CPSC 421/501, Dec 6, 2024

Office hours:

Today, Friday, Dec 6

11 - 11:50 am (in class)

Office hours on Monday, Dec 9

10am - 1pm
2:45 - 4:15 pm
} See piazza and/or website for locations

Give a T.m.: input $x \in \{0,1\}^*$

accepts x if (1) x begins in 0,

and

(2) x has exactly 2 more 0's

than 1's,

and otherwise rejects x.

Meta:

$$L = \{ x \mid x \text{ satisfies (1) and (2) above} \}$$

Intuition! To recognize L, you need to keep track of how many 1's and 0's.

$$L \cap 0^* 1^*$$

$$= \{ 0^{2+n} 1^n \mid n = 0, 1, \ldots \}$$

= non-regular

Regular $\cap$ Regular is regular.

L is non-regular

If L above were regular, then

$$L \cap 0^* 1^*$$

would be a regular $\cap$ regular, hence regular. But, by Myhill-Nerode, we know

$$\{ 0^n 1^n \mid n \in \left\{ \begin{matrix} \mathbb{N} \\ \mathbb{Z} \end{matrix} \right\} \}$$

$$\{ 0^n 1^{n+3} \mid \ldots \}$$

$$\vdots$$

If L_1, L_2 are regular :

(1) $L_1 \cup L_2$ is regular (by NFA)

(2) L_1^{comp} is regular --~

$$L_1 \cap L_2 = \left(L_1^{comp} \cup L_2^{comp} \right)^{comp}$$

DFA #1 $(Q, \Sigma, \delta, q_0, F)$

DFA #2 $(Q', \Sigma, \delta', q_0', F')$

↳ run in parallel :

state set $Q \times Q'$

$\vdots$

L_1 DFA $(Q, \Sigma, \delta, q_0, F)$

L_1^{comp} $(Q, \Sigma, \delta, q_0, Q \setminus F)$

exchange accepting & rejecting

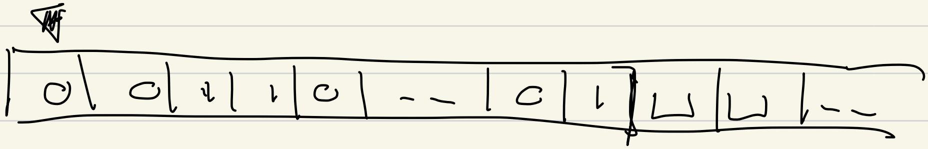
L: start with 0, 2 more 0's than 1's

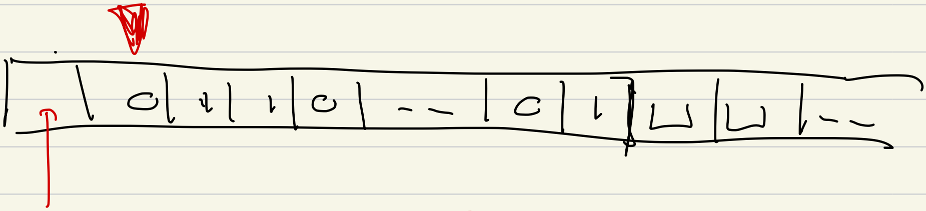
$$T_m = (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$$

$$\delta(q_0, \sqcup) = \delta(q_0, 1) = q_{rej}$$

since we reject any string beginning with something other than 0.

this is an explanation





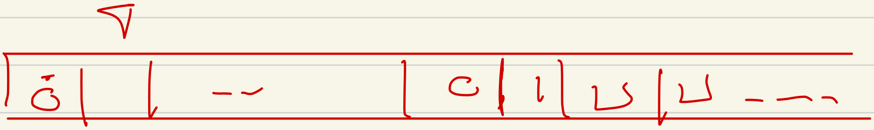
$\overset{\circ}{0}$, start, say $\overset{\circ}{0}$

