CPSC 531F Jan 24

1. Complete proof that

$$H_i(\mathrm{Cone}_p(K_{abs})) \simeq \begin{cases} \mathbb{R} & i=0 \\ 0 & i \geq 1 \end{cases}$$

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2. $H_i(G)$, $i=0,1$, $G=(V,E)$

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3. If K has connected components $K_1, K_2, \ldots, K_r$ then for all i

$$H_i(K) \simeq H_i(K_1) \oplus \ldots \oplus H_i(K_r)$$

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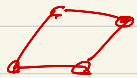
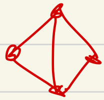
4. Start Mayer-Vietoris

Examples for ③

G:

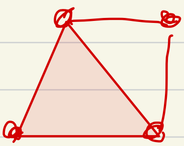


G_1



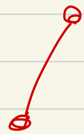
G_2

K:

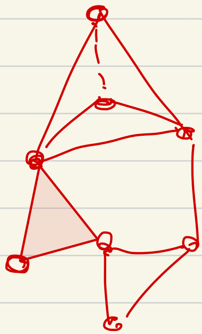


K_1

K_2



K_3



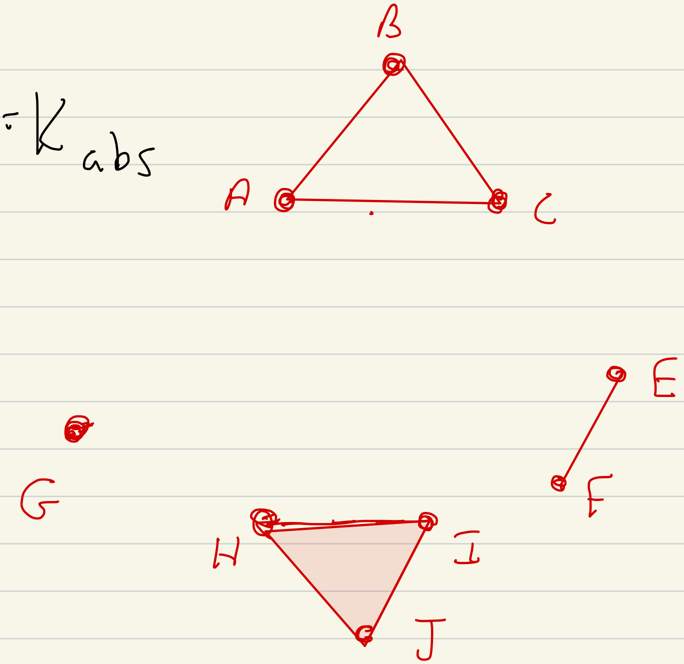
K_4

③ true since

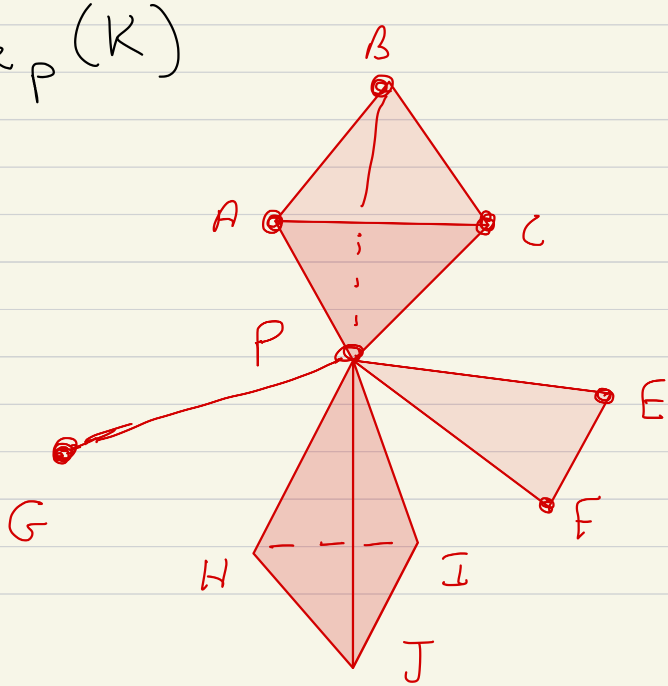
$\partial_0, \partial_1, \dots$ "factor through" the

