \\ \text{doesn't terminates} \end{cases} \begin{array}{l} \text{Alg} \\ \longmapsto \text{"no"} \\ \longmapsto \text{"yes"} \\ \longmapsto \text{"no"} \\ \longmapsto \text{"no"} \\ \longmapsto \text{"loops"} \end{array}$$

Alg

Let valid $\longmapsto \longrightarrow$ don't say yes

accepts $\longmapsto \longrightarrow$ "yes"

rejects $\longmapsto \longrightarrow$ don't say yes

loops $\longmapsto \longrightarrow$ don't say yes

doesn't terminates $\longmapsto \longrightarrow$ don't say yes

$$(Alg \circ u): \sum_{ASCII}^{*} \longrightarrow \begin{array}{c} \text{"yes"} \\ \text{not "yes"} \end{array}$$

$$LangRecBy (Alg \circ u)$$

$$= \left\{ p \sigma_o i \mid p \text{ accepts } i \right\}$$

$$= ACCEPTANCE$$

Thm: NON-ACCEPTANCE

NON-HALTING,   ACCEPTANCE^{Comp}

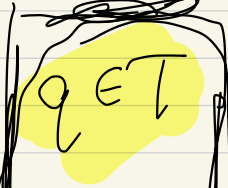HALTING^{Com} are unrecognizable.

NON-ACCEPTANCE =
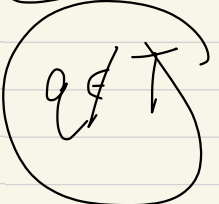
$$\{ p \sigma_o i \mid p \text{ does not accept } i \}$$

$$= \{ p \sigma_o i \mid p \text{ rejects or loops on } i \}$$

$$= \text{REJECTION } \cup \text{ LOOPING}$$

ACCEPTANCE^{Comp} $=$ REJ $\cup$ LOOP $\cup$ NOT PI

	a Py Prg	Rec for WON-ACC	in T
	Alg 1	Alg 2	in T
NON-PYTH-INP	😊	?	$q \in T$
ACCEPTANCE	😊	?	$q \notin T$
REJECTION	😊	😊	$q \in T$
LOOPING	😖	😊	$q \in T$

$$T = \{ q \mid q \text{ does not accept } q \}$$

feed

$q \ G_o \ q$ into Recognizer for

~~Non~~ -Accept

① q accepts q

then $q \notin T$

q rejects q $q \in T$

my Contradiction if

NON -ACC is recog