$$\delta(q_0, x) = (q_{rej}, \text{any}, \text{any})$$

for $x = 1, \sqcup, \dot{0}$

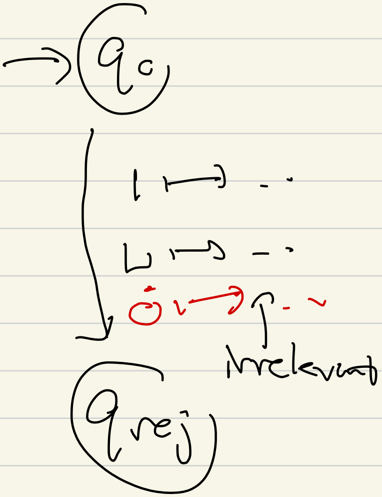
or

$x \neq 0$



Phase: Move until see a 1, mark the 1, maybe $\dot{1}$, then

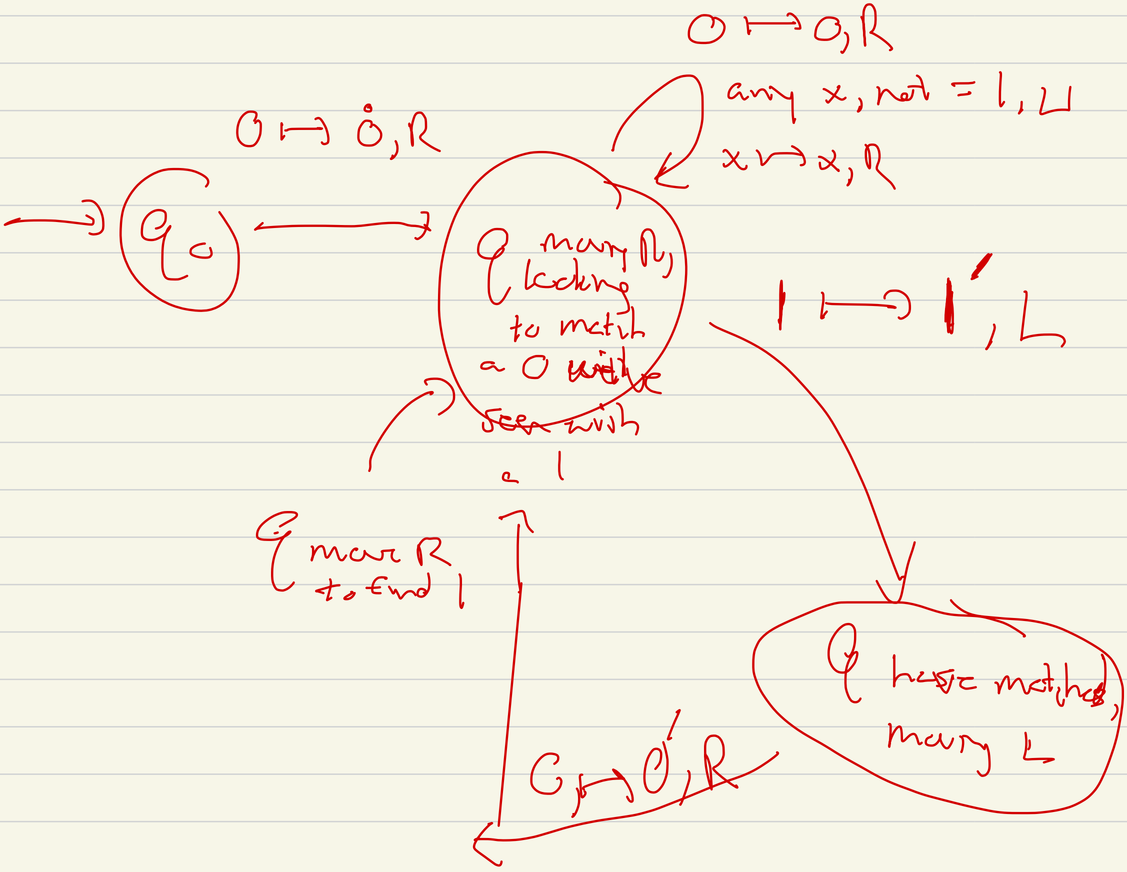
$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$$



2 state:

$$\delta : Q \times \Gamma^2 \longrightarrow Q \times \Gamma^2 \times \{L, R, S\}^2$$

We will use the convention of writing a diagram where we don't write q_{rej}, and any arrow not shown moves to reject.


0 ↦ 0, R
any x, not = 1, ⊔
x ↦ x, R
0 ↦ 0̇, R
q_0
q move R, looking to match a 0 ~~with~~ the ~~seen~~ with a 1
1 ↦ 1', L
q have matched, move L
0 ↦ 0', R
q move R to find 1