CPSC 421/501 Oct 2, 2024

- DFA's and regular language examples: end with "ba" and DIV-BY-3 variants

  DIV-BY-10

- Recall: $L_1 \circ L_2$ and L^*
- Example $\{a^3, a^5\}$ and $\{a^3, a^5\}^*$
- L_1, L_2 regular $\Rightarrow$ so are

$L_1 \cup L_2$, $L_1 \cap L_2$, L_1^{comp}, $L_1 \circ L_2$, L_1^*

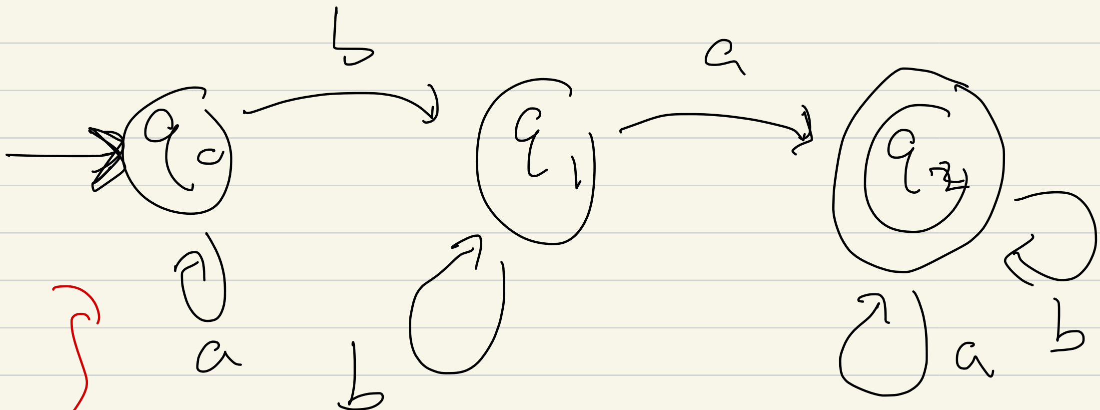
- Oct 3, 11:59 pm: Individual HW 4

  Group HW 3, last 2 questions

Math 344

→ CHEM C124

$L = \{ s \in \{a, b\}^* \mid s \text{ contains "ba" as a substring} \}$

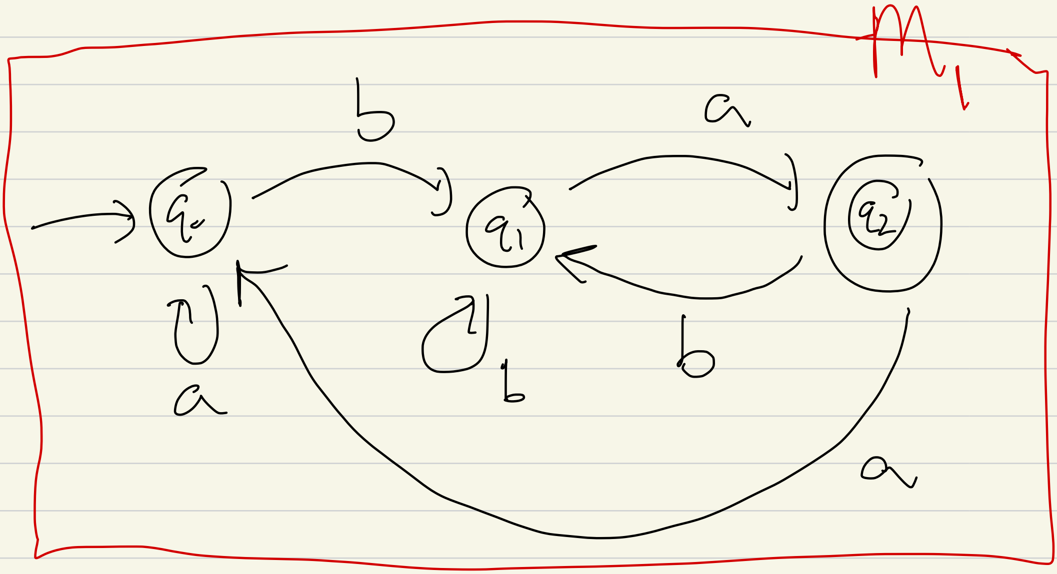


$\rightarrow \bigcirc$ means initial state

q_1 = we've just seen a "b"

this is an accepting state

$L' = \{ s \in \{a,b\}^* \mid s \text{ ends in "ba"} \}$



q_0 = initial

q_1 = we've just seen "b"

q_2 = " " " "ba"

Def: [Sip] L is regular if it is recognized by a DFA.

