CPSC 421/501 Oct 4, 2024

- Last time:

$\{a^3, a^5\}$ looks simple

$\{a^3, a^5\}^* = \{a^0, a^3, a^5, a^6, a^8, a^9, a^{10}, a^{11}, a^{12}, \ldots\}$

(a^0: ε; a^3: L; a^6: $a^3 \circ a^3$)

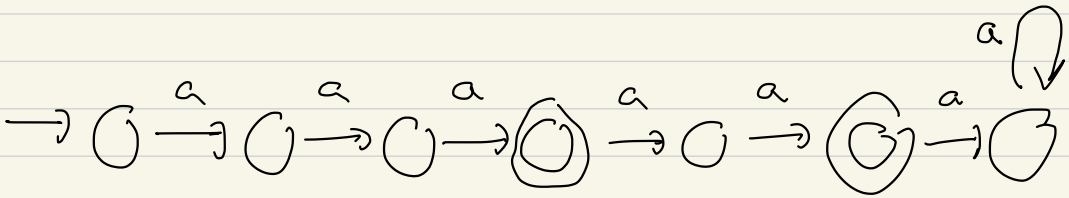
$L^* = L^0 \cup L^1 \cup L^2 \cup \ldots$

$L^2 = L \circ L$

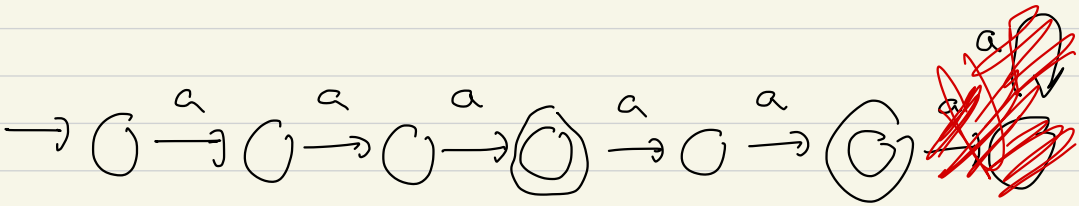
- Today: Thm: Let L be regular. Then so is L^*

- Tool: NFA's (non-deterministic)

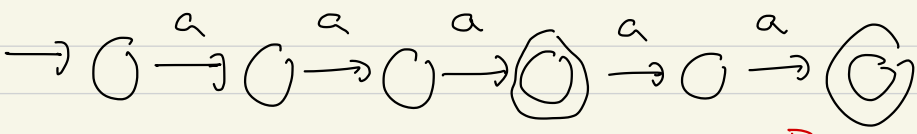
DFA for $\{a^3, a^5\}$:



Wish list for $\{a^3, a^5\}^*$



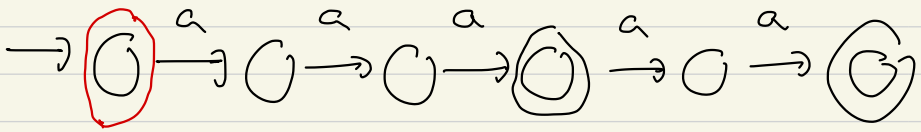
Say we erase

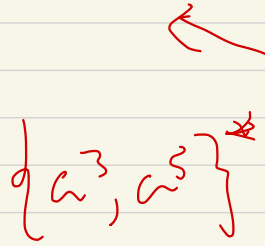


???