$K_1, \dots, K_r$

$K := K_{abs}$ $\qquad \mathbb{R}^2$



$\mathrm{Cone}_p(K)$ $\qquad \mathbb{R}^3$

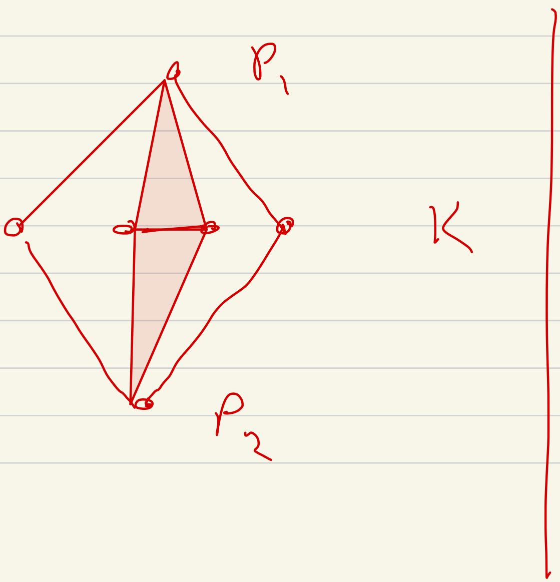


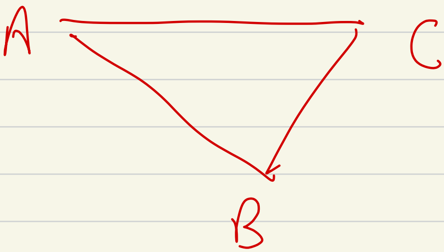
If $K = K_{abs}$ abs. simp complex on V, and $p_1, p_2 \notin V$

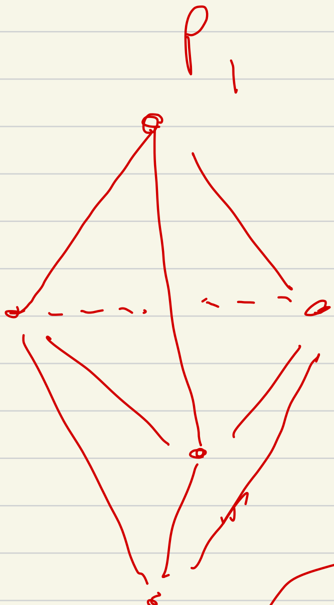
$p_1 \neq p_2$

$\text{Suspension}_{p_1, p_2}(K)$ or SK

$= \text{Cone}_{p_1}(K) \cup \text{Cone}_{p_2}(K)$

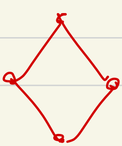




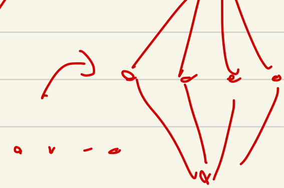


o e t

$H_0 \simeq \mathbb{R}^2$



$H_1 \simeq \mathbb{R}$

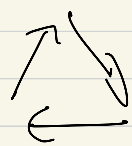


Last time

$$H_i(\Delta) : \quad \begin{array}{ll} i=0 & \mathbb{R} \\ \left.\begin{array}{l} i=1 \\ i=2 \end{array}\right\} & 0 \end{array}$$

$$C_2(\Delta) \xrightarrow{\partial_2} C_1(\triangle) \xrightarrow{\partial_1} C_0(\;)$$

$$\ker \partial_2 = 0$$

$\ker \partial_1 =$ "  " $\cdot \mathbb{R}$

$$= \operatorname{Image}(\partial_2)$$

<u>Thm</u>! $L = \operatorname{Conep}(K) \qquad (\{p\} \notin K)$

then: if $i \geq 1$, $H_i(L) = 0$

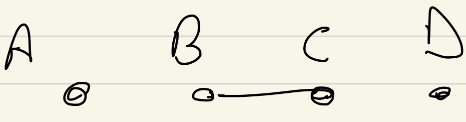
① Let $\tau \in \mathcal{C}_i(L)$, so is a linear combo of terms

$$[u_0, u_1, \ldots, u_i].$$

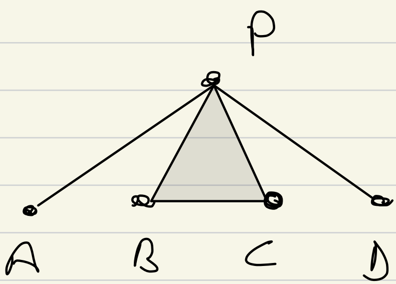
There is $\tau' \equiv \tau \; (\text{Image } \partial_{i+1})$ s.t. τ' only has terms with $u_0 = P$;

② If τ' as above has $\partial_i \tau' = 0$, then $\tau' = 0$.

