CPSC 421/501

- Multi-tape $\leq$ One-tape

- Multi-tape:

Function vs Decision
- Addition / ADDITION
- Multiplication / MULTIPLICATION

- Decide

A_{DFA} = Acceptance for DFA's

Recognize: A_{TM} = " " TM

- Take questions ≈ 10 min

- " " on Wednesday

≈ 20 min

- Midterm: Friday, Room 6

- N.B.: Supplemental Practice for the 2024 midterm now has 7th question on countability.

- All HW good sample midterm problems...

You can have 1 (2-sided) Letter (8.5'' x 11'') sheet of notes, handwritten / computer / reduced

— Level of detail { See solution to HW / Go to office hours

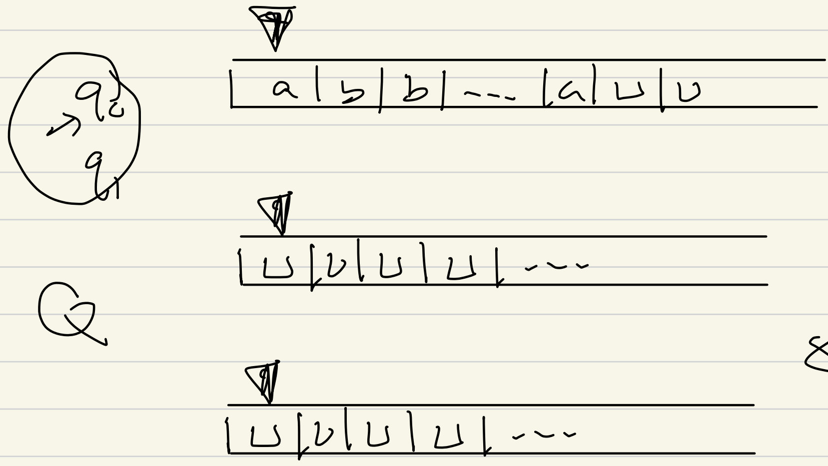
As we delve into multi-tape Turing machines, and more sophisticated algorithms ...

- formal description (i.e. $Q, \Sigma, \Gamma, \delta$ ...)

to

"higher-level" ...

First abstraction: multi-tape Turing machine



tape 1

(input appears here)

tape 2

tape 3

(some finite # of tapes, k,

$$\delta: Q \times \Gamma^k \longrightarrow Q \times \Gamma^k \times \{L, R, S\}^k$$

stay

$$\Gamma = \Sigma \cup \{\sqcup\} \cup \underbrace{\left\{\begin{matrix}\text{any others} \\ \text{you wish} \\ \text{to add}\end{matrix}\right\}}_{\text{finitely many}}$$

Why --

$$\begin{array}{r} {}^{1} \\ 138 \\ +\ 238 \\ \hline 376 \end{array}$$

$\leftarrow$ carry $\leftarrow$ line

$\leftarrow$ one line

$\leftarrow$ another line

$\leftarrow$ another line

four lines $\rightsquigarrow$ 4-tape machine

$$\text{ADDITION} = \left\{ x\#y\#z \;\middle|\; \begin{array}{l} x,y,z \in \{0,1,\dots,9\}^* \\ \text{s.t. } x+y=z \end{array} \right\}$$

Given x, y, z $\Rightarrow$ yes $x+y=z$

no $x+y \neq z$

addition: takes

$$\boxed{1\,2\,5\,1\,7\,\#\,3\,9\,1\,6\,\sqcup\,\sqcup\,\sqcup}$$

returns their sum

function

$$x, y \longrightarrow \boxed{\text{Turing machine}} \xrightarrow{\text{always accept}} \text{write } x+y \text{ on same tape}$$

$$\boxed{\text{Turing machines that compute functions}}$$

ADDITION $\subset \Sigma^*$

$$\Sigma = \{0, 1, \ldots, 9\} \cup \{\#\}$$

MULTIPLICATION / multiplication:

big

$$\begin{array}{r} 3265 \\ \times \quad 121 \\ \hline \end{array}$$

many lines

$$\begin{array}{r} 3265 \\ 65300 \\ +\ 326500 \\ \hline \end{array}$$

finite #

OK

$$\begin{array}{r} {}^{1\,1} \\ 3265 \\ \times \quad\ 2 \\ \hline 6530 \end{array}$$

End of New Content.

Brief Q's about practice

midterm...

$$\{\text{all maps } \mathbb{N} \to \{1,2\}\}$$

$\updownarrow$ 1-1 bijection

to

$$\text{Power}(\mathbb{N})$$

$$f : \mathbb{N} \to \{1,2\} \to \{\text{yes}, \text{no}\}$$

$$1 \mapsto \text{yes}$$
$$2 \mapsto \text{no}$$

So given

$$f : \mathbb{N} \longrightarrow \{yes, no\}$$

$$\{ n \in \mathbb{N} \mid f(n) = yes \}$$

$$\{ \cancel{1}, 2, 3, \cancel{4}, 5, \ldots \}$$

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 & \ldots \\ \downarrow & \downarrow & \downarrow & \downarrow & \\ no & yes & yes & no & \end{array}$$

$\tilde{f}$:	Subset of $\mathbb{N}$ corresp to $\tilde{f}$
$1 \longmapsto$ no	
$2 \longmapsto$ yes	2
$3 \longmapsto$ no	
$4 \longmapsto$ yes	4
$5 \longmapsto$ yes	5
	$\vdots$

$\text{Power}(\mathbb{N})$, $\text{Power}(\Sigma^*)$

are uncountable

$$S \xrightarrow{\text{injection}} T, \quad |T| \geq |S|$$

$$A \xrightarrow{\text{surjection}} B \quad |A| \geq |B|$$

So

$$\bigcirc \xrightarrow{\text{surjection}} \text{Power}(\mathbb{N})$$

(uncountable

must be uncountable

$$\mathrm{Power}(\mathbb{N}) \xrightarrow{\text{injective}} \bigcirc$$

$$\Uparrow$$

uncountable