"halting and rejecting"

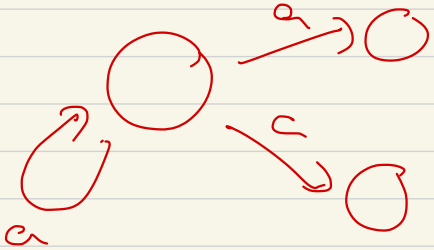
anything after here is rejected



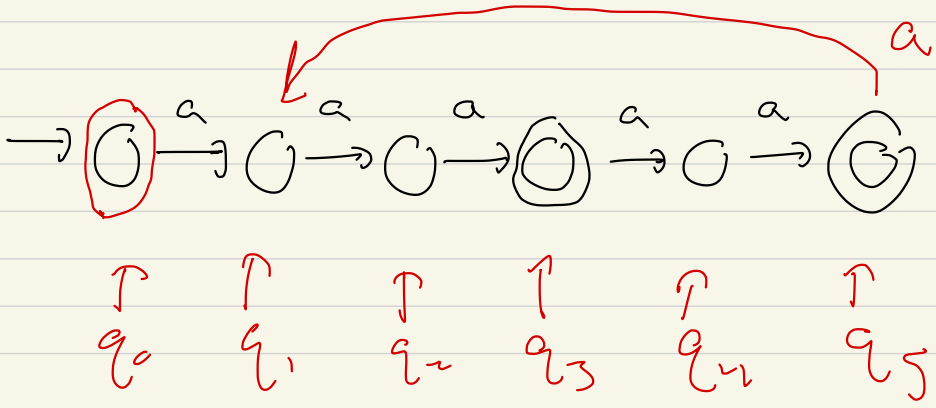


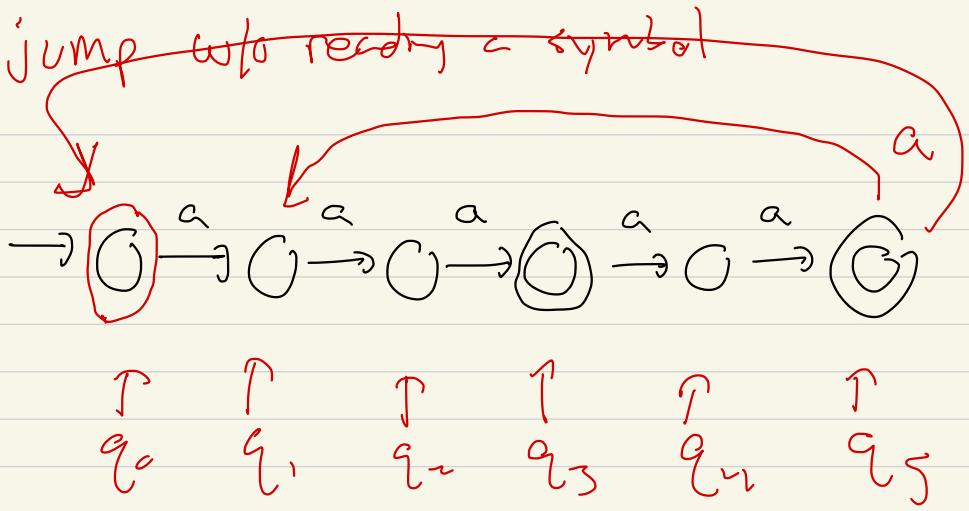
$\{a^3, a^5\}^*$

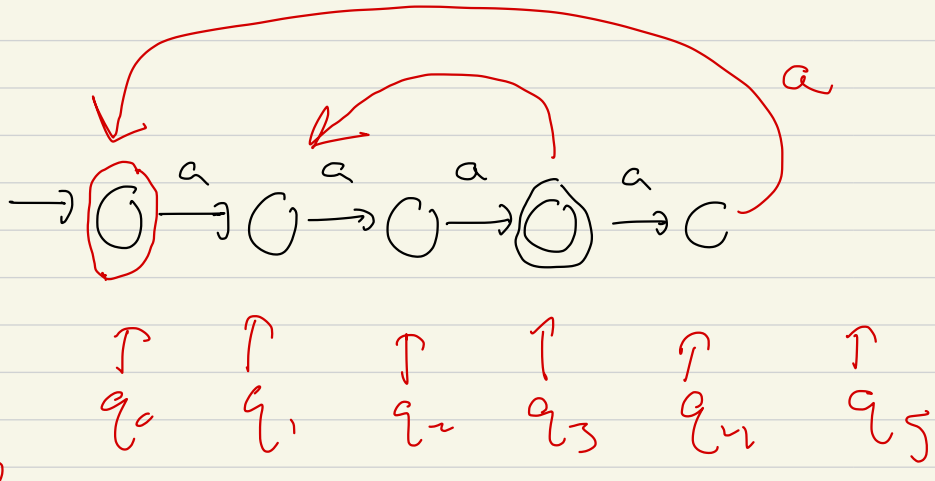
say "non-determinism"



