CPSC 531F

Intro to TDA (topological data analysis).

Topics:

(1) Intro to algebraic topology that is most relevant to TDA, namely simplicial and sigular homology, and Hodge Laplacians in (co)homology.

(2) Applications.

Admin:

- There's no single textbook:

  we're trying to give the minimum amount of algebraic topology needed to understand papers in TDA

- Class notes mostly self-contained, but I'll refer to various textbooks, articles available for free to UBC students.

- Grades:

  ≥ 95 I'll take you as a student. I'll write a nice letter of recom.

  ≥ 90 :... very strong

≥ 85 you're prob not specializing in TDA, Comp Sci Theory w/o more background courses

≥ 80 satisfied the expectations of a grad student

Grading based on

- Homework Problems
- Perhaps a project

Technically (in the past), courses w/o exams had to end on the last day of classes

First 2 weeks should give you an idea of what's expected...

=

I've written some notes to myself, handouts

- Intro to TDA, Point Clouds, and Point-Set Topology ...

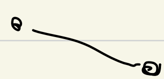
Section 1 overview

Section 2 — details

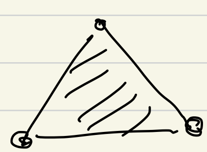
Idea: A <u>simplex</u> in $\mathbb{R}^N$ is the convex hull of a finite set of points in general position:

0-simplex
0-face

1-simplex
1-face

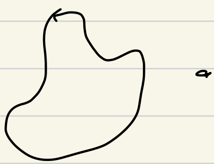


2-simplex
2-face

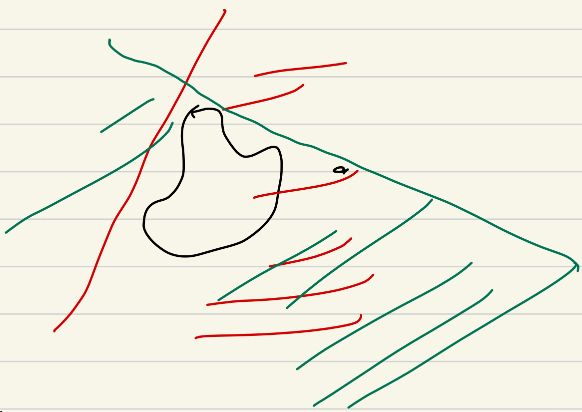


The convex hull of $S \subset \mathbb{R}^N$ is the intersection of all convex sets in $\mathbb{R}^N$ containing S.

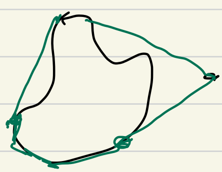
S



convex hull



intersect



If $S = \{\vec{s}_1, \dots, \vec{s}_d\} \subset \mathbb{R}^N$

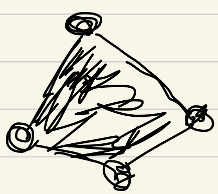
convex hull of $(\vec{s}_1, \dots, \vec{s}_d)$

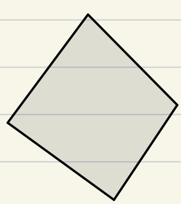
$=$

$\{ \alpha_1 \vec{s}_1 + \alpha_2 \vec{s}_2 + \dots + \alpha_d \vec{s}_d \mid$

real $\alpha_1, \dots, \alpha_d \geq 0,$

$\alpha_1 + \dots + \alpha_d = 1 \}$

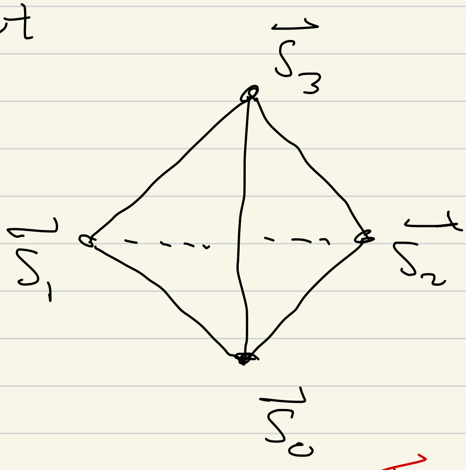




Not general position:

in $\mathbb{R}^2$

but



in $\mathbb{R}^3$

is

should span a 3-dim subspace of $\mathbb{R}^N$

$$S = \{\underbrace{\vec{s}_0, \ldots, \vec{s}_d}_{d+1 \text{ points}}\} \subset \mathbb{R}^N$$ is in

