

CPSC 421/501

Oct 25

- Goal: Build a universal Turing machine; also build a Turing machine that simulates a Python (or a Duck) program.
- Conceptually useful: multi-tape T.m. Use for:
 - PALINDROME
 - ADDITION, MULTIPLICATION
 - addition, multiplication
 - universal T.m.

Midterm 1 week from today

Cover up to Myhill-Nerode
+ Ch 1 [Sip]

Next week!

Monday Wed Friday

Review
10-15 min

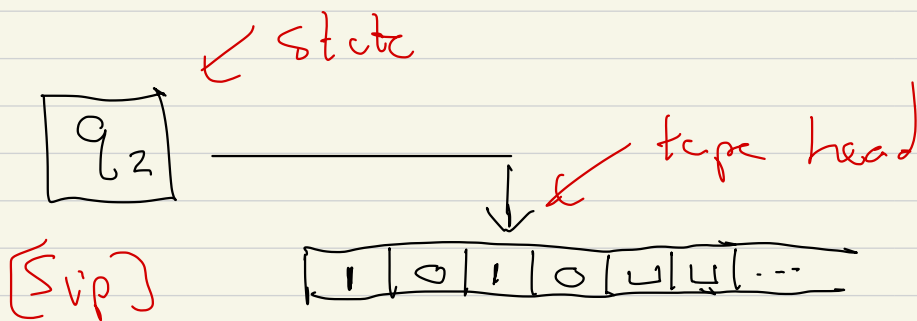
Review
15-20 min

↑
Midterm

Midterm / Exam Review

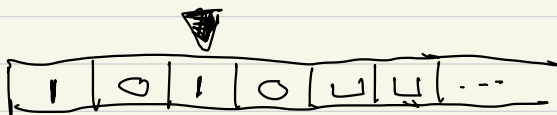
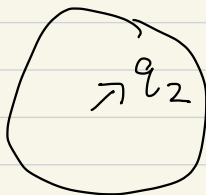
webpage

Note: T.m. configurations can be equivalently written as
 (see Figure 3.4 in [Sip])



OR

(I will write it) (Classical Retro)



OR

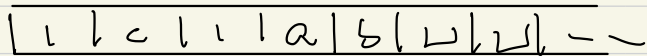
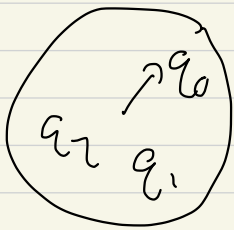
tape head

1 0 1 q_2 0

[Sip]

(most concise)

What is a multi-tape Turing machine?