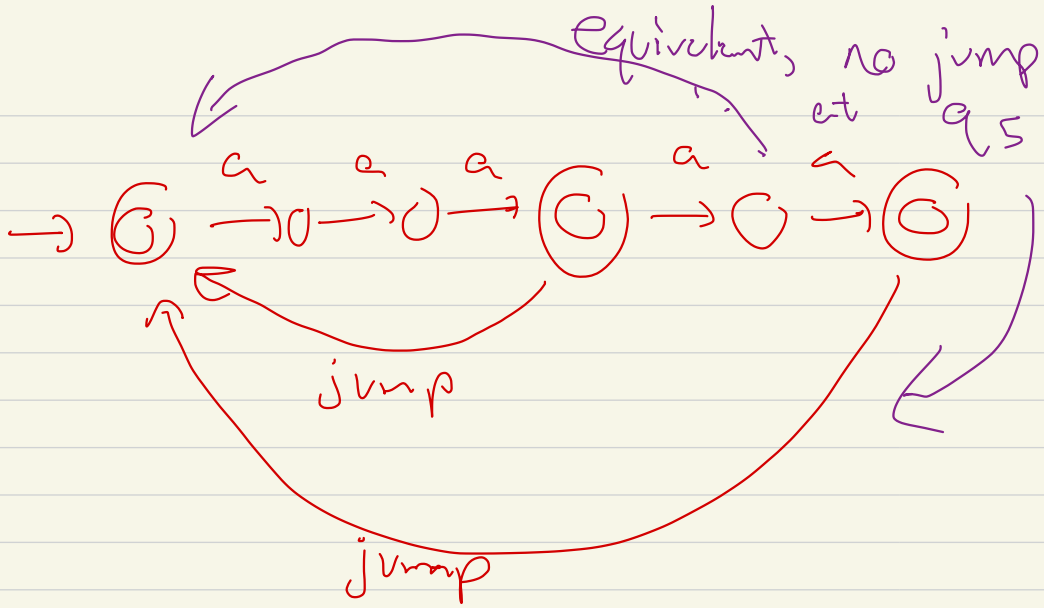
more than one possible next state...




