CPSC 421/501 Nov 18, 2024

- Some Boolean algebra: any function has a CNF, DNF

- To any Boolean formula φ in CNF form, there is a formula $\tilde{\varphi}$ in 3CNF, s.t.
  - $|\langle\tilde{\varphi}\rangle| \leq O(|\langle\varphi\rangle|)$

    small constant
  - $\varphi \in SAT \iff \tilde{\varphi} \in SAT$

- Carry on with Cook-Levin

- Next HW 9, to be assigned today.

- Midterms coming back on Wednesday (or Thursday)

- No new material on Dec 4 or 6

Final on Dec 10

Boolean algebra:

Boolean function on n-variables:

we mean

$$f : \{F, T\}^n \to \{F, T\}$$

another world

$$f : \{0, 1\}^n \to \{0, 1\}$$

$0 \leftrightarrow$ false $= F$

$1 \leftrightarrow$ true $= T$

A function $f = f(n) : \mathbb{N} \to \mathbb{R}$

$\mathbb{Z}_{\geq 0} \to \mathbb{R}$

is said to belong to

$O(g(n))$, where $g : \mathbb{Z}_{\geq 0} \to \mathbb{R}$

if there exists n_0, C s.t.

$$\boxed{\begin{gathered}\text{for all } n \geq n_0, \\ |f(n)| \leq C\, g(n)\end{gathered}}$$

e.g.,

$$\boxed{3n^2+n = O(n^2)}$$

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something = O (something else)

ireal values

something $\leq$ C (something else)