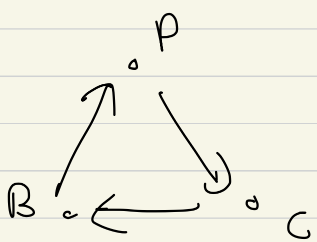
$$\text{Image}(\partial_{i+1}) = \text{Ker}(\partial_i), \; i \geq 1$$



K



$L = \mathrm{Cone}(K)$

$\partial_1 \Bigg($  $\Bigg) = 0$

$$\| $$

$$[B,P] + [P,C] + [C,B]$$

$$= -[P,B] + [P,C] + [C,B]$$

get something = mod $\mathrm{Im}\,\partial_{i+1}$ with only P's

Here

$$[C,B] \rightsquigarrow [P,C,B] \in C_2(K)$$

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want

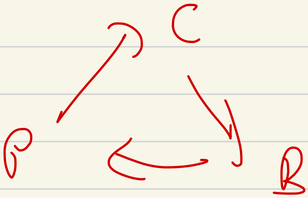
$$\underbrace{\partial_2 [P,u_0,u_1)}_{\text{by def}} \doteq [C,B] + \text{stuff with a letter } P \text{ in each}$$

$$[\hat{P},u_0,u_1) - [P,\hat{u}_0,u_1] + [P,u_0,\hat{u}_1]$$

$$[u_0,u_1] - [P,u_1] + (P,u_0]$$

$$\begin{tikzcd}
 & u_0 \arrow[dr] & \\
P \arrow[ur] & & u_1 \arrow[ll]
\end{tikzcd}$$

$$\partial[P, C, B] = [C, B] - [P, B] + [P, C]$$



$$\underbrace{[C,B] - [P,B] + [P,C]}_{} \in \underbrace{\mathrm{Im}(\partial_2)}_{B_1}$$

$$[C,B] = [P,B] - [P,C] \pmod{B_1}$$

More generally $u_0, \dots, u_i \not\ni P$

$$[u_0, \dots, u_i]$$

$$\partial_{i+1}[P, u_0, \dots, u_i] = [u_0, \dots, u_i] \pm \text{stuff with } P\text{'s}$$

Q: What happened to

$$\partial_1\left(\begin{tikzcd} & P \arrow[dr] & \\ B \arrow[ur] & & C \arrow[ll] \end{tikzcd}\right) = 0$$

$$\|$$

$$[B,P] + [P,C] + [C,B]$$

$$= -[P,B] + [P,C] + \underbrace{[C,B]}_{\text{mod } B_1 \text{ or } \mathrm{Im}(\partial_2)}$$

$$= -[P,B] + [P,C] + [P,B] - [P,C]$$

Question: Say we have

$$30[P,A] + 12[P,B] + 17[P,C] \xrightarrow{\partial_1} 0$$

$$\partial_1\left(\searrow \downarrow \swarrow\right) = \text{stuff with } p_j'$$

$$+\ 30[A] + 12[B] + 17[C]$$

$$\neq 0$$

Similarly: if

$$\sum \alpha_{p,u_1,\dots,u_i}[p,u_1,u_2,\dots,u_i] \in C_i(K)$$

and $\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}$

$$\xmapsto{\partial_i} 0$$

$$\partial_i\left(\sum \alpha_{p,u_1,\dots,u_i}[p,u_1,u_2,\dots,u_i]\right)$$

$$= \text{stuff with } p_r' + \underbrace{\sum \alpha_{p,u_1,\dots,u_i}[u_1,\dots,u_i]}_{\longrightarrow \text{ has to } = 0}$$