<u>general position</u> if:

(1) S does not lie on any

$d-1$ dimensional affine subspace

" " plane in $\mathbb{R}^N$

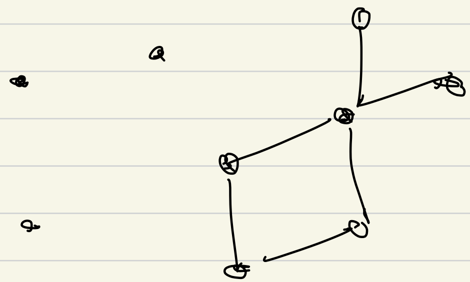
(2) $\vec{s}_1 - \vec{s}_0, \ldots, \vec{s}_d - \vec{s}_0$ these

span a d dimensional subspace of $\mathbb{R}^N$

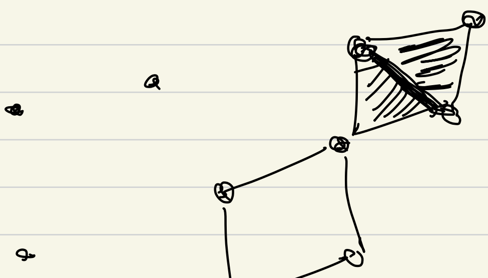
(3) the vectors $\vec{s}_1 - \vec{s}_0, \ldots, \vec{s}_d - \vec{s}_0$

are linearly independents

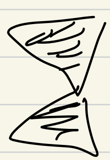
A simplicial complex in $\mathbb{R}^N$ is a (finite) set of simplices!



"graph"



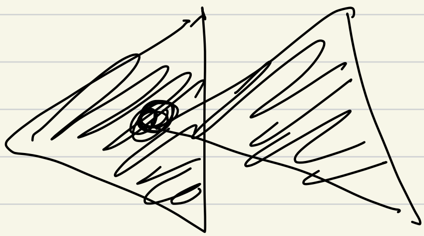
2-dim simplex

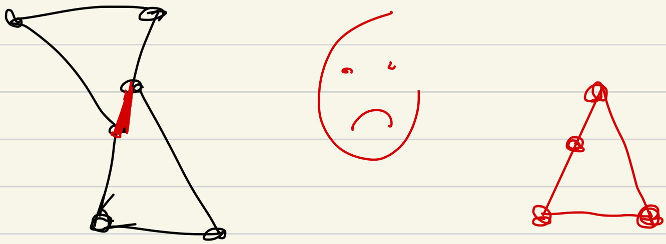


3 vertices $\to$ 2-dim simplex,

2-dim faces

but not





If you have a simplex :

$S = \text{Convex Hull}(\vec{s}_0, \dots, \vec{s}_d)$, a

<u>face</u> of S (i-dimensional face)

is $\text{Convex Hull}(\vec{s}_0{}', \dots, \vec{s}_i{}')$

where $\{\vec{s}_0{}', \dots, \vec{s}_i{}'\} \subset \{\vec{s}_0, \dots, \vec{s}_d\}$

Def A collection of simplices K in $\mathbb{R}^N$ is a simplicial complex if

(1) $S \in K$, S' is a face of S, then $S' \in K$

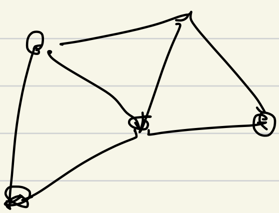
(2) $S_1, S_2 \in K$, then $S_1 \cap S_2$ is ~~in K~~ (if $S_1 \cap S_2 \neq \emptyset$).

