CPSC 421/501, Oct 9, 2024

Remark: Is passing from an NFA to DFA really so bad?

<u>Thm</u>: L is regular $\Longleftrightarrow$ L is described by a regular expression.

Start: Non-regular languages:

- $\Sigma = \{a\}$ and eventual periodicity
- Myhill-Nerode

Say you have

NFA: $(Q, \Sigma, \delta, q_0, F)$

$\rightsquigarrow$

DFA: $(\text{Power}(Q), \Sigma, \tilde{\delta}, \tilde{q}_0, \tilde{F})$

$$|\text{Power}(Q)| = 2^{|Q|}$$

$$Q = \{q_0, q_1, \ldots, q_7\}$$

$$|\text{Power}(Q)| = 2^{|Q|} = 2^8 = 256$$

Homework: Things aren't so bad...

in Python, given an

NFA with 20 states

you don't need 2^{20} "states"

to implement the NFA

There is a sneaky way to

implement an NFA with

state set Q so that in

terms of time and space

you don't see a $2^{|Q|}$

(poly in $|Q|$ suffices)

Thm: If L_1, L_2 are regular, then so are L_1^*, $L_1 \cup L_2$, $L_1 \circ L_2$.

Thm: Let $L \subset \Sigma^*$, Σ alphabet.

Then

L is described by a regular expression

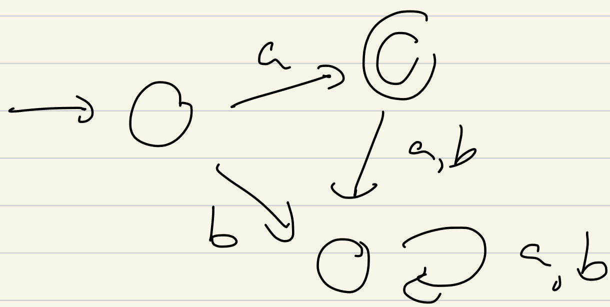
$\iff$

L is regular (reconongized by a DFA or NFA)