Formally a DFA
(deterministic finite automaton)
consists of

Q = "set of states" (finite)

Σ = alphabet

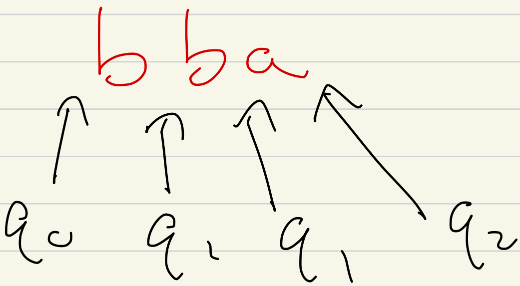
$\delta : Q \times \Sigma \longrightarrow Q$

[e.g. $q_0 \xrightarrow{b} q_1$: $\delta(q_0, b) = q_1$]

q_0 = initial state

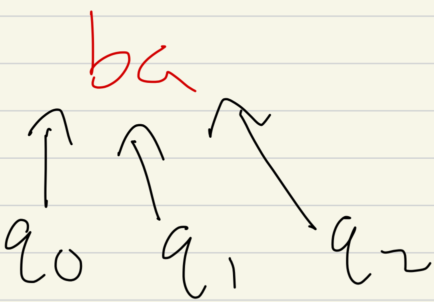
F = set of accepting (final) states, $F \subset Q$

"Language recognized by a DFA": strings wind up in a state in F.



$q_2 \in F$

we accept "bba"



Special cases "a", "b", ε

Rem: ε accepted iff initial state lies in F

DIV-BY-3 :

$\Sigma_{digits} = \{0, 1, \ldots, 9\}$

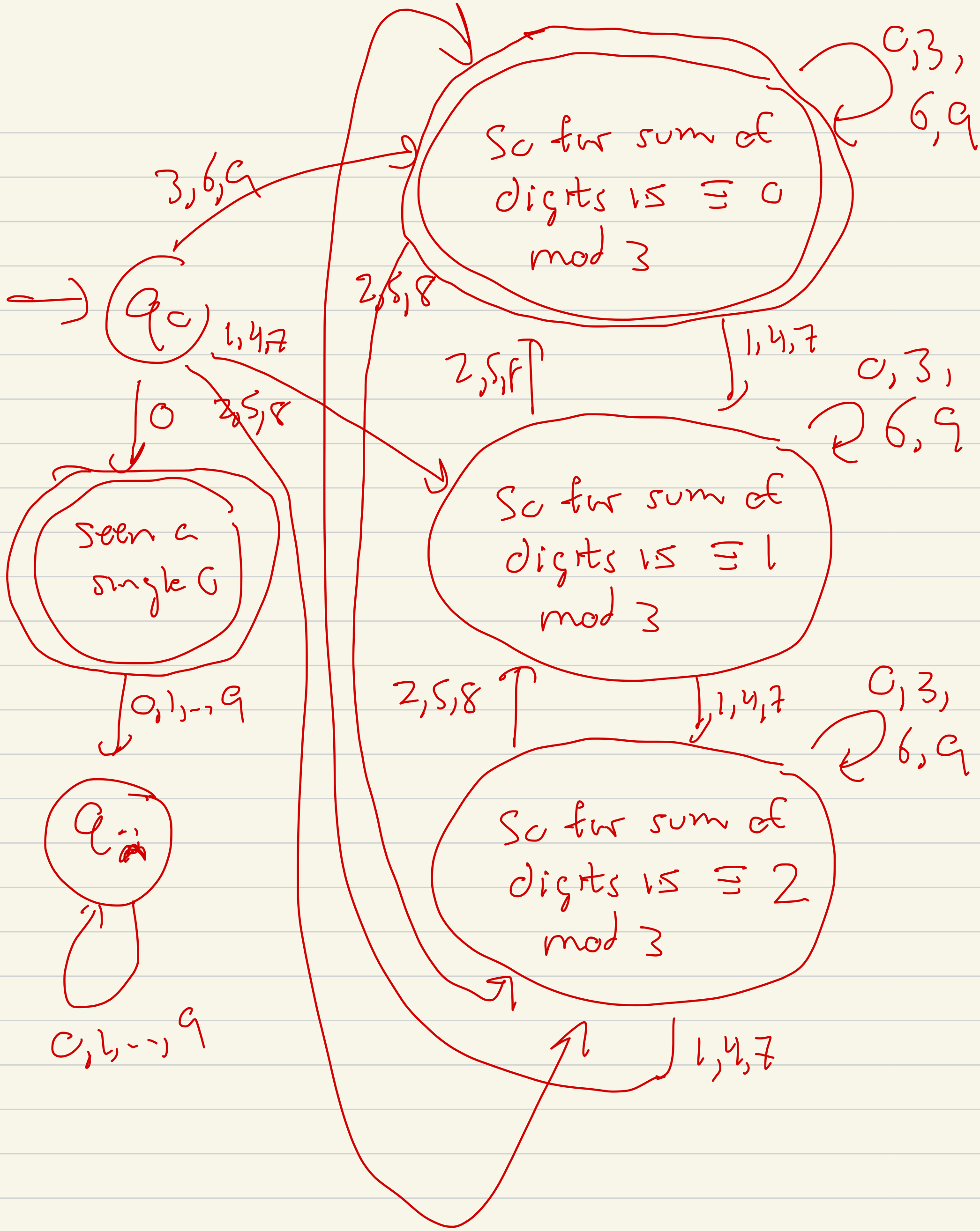
DIV-BY-3 :