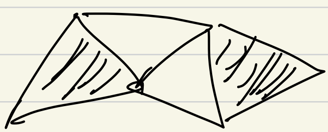
a face of both S_1 and of S_2

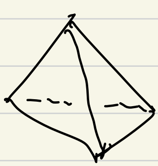
[Corrected on Jan 7]

e.g.,

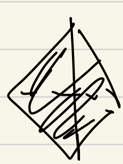
union of points





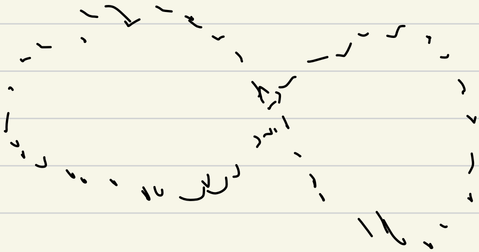


filled in



TDA:

$P \subset \mathbb{R}^2$



P finite subset.

Def: A point cloud in $\mathbb{R}^N$ is just a finite subset, P, in $\mathbb{R}^N$.

Idea:

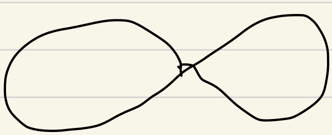
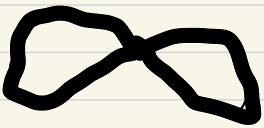


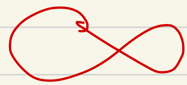
figure 8

← could be a curve in $\mathbb{R}^2$

or $\hat{X}$



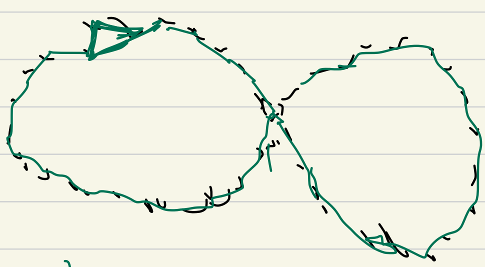
might be a "thickening" of



"2 dimensional object"

$X, \breve{X} \subset \mathbb{R}^2$

So ...



turn P into a simplicial complex ... ?

A simplicial complex in $\mathbb{R}^N$

is K = set of simplices in $\mathbb{R}^N$

s.t.
- $S \in K \Rightarrow \{\text{all faces of } K\} \subset S$
- $S_1, S_2 \in K \Rightarrow \cancel{S_1 \cap S_2 \in K}$

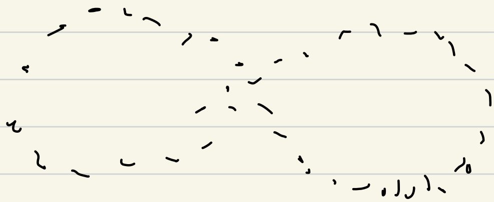
$S_1 \cap S_2$ is both a face of S_1 and S_2

or $S_1 \cap S_2 = \emptyset$.

Draw picture

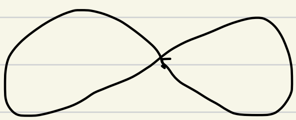
Corrected on Jan 7

Given a "point cloud"

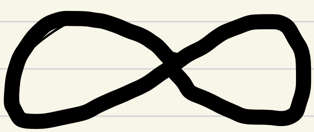


$P \subset \mathbb{R}^2$

looks like a "sample" from

$X =$  in $\mathbb{R}^2$

or

$\tilde{X} =$  thickened figure 8 in $\mathbb{R}^2$

To any <u>topological space</u>, X, we associate its "homology groups,"

$d+1$ points $\Rightarrow$ d-dim thing

$\subset \mathbb{R}^N$

if $N \geq d$

 $\longrightarrow$ lies in $\mathbb{R}^2$

 has to lie in $\mathbb{R}^3$

N data points : something $\mathbb{R}^N$