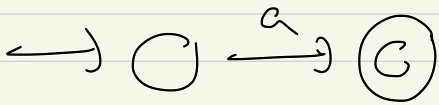
L is described by

$$\left(\left((a) \cup (bbb)\right)^* \cup aaa\right)^* \cup \varepsilon$$

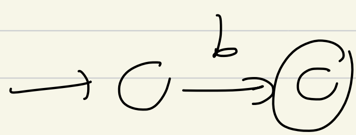
$\{a\}$ has a DEA



$\{a\}$ has an NEA

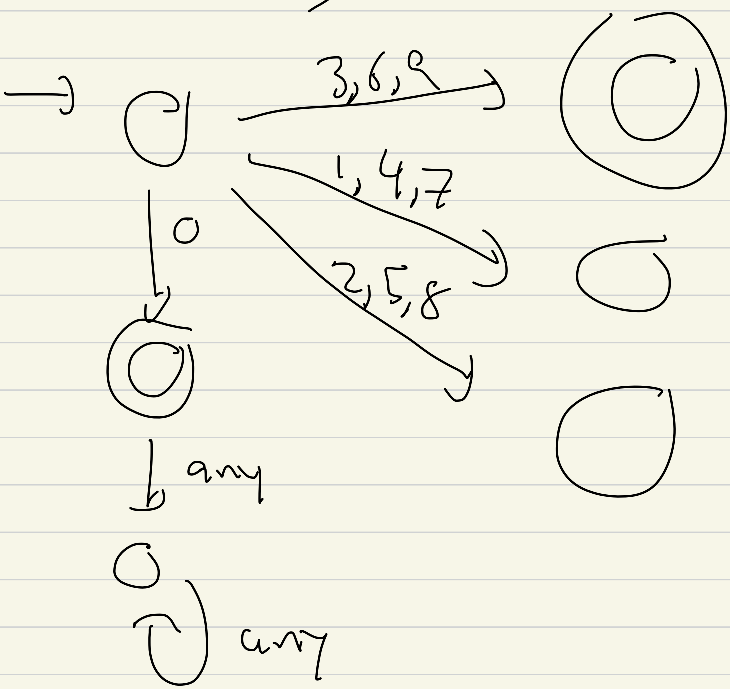


$\{b\}$ has an NEA



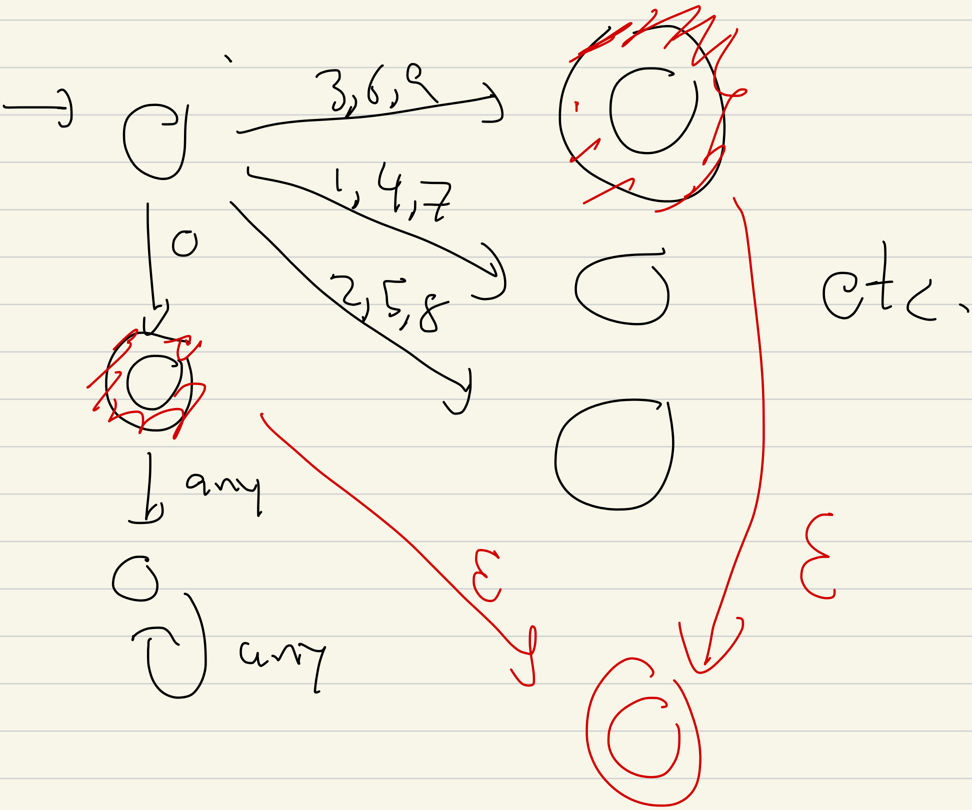
Given a DFA, there is a regular expression that describes the language recognized by the DFA.