$\{0, 3, 6, 9, 12, 15, 18, \ldots\}$

$712395 \overset{?}{\in} \text{DIV-BY-3}$

$0 \in \text{DIV-BY-3}$

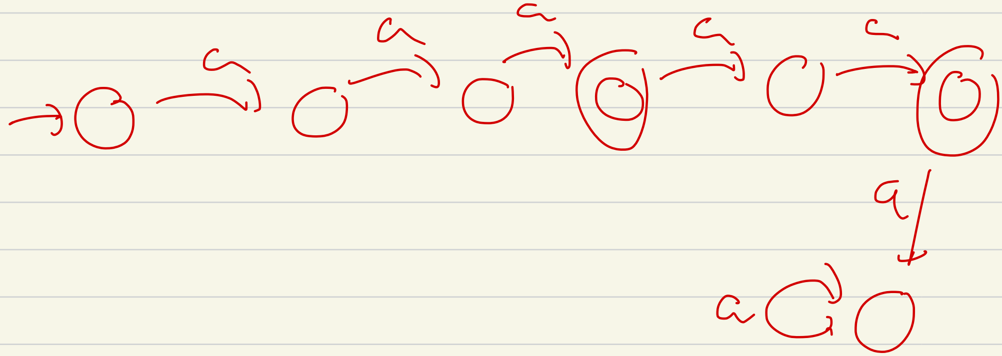
$03 \notin$ " " "



So far sum of digits is ≡ 0 mod 3
So far sum of digits is ≡ 1 mod 3
So far sum of digits is ≡ 2 mod 3
Seen a single 0
q_0
0
3,6,9
1,4,7
2,5,8
0,3,6,9
0,1,...,9

$\Sigma = \{a\}$

$L = \{a^3, a^5\}$



Thm: If L_1, L_2 are regular, then

$L_1 \circ L_2$ and L_1^*

are also regular

$L_1 \circ L_2$

$= \{ s_1 \circ s_2 \mid s_1 \in L_1, s_2 \in L_2 \}$

$L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \ldots$

$L^2 = L \circ L$

$L^3 = L \circ L \circ L \quad \ldots$

Question: What is

$\{a^3, a^5\}^* = \{ \varepsilon, a^3, a^5,$

$a^3 a^3, a^3 a^5, a^5 a^3, a^5 a^5, \ldots$

$= \{ a^0, a^3, a^5, a^6, a^8, a^9,$

$a^{10}, a^{11} = a^{3+3+5}, a^{12},$

$a^{13}, a^{14}, \ldots \}$

$\{a^5, a^7\}^* = \{$

$\rightsquigarrow$ NFA