CPSC 421/501 Oct 18, 2024

- Myhill-Nerode: $\Sigma = \{a, b\}$

- $\{a^n b^n \mid n \in \mathbb{N}\}$

- $(a \cup b)^* a$, i.e., $\Sigma^* a$

- $\Sigma^* a \Sigma^k$, $k = 0, 1, 2, \ldots$

- Next week:

  Start Ch. 3, Turing machines

- Midterm materials to appear

$\Sigma = \{a, b\}$

$L = \{a^n b^n \mid n \in \mathbb{N}\}$

$= \{ab, a^2b^2, a^3b^3, \dots\}$

Claim: L is not regular.

Recall: $s \in \Sigma^*$, $L \subset \Sigma^*$

$\text{AccFut}_L(s) = \{s' \in \Sigma^* \mid s \circ s' \in L\}$.

s	$\mathrm{AccFut}_L(s)$
ε	$\{s' \mid \varepsilon \circ s' \in L\}$ $= \{s' \mid s' \in L\} = L$ $= \{ab, a^2b^2, a^3b^3, \dots\}$
b	$\{s' \mid b \circ s' \in L\} = \emptyset$
b^2	$\{s' \mid b^2 s' \in L\} = \emptyset$
b^3	$\emptyset$
beb	$\emptyset$

s	$\mathrm{Accful}_L(s)$
a	$\{s' \mid a \cdot s' \in L\}$ $= \{b, ab^2, a^2b^3, \ldots\}$
a^2 ⋮	$\{s' \mid a^2 s' \in L\}$ $= \{b^2, ab^3, a^2b^4, \ldots\}$
a^k $(k = 0, 1, 2, \ldots)$	$\{b^k, ab^{k+1}, a^2b^{k+2}, \ldots\}$

1. Note

$$b^k \in \mathrm{Accfut}_L(a^k)$$

but

$$b^k \notin \mathrm{Accfut}_L(a^{k'}), \quad k' \neq k$$

2. Note: the shortest string in $\mathrm{Accfut}_L(a^k)$ is b^k

3. Note: the longest string in $\mathrm{Accfut}_L(a^k)$ is . . .

Also

$$\mathrm{Accfut}(ab) = \{\varepsilon\}$$

$$\mathrm{Accfut}(a^2b) = \{b\}$$

$$\vdots$$

$$\mathrm{Accful}(a^k b) = \{b^{k-1}\}$$

all singletons

(sets of size one), all different

So

$$\{ \mathrm{Accful}_L(s) \}_{s \in \Sigma^*}$$

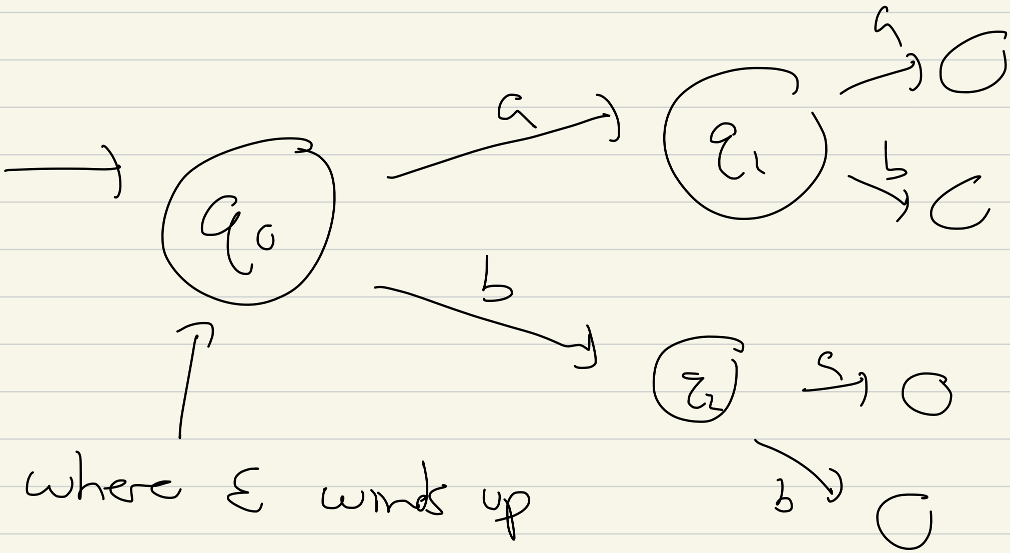
is infinite, i.e.

as $s \in \Sigma^*$,

$$\mathrm{Accful}_L(s)$$

has ∞-many values.