jump w/o reading a symbol
a
a
a
a
a
a
q_0
q_1
q_2
q_3
q_4
q_5

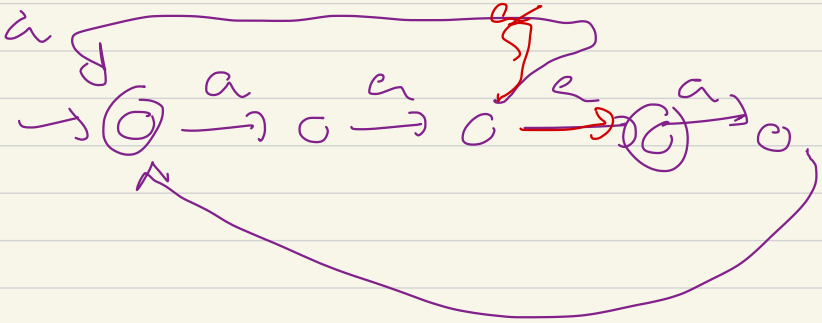

a
a
a
a
a
q_0
q_1
q_2
q_3
q_4
q_5

word $a^3\,a^3$



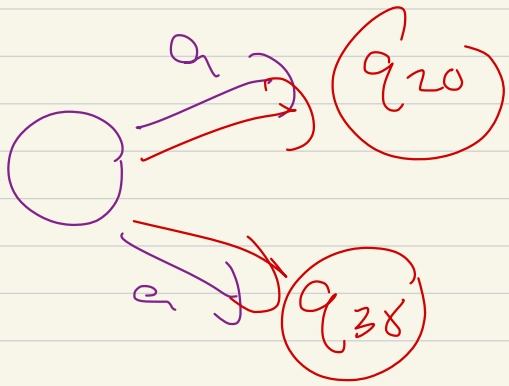
From any final (accepting) state

we can jump to q_0



but we need to accept $a^3 a^3$

Clarify! Under non-determinism



we'll say that string $s \in \Sigma^*$ is accepted by this strange machine if the exists a path through it that lands on an accepting state

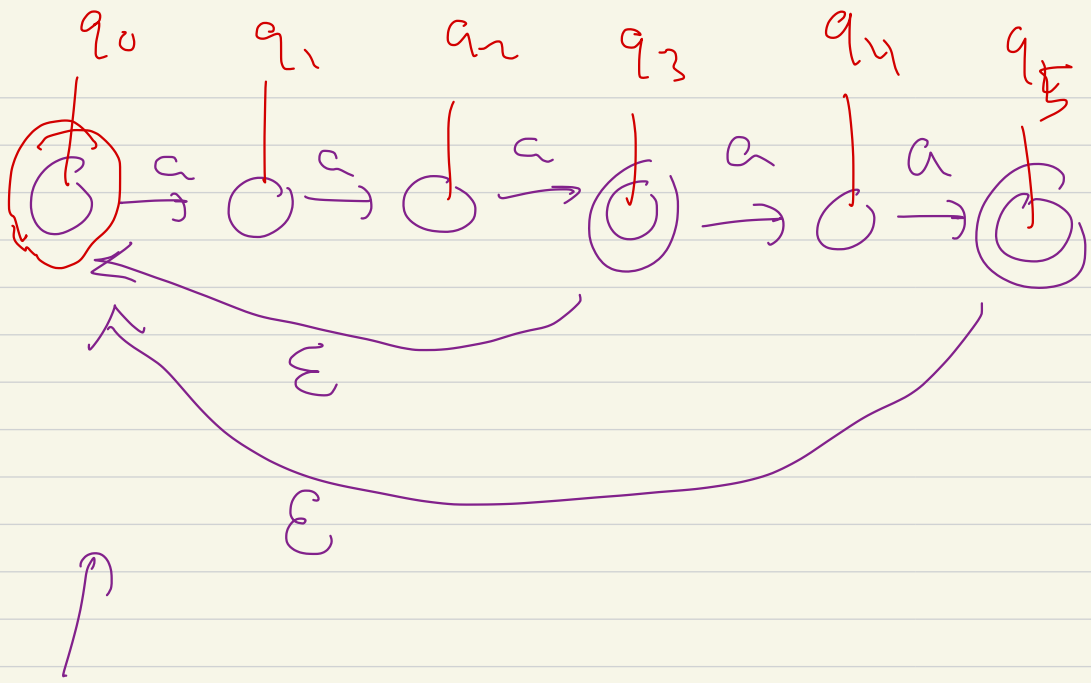
Section 1.2 of [Sip]

Def: An NFA is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ where

$$\delta : Q \times \Sigma_\varepsilon \to \text{Power}(Q)$$

where $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$,

ε means "jump w/o reading a symbol"



this is an NFA that recognizes

$$\{a^3, a^5\}^*$$

Thm: If L is regular, then L^* is ... the set of accepting strings of some NFA ...

So what?

Thm: For each NFA, there is a equivalent DFA, i.e. the NFA & DFA accept the same set of strings

start

a a a a a a a a a a a ... a