DIV-BY-3



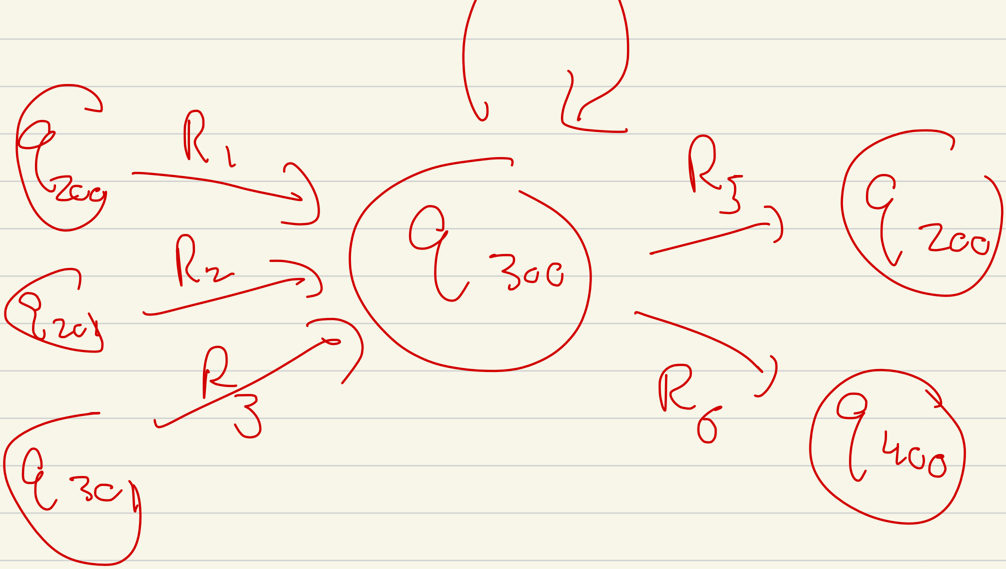
etc.

Step 1: Create equivalent NFA with a single accepting state



(We eliminate each middle state (not initial, not unique final)

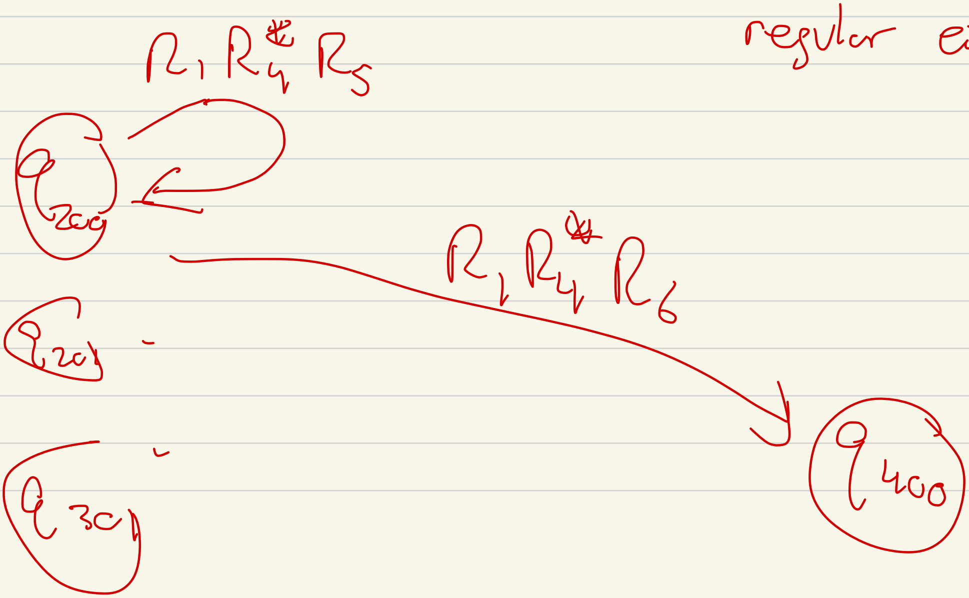
Idea



Generalized NFA

$R_1, \dots, R_5$

regular expressions



Word of caution about

DIV-BY-3 ...

<u>Thm:</u> The language

$$\{ a^{n^2} \mid n \in \mathbb{N} \}$$

$$= \{ a, a^4, a^9, a^{16}, a^{25}, \ldots \}$$

is not regular.