More systematically



where ε winds up

You can merge states

with Myhill-Nerode

$\text{AccFut}_L(s)$

$$L = \Sigma^* a \qquad (\Sigma = \{a, b\})$$

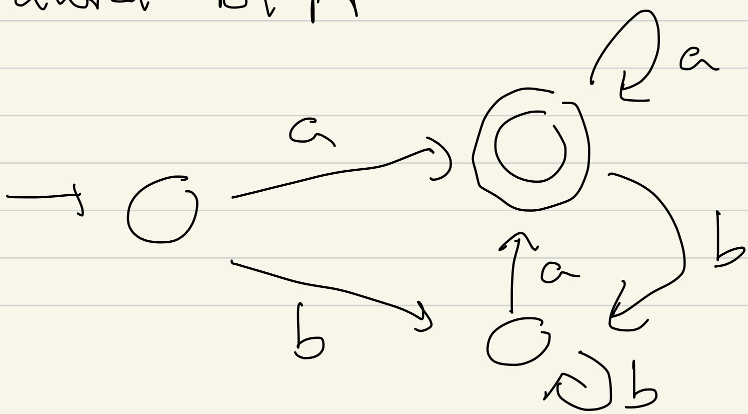
$$(a \cup b)^* a$$

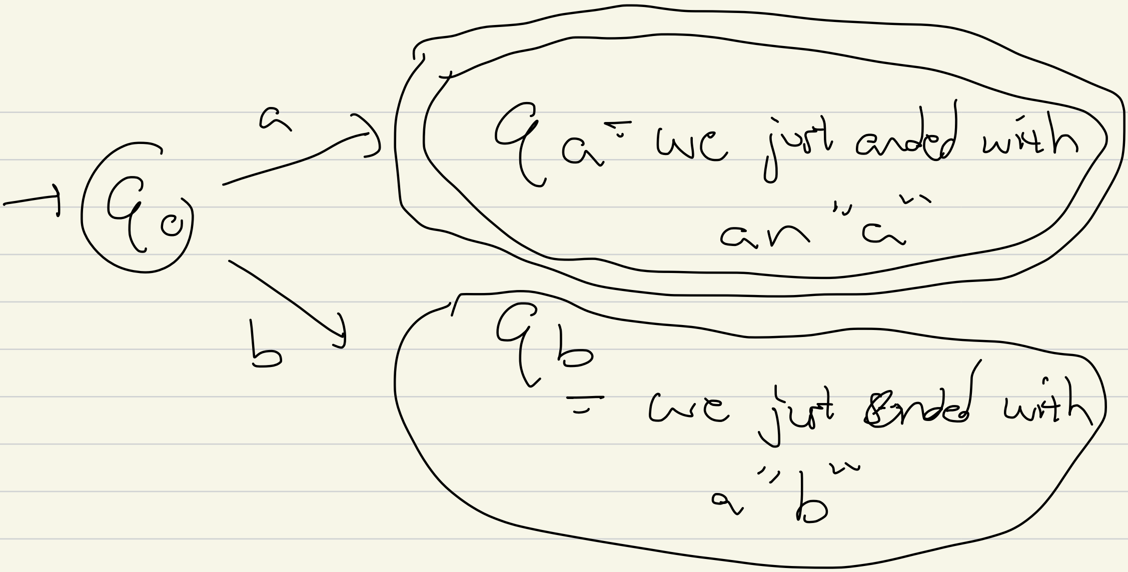
often abbreviated $a \cup b$ as Σ

