CPSC 421/501 Sept 25, 2024

(1) Use: "L is decidable $\Leftrightarrow$ L and L^{comp} are recognizable" to give undecidable languages.

(2) More undecidable and unrecognizable languages via reductions

(3) Paradoxes (Russell, "Berry", write about themself)

(4) Start: Ch. 1 [Sip], Regular Languages

Some examples for today!

Decidable: PRIMES

Undecidable but recognizable:

ACCEPTANCE, HALTING,

$\{ p \mid p \text{ accepts } \geq 1 \text{ input in } \Sigma^*_{ASCII} \}$

$\{ p \, \sigma_0 \, \ell \mid p \text{ reaches line number } \ell \text{ on } \geq 1 \text{ input} \}$

Unrecognizable

NON-ACCEPTANCE

$\{ p \mid p \text{ accepts no inputs} \}$

$\{ p \mid p \text{ accepts exactly 3 inputs} \}$

Note: "L is decidable $\Leftrightarrow$

(1) L and L^{comp} are recognizable"

implies:

"L is unrecognizable $\Rightarrow$ $\Leftarrow$? (Homework)

(2) L and L^{comp} are undecidable

(2) ! $ACCEPTANCE^{Comp}$ is unrecognizable

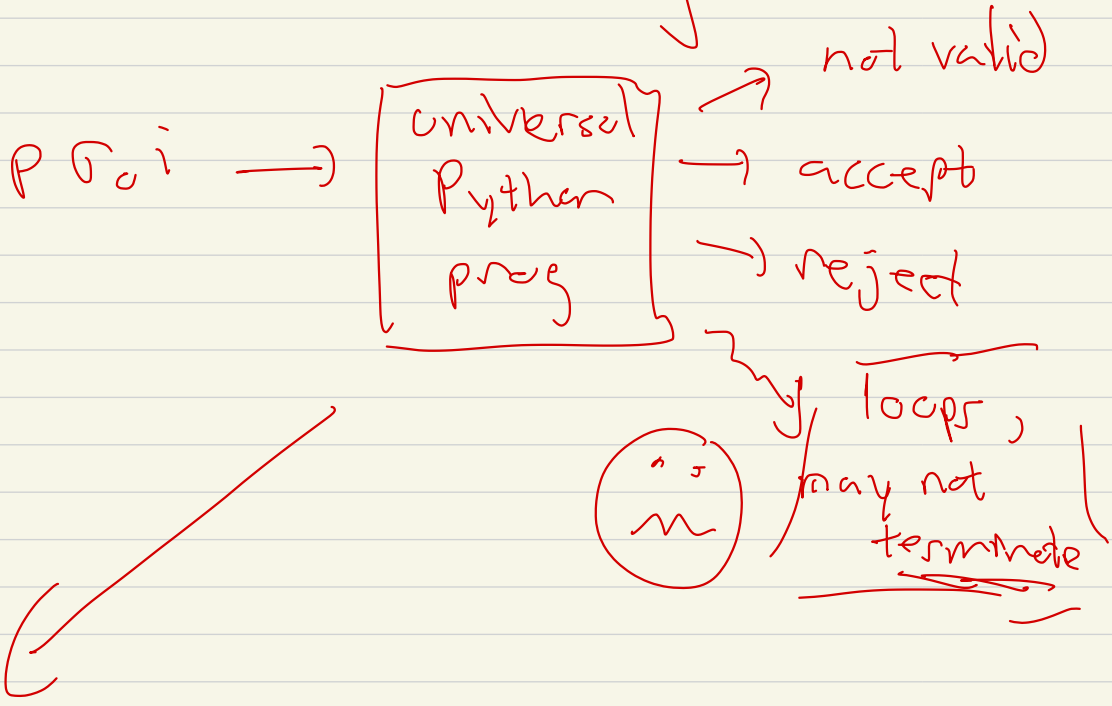
$\Rightarrow$

ACCEPTANCE undecidable

$= \{p\sigma_0 i \mid p \text{ accepts } i\}$

ACCEPTANCE = $\{\langle p, \sigma, i \rangle$

p accepts $i\}$



accept ⟶ return "yes"

otherwise ⟶ don't return "yes"

This recognizes ACCEPTANCE

This "algorithm" can be coded in Python, gives $\alpha \in \Sigma^*_{ASCII}$ s.t.

$$\text{ACCEPTANCE} = \text{LangRecBy}_{\text{Python}}(\alpha)$$

Rem: Since ACCEPTANCE is undecidable, α above cannot { return "yes" / return "no" } on all inputs. (G.H.)

$L \overset{\text{def}}{=}$

$$\{ p \mid p \text{ accepts} \geq 1 \text{ input in } \Sigma^*_{\text{ASCII}} \}$$

$$\{ p \mid p \text{ is a valid Python prog and } \exists i \in \Sigma^*_{\text{ASCII}} \text{ s.t. } p \text{ accepts } i \}$$

there exists

Is L recognizable?

