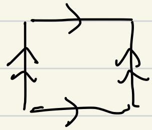
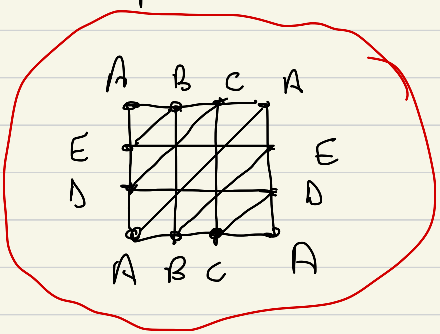
CPSC 531F March 19

- Today: Nash Equilibria } give intuition

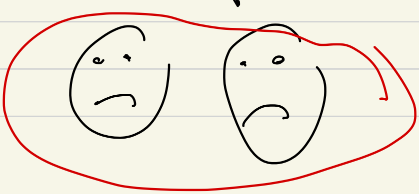
- Friday: Δ-complexes: like simplicial complexes, but more general

Torus: 

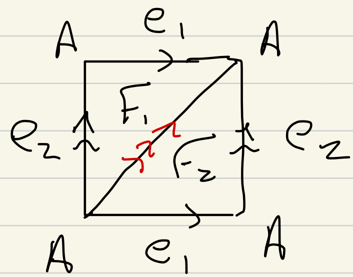
simplicial complex



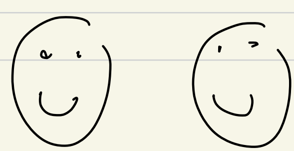
18 2-simplices



Δ-complex



2 2-simplices



- Next week:

$f: X \to Y$ gives $f_*: H_i(X) \to H_i(Y)$

$f: K^{abs} \to L^{abs}$ gives $f_*: H_i^{simp}(K^{abs}) \to H_i^{simp}(L^{abs})$

Homotopy equivalence of maps f, g

- The rest of the course:

Bar codes $\leftarrow$ main things that TDA measures

Homework:

B.1 - B.7 not to hand in,

(maybe B.4 if you insist,

maybe others...)

Other section B exercises so far:

✓ B.13 - B.14 wedge sum

B.18 - B.19 haven't yet gotten there

now B.20 - B.25 applications of

Brouwer fixed point theorem

Intuition Behind Nash Equilibrium

"0-sum" games (2 player)

Rock-scissors-paper

Player 1 \ Player 2	R	P	S
R	$(0,0)$	$(-1,1)$	.
P	$(1,-1)$	$(0,0)$	.
S	.	.	$(0,0)$

$(1,-1)$: Reward/payout is $\begin{cases} 1 & \text{the player 1} \\ -1 & \text{" " 2} \end{cases}$

Cooperative Game:

Driving Game

	Player	R	L
Player 1			
Drive on the right side of road		(c, 0)	(-1, -1)
- - - left		(-1, -1)	(c, 0)

2 player game:

	Player 2 Option: 1	2	...	n_2
Player 1				
Option 1	(,)	(,)		
Option 2	(,)	...		
$\vdots$		$\ddots$		
Option n_1				

A k-player game

player 1 $\quad n_1$ options
strategies

player 2 $\quad n_2$ "

⋮

player k $\quad n_k$

For each $i_1 \in [n_1]$, $i_2 \in [n_2]$, ...

we have, for each $p \in [k]$

Payout
Reward $\left(p, i_1, \ldots, i_k \right)$

Game: $k, n_1, \ldots, n_k \geq 1$ integers

Reward / Payout: $[k] \times [n_1] \times \ldots \times [n_k]$

$$\longrightarrow \mathbb{R}$$

Mixed strategies:

$\vec{p}_1 \in \mathbb{R}^{n_1}$, stochastic, $\vec{p}_1 \in \Delta^{n_1 - 1}$

$\vec{p}_2 \in \Delta^{n_2 - 1}, \ldots, \vec{p}_k \in \Delta^{n_k - 1}$

$$\begin{pmatrix}\text{Reward to}\\ \text{player } j\end{pmatrix}(\vec{p_1},\dots,\vec{p_k})$$

$$= \sum_{\substack{i_1\in[n_1]\\ i_2\in[n_2]\\ \vdots}} \text{Reward}(j;\, i_1,\dots,i_k)\cdot p_1(i_1)\,p_2(i_2)\cdots p_k(i_k)$$

$(\vec{p_1},\dots,\vec{p_k})$ is a <u>Nash equilibrium</u> if for all

$j = 1,\dots,k$

$$\begin{pmatrix}\text{Reward}\\ \text{to } j\end{pmatrix}\left(\vec{p}_1, \ldots \vec{p}_k\right)$$

$$\geq$$

$$\underset{\text{to } j}{\text{Reward}}\left(\vec{p}_1, \ldots, \vec{p}_{j-1}, \vec{q}_j, \vec{p}_{j+1}, \ldots, \vec{p}_k\right)$$

for all $\vec{q}_j \in \mathbb{R}^{n_j}$, stochastic

<u>Thm</u> (Nash): For any k-player game, there exists at least one Nash equilibrium.

Say player 1 can't improve strategy <u>with player 2, ..., k fixed.</u>

	Reward
Player 1	
Option 1	$a_1 \in \mathbb{R}$
Option 2	$a_2 \in \mathbb{R}$
$\vdots$	$\vdots$
Option n_1	$a_{n_1} \in \mathbb{R}$

If player 1 plays

$\vec{p} = (p_1, p_2, \ldots, p_n)$ (stochastic)

Player 1 reward:

$$p_1 a_1 + p_2 a_2 + \cdots + p_n a_n$$

You have to play $\vec{p}$

s.t.

$$p_\ell > 0 \Rightarrow a_\ell = \max_i (a_i)$$

Could be $a_1 = a_2 = \cdots = a_n$

$\Rightarrow$ any $\vec{p}$ works

Gradient search:

Nash's version:

Reward To Swich from $\vec{p}$ to $\vec{e}_i$

$\vec{e}_i = (0, 0, \ldots, 1, 0, \ldots 0)$

pos i

$$\underbrace{\max\left(\text{Reward}(\vec{e}_i) - \text{Reward}(\vec{p}), 0\right)}_{R_i}$$

$$\text{Reward Vector} \stackrel{\text{def}}{=} (R_1, R_2, \ldots, R_{n_1})$$

$$a_1 = a_2 = 2$$

$$a_3 = 1$$

$$a_4 = a_5 = \ldots = -1$$

$$\vec{p} = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, 0, \ldots 0\right)$$

$$\text{Reward}(\vec{p}) = \frac{2}{5} + \frac{2}{5} + \frac{1}{5} - \frac{1}{5} - \frac{1}{5} = \frac{3}{5}$$

$$(R_1, R_2, R_3, R_4, R_5, \ldots) = \left(2 - \frac{3}{5}, 2 - \frac{3}{5}, 1 - \frac{3}{5}, 0, 0, \ldots\right)$$

$$\left(\frac{7}{5}, \frac{7}{5}, \frac{2}{5}, 0, 0 \ldots \right)$$

We move

$\vec{p}$ to

$$\text{Stochastic}\left(\vec{p} + \varepsilon \, \text{Reward}(\vec{p}) \right)$$

$= \ldots$