Regular Expression

$$= \{ s \mid s \text{ ends in } a \}$$

Natural DFA





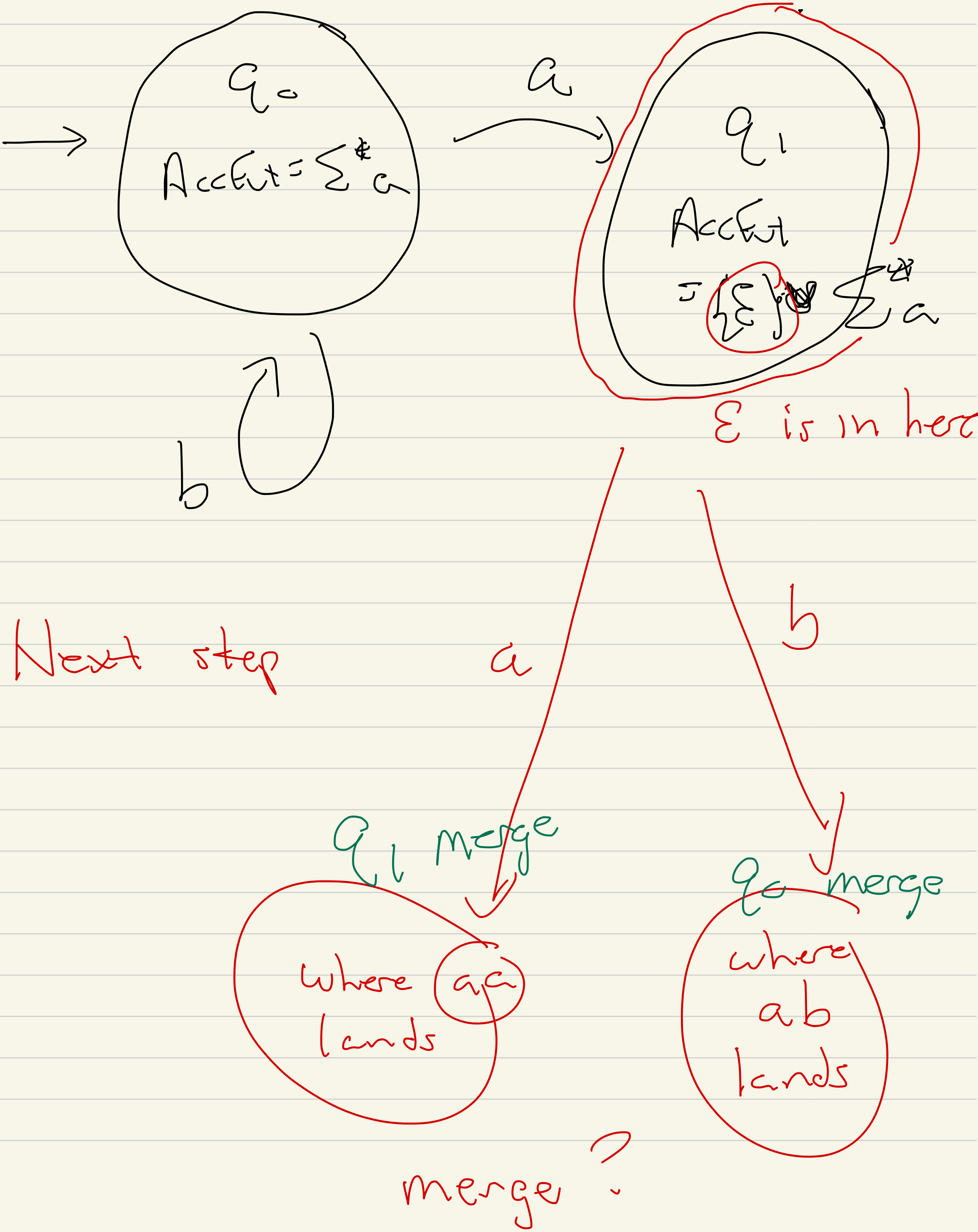
	s	$\text{Accept}_L(s)$
q_0	ε	L described by $\Sigma^* \circ a$
q_1	a	$\{\varepsilon\} \cup (\Sigma^* a)$
q_0	b	$\Sigma^* a$
q_1	aa	$\{\varepsilon\} \cup \Sigma^* a$
q_0	ab	$\Sigma^* a$

