CPSC 421/501 Oct 16, 2024

- Myhill-Nerode theorem:
  - General definition

$$\mathrm{AccFut}_L(S) = \{ S' \mid SS' \in L \}$$

  - Myhill-Nerode Theorem Part I:

$\{\mathrm{AccFut}_L(S)\}_{S \in \Sigma^*}$ gives DFA

lower bound

  - Examples: $\{ a^{n^2} \mid n \in \mathbb{N} \}$,

$\{ a^n b^n \mid n \in \mathbb{N} \}$, $a^* b^*$,

$\{a^{2n} \mid n \in \mathbb{N}\} \cup \{a\}$,

etc.

- Myhill-Nerode Theorem Part II:

  Part I is an equality

- See handout:

  Non-Regular Languages and the Myhill-Nerode theorem

- Not cover 1.4 [Sip] as such ("pumping lemma")

Recall:

$$L = \{a^{2n} \mid n \in \mathbb{N}\} \cup \{a\}$$

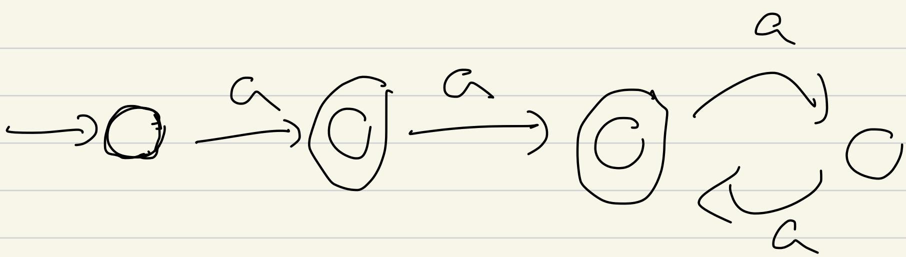
$$\{a^2, a^4, a^6, \dots\} \cup \{a\}$$

Eventual Period of L:

$a^0 \notin L$, $a^2, a^4, a^6, \dots \in L$

$a \in L$, $a^3, a^5, a^7, \dots \notin L$

So eventual period is 2



path length 2

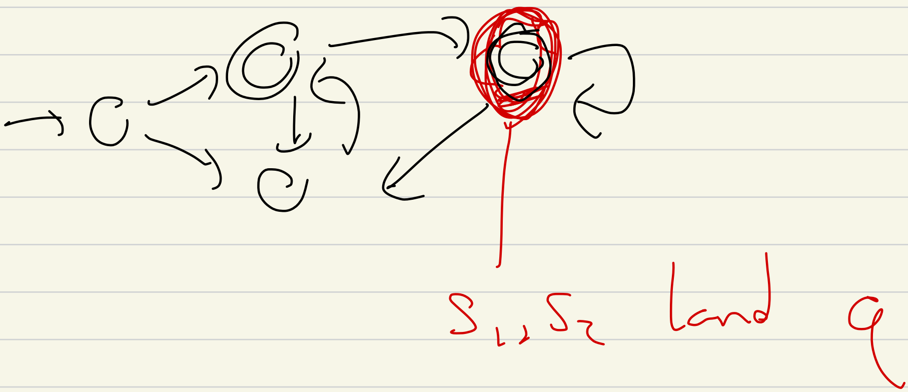
cycle of length 2

states = 4, and we can't recognize L with 3 states or fewer.

$$L = \{ a, a^2, a^4, a^6, a^8, \dots \}$$

$$\mathrm{AccFut}_L(s_1) = \mathrm{AccFut}_L(s_2)$$

Idea: say s_1, s_2 land in the same state of a DFA



then, for any $s' \in \Sigma^*$

$s_1 s'$, $s_2 s'$ both land in the state q for some $q \in Q$.

So....

Define for each $s \in \Sigma^*$

$$\text{AcceptingFutures}_L(s)$$

$$= \{ s' \mid s \cdot s' \in L \}.$$

Then if s_1, s_2 land in same state, then

$$\text{AccFut}_L(s_1) = \text{AccFut}_L(s_2)$$

s	$\text{AccFut}_L(s)$
ε	$L = \{a, a^2, a^4, a^6, a^8, \dots\}$

a	$\{\varepsilon, a^1, a^3, a^5, a^7, \ldots\}$
a^2	$\{\varepsilon, a^2, a^4, a^6, a^8, \ldots\}$
a^3	$\{a, a^3, a^5, a^7, \ldots\}$
a^4	$\{\varepsilon, a^2, a^4, a^6, \ldots\}$
a^5	$\{a, a^3, a^5, \ldots\}$

all give different AccFut

are repeats

Claims: Since

$$\left.\begin{array}{l} \mathrm{AccFut}_L(\varepsilon) \\ \mathrm{AccFut}_L(a) \\ \mathrm{AccFut}_L(a^2) \\ \mathrm{AccFut}_L(a^3) \end{array}\right\}$$ are different subsets of $\Sigma^* = \{a\}^*$

ε, a, a^2, a^3 must land in different states of any DFA recognizing L

(If ε and a^2 landed in same state
$\varepsilon \circ a \in L$ $\quad$ $a^2 \circ a \notin L$ ☹)

Textbook Exercise :

$s_1 \sim s_2$ with resp. to L

s_1 is equivalent to s_2 wrt L

means

$$\mathrm{Accept}_L(s_1) = \mathrm{Accept}_L(s_2)$$

means

$$\forall s' \in \Sigma^*, \quad s_1 s' \in L \Leftrightarrow s_2 s' \in L$$

$$L = \{ a^{n^2} \mid n \in \mathbb{N} \}$$

$$= \{ a, a^4, a^9, a^{16}, a^{25}, \ldots \}$$

s	$\mathrm{Accept}_L(s)$
ε	$\{ a, a^4, a^9, \ldots \}$
a	$\boxed{\{ \varepsilon, a^3, a^8, a^{15}, \ldots \}}$
a^2	$\{ a^2, a^7, a^{14}, \ldots \}$
a^3	$\ldots$
a^4	$\boxed{\{ \varepsilon, a^5, \ldots \}}$

a^9 $\left\{ \varepsilon, a^{16-9} = a^7, \ldots \right\}$

a^{16} $\left\{ \varepsilon, a^{25-16} = a^9, \ldots \right\}$

$\vdots$

a^{m^2} $\left\{ \varepsilon, a^{(m+1)^2 - m^2} = a^{2m+1}, \ldots \right\}$

So

$\mathrm{Accept}_L(a^{m^2})$

$= \left\{ \varepsilon, a^{2m+1}, \ldots \right\}$

these are different for $m = 1, 2, 3, \ldots$

hence

$$\mathrm{Accfut}_L(s)$$

for this L, there are

∞ - many possibilities

$\Rightarrow$ L is not regular

Next time . . . .

$\Sigma = \{a, b\}$ (!)