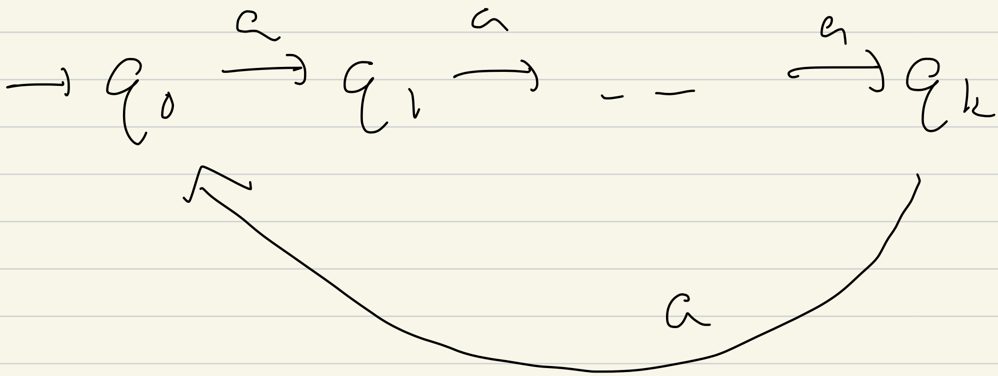
Say $\Sigma = \{a\}$, what does any DFA "look like"?

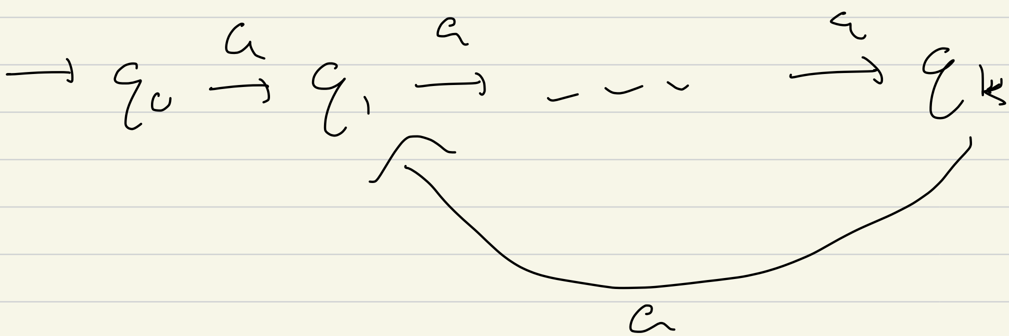
$$\to q_0 \xrightarrow{a} q_1 \xrightarrow{a} q_2 \xrightarrow{a} q_3 \xrightarrow{a} q_4$$

finitely many states $\Longrightarrow$

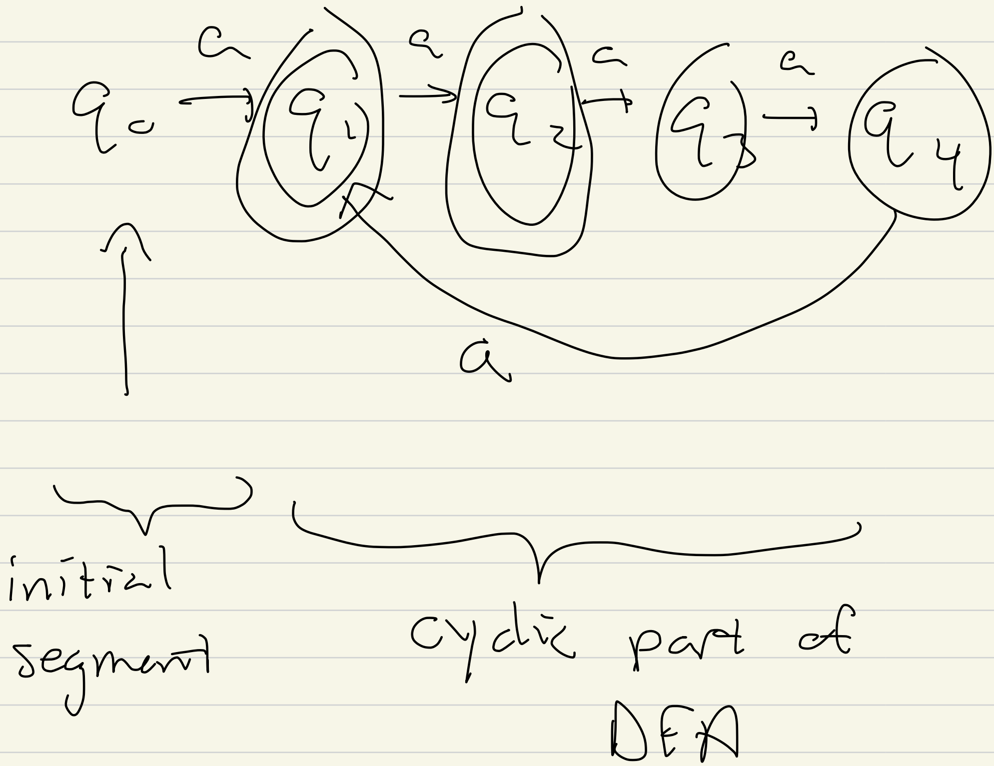
$$\begin{tikzcd}
\arrow[r] & q_0 \arrow[r, "a"] & q_1 \arrow[r, "a"] & \cdots \arrow[r, "a"] & q_k \arrow[l, bend left=60, "a"]
\end{tikzcd}$$