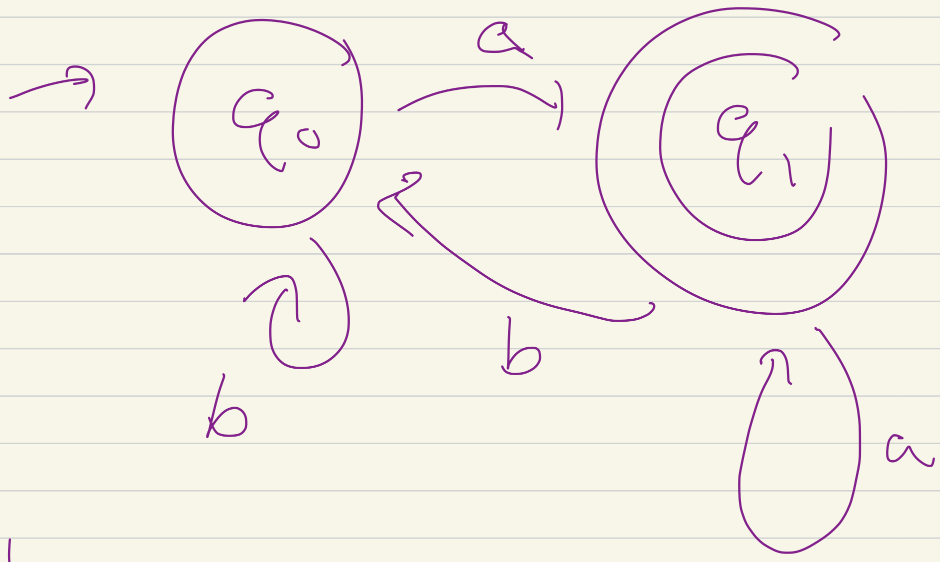
merge

merge

merge

merge


q0
Accept = Σ*a
a
b
q1
Accept = {ε} ∪ Σ*a
ε is in here
Next step
a
b
q1 merge
q0 merge
where aa lands
where ab lands
merge ?



q_0 for ε → anything ending in a

→ anything ending in b

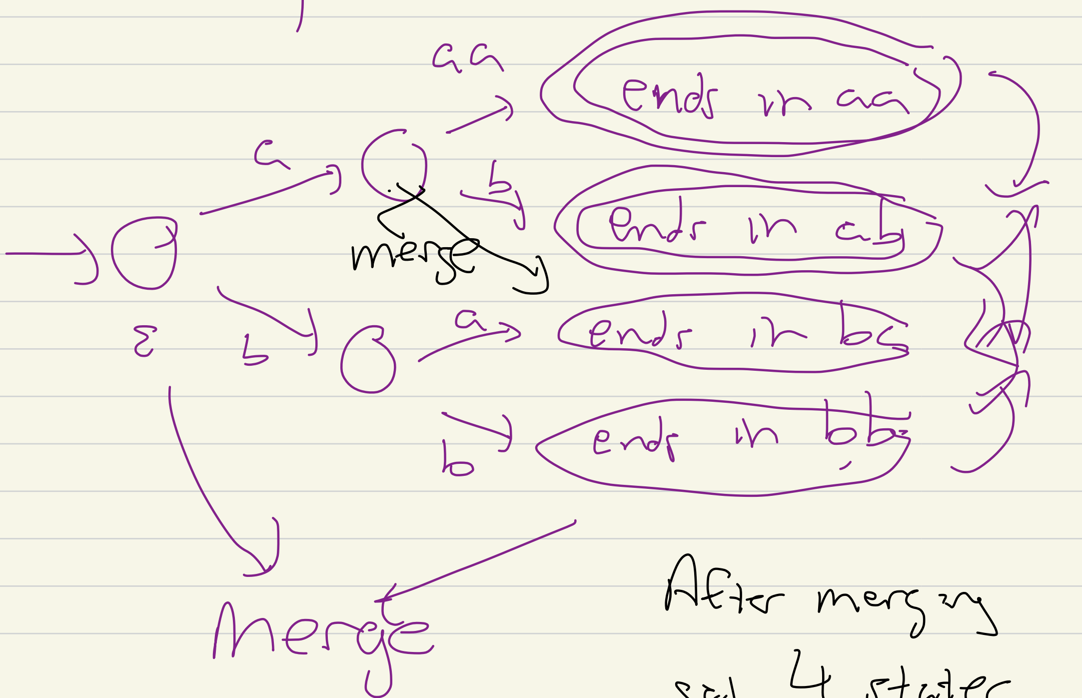
intuitive for Joel

Now!

$L = \Sigma^* a \Sigma$

$= \{$ strings with ≥ 2 symbols, second to last is an a $\}$

Intuitively



After merging get 4 states

What about an NFA ...?

" " $\Sigma^* a \Sigma^k$

$k = 0, 1, 2, 3, \ldots$