Yes:

Idea

$$\Sigma^*_{\text{ASCII}} = \{ i_1, i_2, i_3, \ldots \}$$

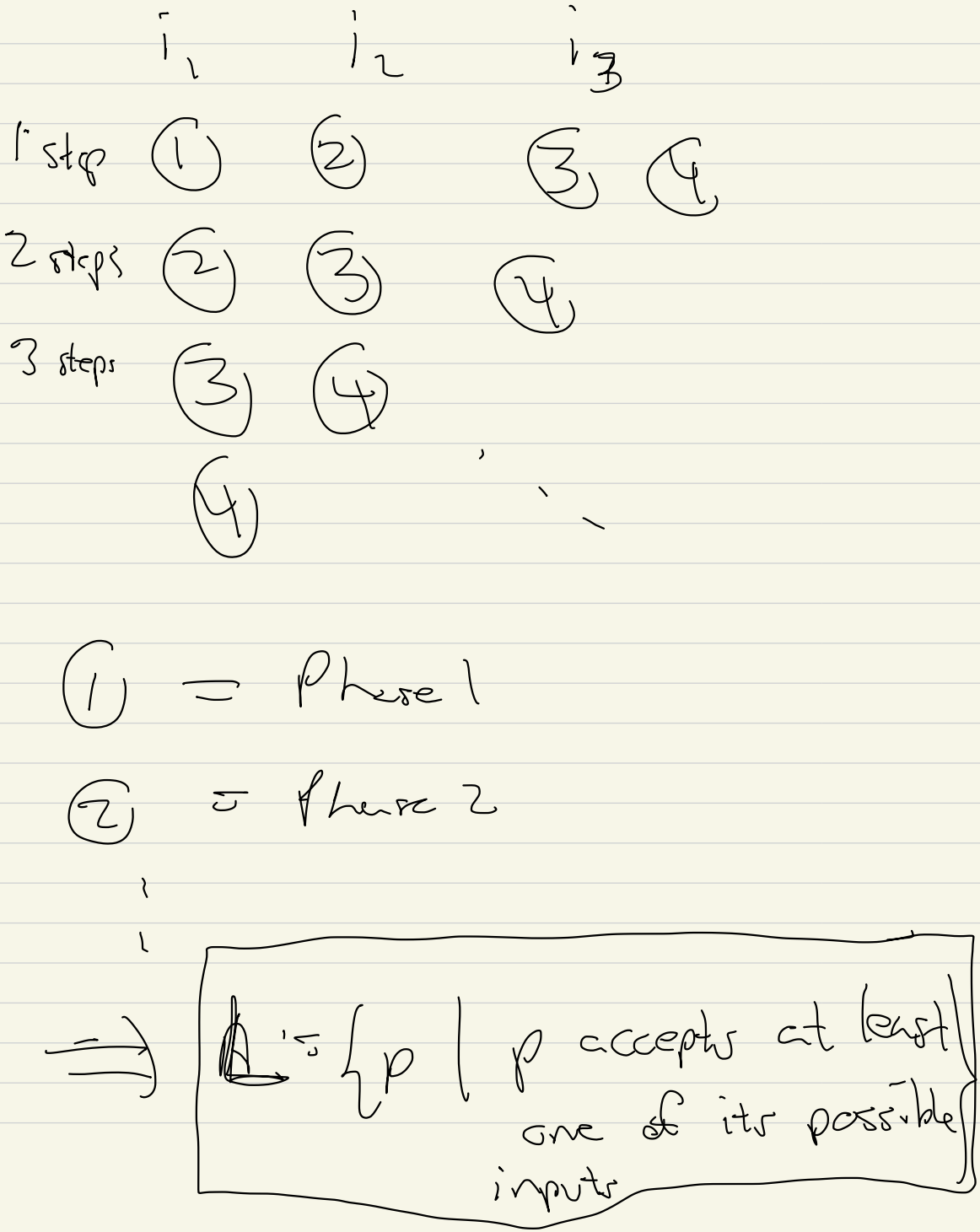
So ---

have universal Python prog!

Phase 1: Run univ Py prog on one step of i_1

Phase 2: One step i_2

Phase 3: 2 steps in total i_1


i_1
i_2
i_3
1 step
2 steps
3 steps
1
2
3
4
4
3
4
4
1 = Phase 1
2 = Phase 2
$A := \{p \mid p$ accepts at least one of its possible inputs

Equiv

Prog 1 locks at p on input i_1

Prog 2 " " " " " " i_2

Prog 1 Prog 2 — — —

One step

Two steps

,

,

,

then L is recognizable.

Why is L undecidable??

We know ACCEPTANCE is undecidable.

Reduction!

L is decidable $\Longrightarrow$ ACCEPTANCE is decidable

Say L is decidable.

You're given $\langle M, w \rangle$, is it in ACCEPTANCE

Is $p\sigma_0 i \in$ ACCEPTANCE