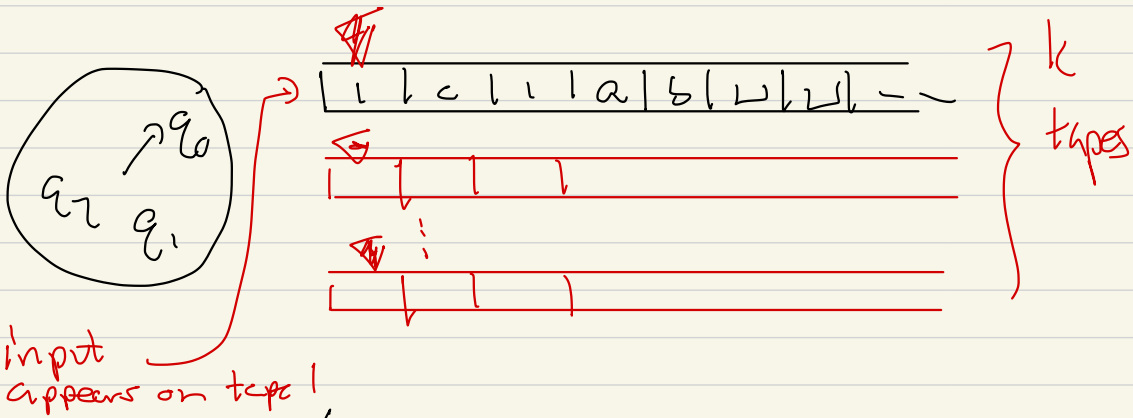
set of tape symbols

$$T.m. = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$$

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

Γ contains Σ , L , and maybe other symbols

What is a multi-tape Turing machine?
 fix $k \in \mathbb{N}$



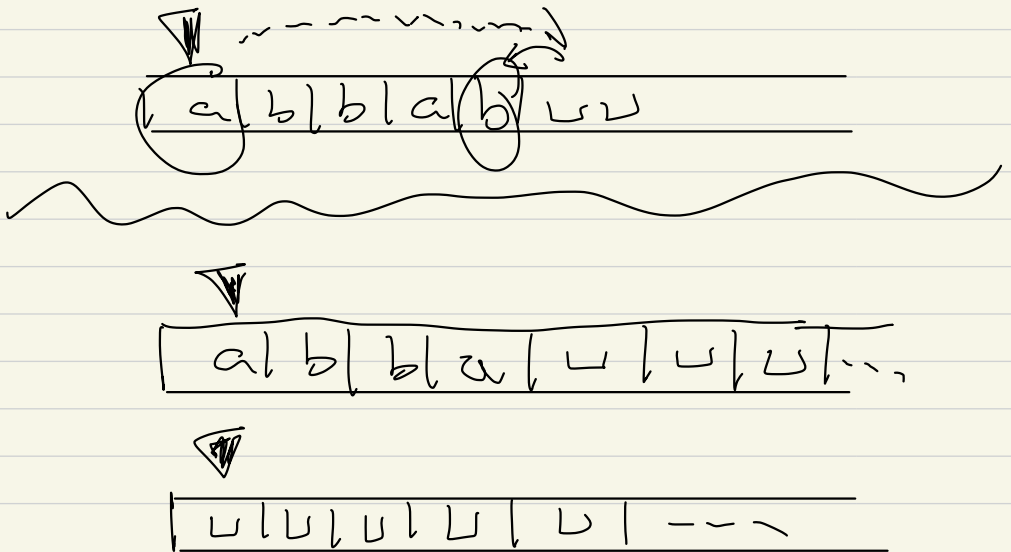
$$T.M. = (Q, \Sigma, \Gamma^k, \delta, q_0, q_{acc}, q_{rej})$$

$$\delta : Q \times \Gamma^k \rightarrow Q \times \Gamma^k \times \{L, R, S\}^k$$

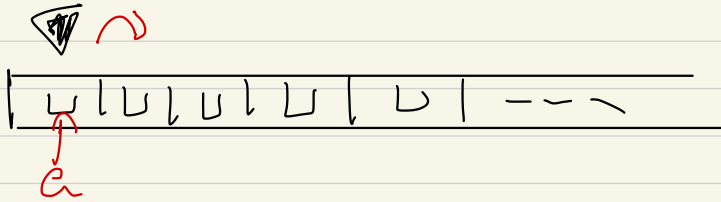
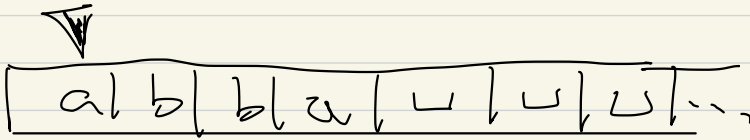
L → move left
 R → " right
 S → stay

Remark: In a 1-tape T.m.,
we can always forgo S -stay

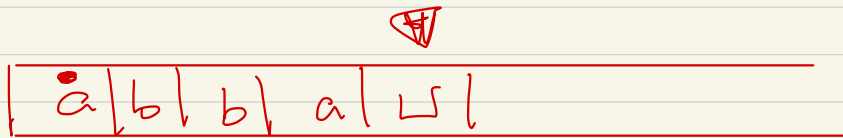
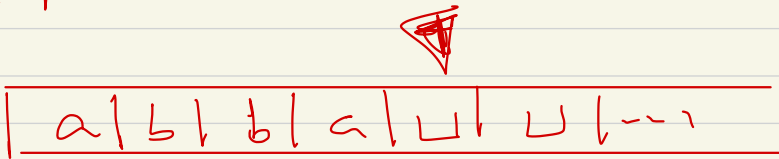
2-tape: recognize PALINDROME
(of w , built 1-tape "machine;
on input size n , it took $O(n^2)$
time)



step 1: move both tape head



Phase 1



$q_0 = \text{initial}$, moves right + copies
the word

$\Sigma = \{a, b\}$, $\Gamma = \{a, b, \sqcup, \overset{\circ}{a}, \overset{\circ}{b}, \dots, ?\}$

\dot{a}, \dot{b} mean these are a, b
respectively, but \ddot{a}, \ddot{b} at
cell l .

$$\delta(q_0, (a, \sqcup)) = (q_1, (a, \dot{a}), (R, R))$$

$$\delta(q_0, (b, \sqcup)) = (q_1, (b, \dot{b}), (R, R))$$

$$\delta(q_0, (\sqcup, \sqcup)) = (q_{acc}, \text{irrelevant for the rest})$$

(special case of ϵ as input)

q_0 = initial state (write a \circ)

q_1 = we still write input to

tape Z , but no longer
at cell l , we've already
written a \circ over a, b in

Phase 1



a | b | b | a | U | U | ...



• a | b | b | a | U |

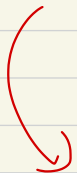
→ → →
q₀ q₁ ...

$$\delta(q_1, (a, U)) = (q_1, (a, a), (R, R))$$

e.g.



a | a | ... | U U U ...



• a | U U U



a | b | ...



a | U U ...

$$\delta(q_1, (b, \sqcup)) = (q_1, (\sqcup, b), (R, R))$$

$$\delta(q_1, (\sqcup, \sqcup)) = (q_2, (\sqcup, \sqcup), (S, L))$$

q_2 means we move tape head 2
to cell 1 (on tape 2)

$$\delta(q_2, (\sqcup, a)) = (q_2, (\sqcup, a), (S, L))$$

$$\delta(q_2, (\sqcup, b)) = (q_2, (\sqcup, b), (S, L))$$

$$\delta(q_2, (\sqcup, \dot{a}))$$

$$\delta(q_2, (\sqcup, \dot{b}))$$

Phase 2

90

a|b|b|a|U|U|

← ← ←

a|b|b|a|U|

Phase ← a

or a|U

a

a