SUPPLEMENTAL T/F QUESTIONS, CPSC 421/501, FALL 2024

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In 2024, we summarized what Chapter 9 tells us about P versus NP in a single class (Dec 2); some of this relates to material we covered in Chapter 8, and part relates to oracle Turing machines in Chapter 3, which we did not cover. You should know how to answer the following true/false questions.

(1) Mark the following questions as true or false. The phrase "as of today," refers to published results as of today that are commonly accepted in the theoretical computer science community.

(a) As of today, December 4, 2024, we don't know whether or not $\mathrm{P} = \mathrm{NP}$.
(b) As of today, December 4, 2024, we don't know whether or not there is a polynomial time reduction 3COLOUR $\le_{\mathrm{P}}$ 2COLOUR.
(c) As of today, December 4, 2024, we don't know whether or not there is a polynomial time reduction 2COLOUR $\le_{\mathrm{P}}$ 3COLOUR.

(d) If A is any PSPACE-complete language, then $\mathrm{P}^A \neq \mathrm{NP}^A$.
(e) If A is any PSPACE-complete language, then $\mathrm{P}^A = \mathrm{NP}^A$.
(f) There exists a language, B, such that $\mathrm{P}^B = \mathrm{NP}^B$.

(g) If you believe that you have proven that $\mathrm{P} \neq \mathrm{NP}$, and your supposed proof also shows that $\mathrm{P}^A \neq \mathrm{NP}^A$ for any language A, then your proof contains an error.
(h) If you believe that you have proven that $\mathrm{P} = \mathrm{NP}$, and your supposed proof also shows that $\mathrm{P}^A = \mathrm{NP}^A$ for any language A, then your proof contains an error.
(i) If you believe that you have proven that for all positive $x, y, z \in \mathbb{N}$ we have $x^3 + y^3 \neq z^3$, and your supposed proof also shows that the same is true for all positive $x, y, z \in \mathbb{R}$, then your proof contains an error.

(j) If $\mathrm{P} = \mathrm{NP}$, then there are polynomial size circuits to determine membership in 3COLOUR.
(k) A Boolean formula can be viewed as a special case of a Boolean circuit.
(l) A Boolean circuit can be viewed as a special case of a Boolean formula.
(m) In 1961, Subbotovskaya proved that the XOR of n Boolean variables (i.e., the Parity function of n Boolean variables) requires formula size $\ge cn^{3/2}$ for some $c > 0$.

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(n) As of today, December 4, 2024, no one has identified a language in NP that provably requires formula size $\geq n^{3.000001}$ on inputs of length n with n sufficiently large.

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