if something $\geq n_0$

$|\langle \hat{\varphi} \rangle| \leq f(|\langle \varphi \rangle|)$ some f

$f(n) \in O(n)$

Also defined

$$f(n) = o(g(n))$$

$$f(n) = 10^{10^{10^{10}}} \cdot n^2$$

then

$$f(n) \in O(n^2)$$

$$\in \underbrace{OO}_{\text{uh-oh}}(n^2)$$

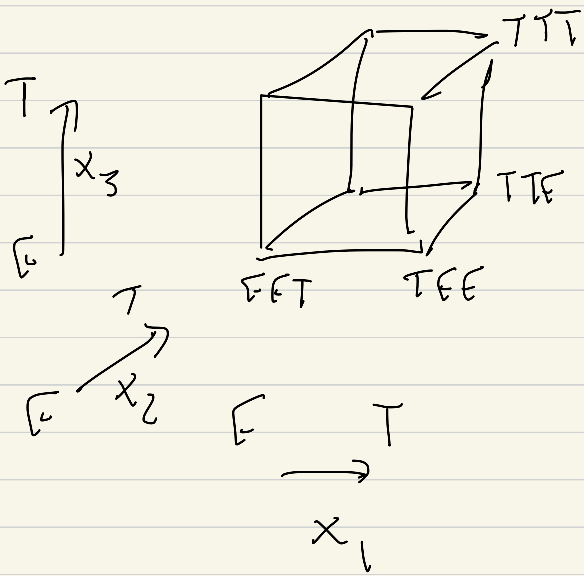
Udi Manber

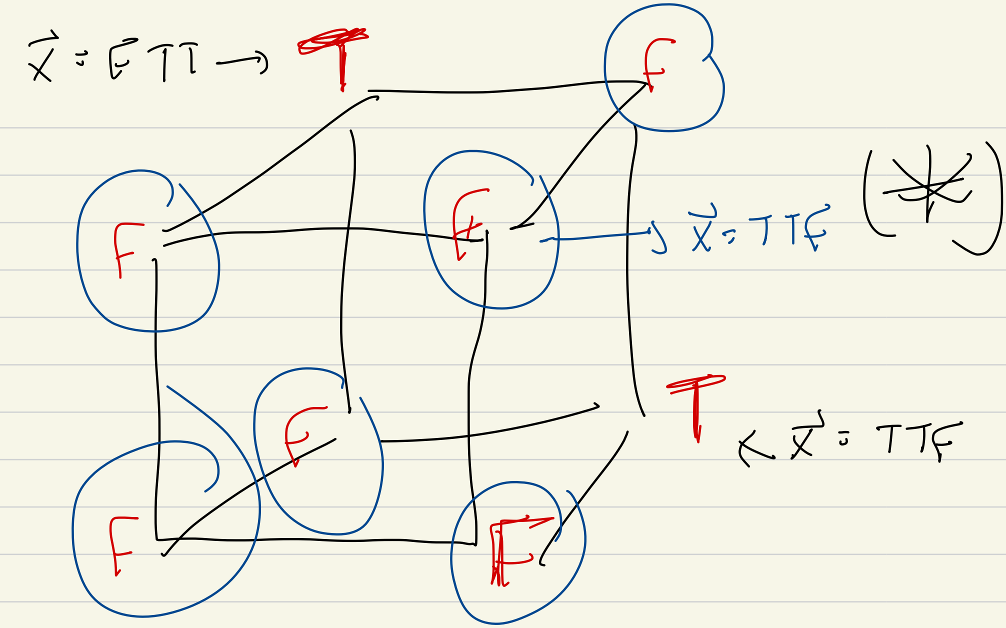
CNF:

$$f = f(\vec{x}) = f(x_1, \ldots, x_n)$$

$$f : \{F, T\}^n \to \{F, T\}$$

$n = 3$





$$f : \{F, T\}^3 \to \{F, T\}$$

$$f(T, T, F) = T$$

$$f(F, T, T) = T$$

$$f \text{ otherwise } = F$$

f is true if $x_1 = T, x_2 = T, x_3 = F$

OR

if $x_1 = F, x_2 = T, x_3 = T$

otherwise not

formula

$$(X_1 \text{ AND } X_2 \text{ AND } \neg X_3)$$

OR

$$(\neg X_1 \text{ AND } X_2 \text{ AND } X_3)$$

canonical
disjunctive normal form

$n = 3$, # vars in each clause $= n$

AND $\wedge$ conjuction

OR $\vee$ disjuction

DNF

( ) or ( ) or ... or ( )

each clause

$(x_{i_1}$ AND $\neg x_{i_2}$ AND x_{i_3} AND ... $)$

$x_1, \ldots, x_n$ variable

<u>literal</u> $x_1, \neg x_1, x_2, \neg x_2, \ldots, x_n, \neg x_n$

DNF is <u>any</u> disjunction (OR, $\vee$) of clauses, each of which is a conjunction (AND, $\wedge$) of literals, $x_1, \neg x_1, \ldots, x_n, \neg x_n$.

Possible exercise: if φ Boolean formula is there $\hat{\varphi}$ s.t. $\varphi \in SAT \iff \hat{\varphi} \in SAT$ and $|\langle \hat{\varphi} \rangle| = O(|\langle \varphi \rangle|)$ William Chu

φ ~~[illegible]~~ $\in$ $\hat{\varphi}$ in 3CNF

CNF = a DNF with AND's OR's interchanged

3CNF = CNF where all clauses have at most 3 literals (or exactly 3 literals)

$$SAT = \{ \langle \varphi \rangle \mid \varphi \text{ is a satisfiable Boolean formula} \}$$

$$3SAT = \{ \text{''} \mid \text{'' '' '' '' '' '' and } \varphi \text{ is in 3CNF} \}$$

f in (canonical) CNF

in example (*)

i.e.

$f = (\neg x_1 \text{ OR } \neg x_2 \text{ OR } \neg x_3)$ AND

$(\neg x_1 \text{ OR } \neg x_2 \text{ OR } x_3)$ AND

$(\qquad\qquad)$ AND

$(\qquad\qquad)$ AND

$(\qquad\qquad)$ AND

$(\qquad\qquad)$

If $\neg f$ in DNF

$f \rightsquigarrow$ CNF

$$\neg f = (x_1 \text{ AND } x_2) \text{ or } (x_3 \text{ AND } \neg x_4)$$

$$f = (\neg x_1 \text{ or } \neg x_2) \text{ AND } (\neg x_3 \text{ or } x_4)$$