So DFA only visit:





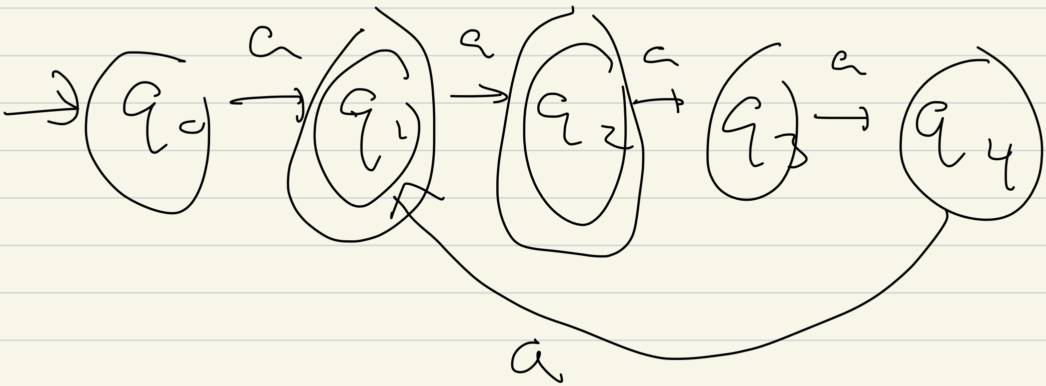
For example



If cycle in the DFA is of length p $(=1,2,3,\ldots)$ then for n sufficiently large,

$$a^n \in L \iff a^{n+p} \in L$$

For example



$$L = \{ a, a^5, a^9, a^{13}, a^{17}, a^{21}, \ldots, a^2, a^6, a^{10}, a^{14}, \ldots \}$$

$$\begin{tikzcd} q_0 \arrow[r, "a"] & q_1 \arrow[r, "a"] & q_2 \arrow[r, "a"] & q_3 \arrow["a", loop right] \end{tikzcd}$$

cycle length, p, can be of length 1.

<u>Def</u> Let $L \subseteq \{a\}^*$. We say that L is <u>eventually periodic</u> if for some p

$$a^n \in L \iff a^{n+p} \in L$$

for n sufficiently large.

The smallest possible $p \in \mathbb{N}$ is called the <u>period</u> of L.

<u>Remark</u>: If

$$L = \{ a^{n_1}, a^{n_2}, a^{n_3}, \dots \}$$

$0 \leq n_1 < n_2 < n_3 < \dots$

if

$$\lim_{i \to \infty} (n_{i+1} - n_i)$$

is infinite, L is not <u>eventually periodic</u>.

<u>Rem</u>: If L is not eventually periodic, then L is not regular

$(\Sigma = \{a\})$