Introduction to Hawking Radiation

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Physics 407

December 18

Abstract

This article will give a biref introduction to the Hawking Radiation effect and its implications on the quantum theory of gravity.

1 Introduction

Hawking Radiation is the emission of particles by a blackhole as a result of quantum mechanical effects. [1] It was discovered by Stephen Hawking in 1975. It was an important discovery in the study of gravity and black holes, and has many interesting implications.

In this article, we will first do a brief review of classical general relativity, then introduce the Hawking radiation and its implications.

2 Classical General Relativity and Black Hole theory

2.1 Einstein's Equation and Schwarschild Spacetime

General Relativity predicts that mass influences the space-time around it. Gravity is really geometry. The geometry of the space-time is governed by Einstein's Equation:

$$R_{ab} - \frac{1}{2}g_{ab}R = 8\pi T_{ab} \tag{1}$$

One of the simplest solutions of Einstein's Equation is the Schwarschild spacetime, which is the geometry outside a static spherical star. The geometry can be specified by the geodesic:

$$ds^2 = -(1 - \frac{2GM}{c^2r})(cdt)^2 + (1 - \frac{2GM}{c^2r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{2}$$

Or in geometrical units(where $G = C = 1$):

$$ds^2 = -(1 - \frac{2M}{r})dt^2 + (1 - \frac{GM}{r})^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{3}$$

2.2 Gravitational Collapse and Black Hole

A stable star is in an equlibrium between the attractive force of gravity and the pressure from burning hydrogen, helium etc. When it is out of stuff to burn, there are several possible outcomes. If the mass of the star is small, gravity will be balanced by the degeneracy pressure of electrons or neutrons, and the star will become a white dwarf or neutron star respectively. But if the mass of the star is greater than about 3 solar massess, the gravity will dominate over the degeneracy pressure, and the star will collapse forever. We call it a black hole.

The spacetime is very curved near a black hole, so that inside the Schwarschild radius($2M$) of the black hole, nothing can escape its gravity, not even light. Everything inside must collapse into the black hole. We call the sphere at radius $2M$ the even horizon of the black hole. Observers ouside the event horizon cannot observe evens inside the horizon. If we see the even horizon as the boundary of the black hole, then the black hole only absorbs particles and never emits particles. Its temperature is 0.

The spacetime for a static black hole is the Schwarschild spacetime. Rotating black holes correspond to Kerr spacetime. The No-hair theorem states that stationary black holes are characterized by mass M, angular momentum J, and electric charge Q. It does not depend on other information of the star before the collapse. If mass is dropped into a black hole, the mass of the black hole increases, so does the area of the black hole.

3 Hawking Radiation

In 1975 Hawking made a shocking discovery that black holes does emit particles, and the spectrum is just like a black body radiation with tempearature

$$T = \frac{\kappa}{2\pi}. \tag{4}$$

κ is the surface gravity of the black hole. This result is derived by applying quantum field theory on the spacetime around a black hole.

In quantum field theory, ther are constantly particle-antiparticle pairs created and annihilated in vacuum. Althogh vacuum has zero energy, the uncertanty principle has $\Delta E \Delta t = h$, so particle-antiparticle pairs can be created and quickly annihilated. One particle in a pair has positive energy, by the conservation of energy the other particle has negative energy. Since normally particles have positive energies, the negative energy particle is only a virtual particle and can not live long, it has to annihilate with its counterpart.

Now we consider the vacuum in the spacetime around a black hole, especially near the even horizon. If a particle pair is created just outside the event horizon of a black hole, The negative energy particle may fall inside the event horizon and the positive energy particle may escape from the black hole. Inside the horizon of the blackhole, the energy component of the momentum 4-vector of the particle becomes spacelike. It is now a momentum component, which can be negative. So the virtual particle can stay as a real particle inside the horizon of the black hole. Since the black hole absorbed a negative energy particle, its mass will decrease. From an observer far away, it would seem that the black hole is emitting particles.

The original derivation of the Hawking Radiation effect was done by applying quantum field theory on a spacetime before and after the collapse into a black hole. Hawking's calculation involves taking the positive frequncy mode function corresponding to a particle state at late times, propagating it backwards in time, and determining its positive and negative frequency parts in the asymptotic past. [3] The result is the blackbody spectrum of the black hole's radiation.

The bigger the black hole, the smaller its temperature. For a black hole with a few solar masses, its temperature is only one ten millionth of a degree Kelvin, which is much less than the cosmic background radiation(about 2.7 Kelvin). Such black holes would emit even less than they absorb. But for primordial black holes, which was formed in the early stages of universe and has a much smaller mass, their temperature would be much higher, and the blac hole would eventually vanish. A primordial black hole with a mass of a thousand million tons would have a lifetime roughly equal to the age of universe. Since primordial black holes, if they exist, are very scarce and hard to detect, it would be very hard for us to observer Hawking radiation directly. [4]

Since black holes has temperatures, it's natural to assign other thermal properties to them. It turns out that the entropy of the black hole is proportional to the area of the black hole. Since the area will shrink after emission of particles, the entropy of the black hole will also decrease. But since the radiation gives more entropy to the universe, the second law of thermodynamics is not violated.

An important consequence of Hawking radiation is information loss. Since a black hole is characterized by its mass, angular momentum and charge only, information is lost when a star collapse into a black hole, or when mass is dropped into a black hole. In classical theory it didn't really matter, because one could say that all the information was still inside the black hole. But If we apply quantum theory, black holes emit particles and eventrually vanish by the Hawking radiation effect, the information is truly lost. Hawking argues that this loss of information introduces a new level of uncertanty in physics over and above the usual uncertanty principle of quantum theory, while Penrose thinks that the loss of information compliments the uncertainty principle.[2]

Another interesting implication is Hawking's theory that black holes are the same as white holes. Classically white holes are black holes running backwards in time. White holes only emits mass and never absorbs. But now since black holes can also emit particles, white holes can absorb particles, too. Hawking suggests that a small black hole that is sending large amounts of radiation is just the same as a white hole.[2]

4 Conclusion

The discovery of Hawking radiation sparked research into the quantum theory of gravity. Today many researchers are working on unifying two major braches of modern physics, general relativity and quantum theory.

References

[1] Definition of Hawking Radiation, About Physics Dictionary, http://physics.about.com/library/dict/bldefhawkingradiation.htm

[2] Hawking and Penrose, The Nature of Space and Time, 1996.

[3] Robert W. Wald, The Thermodynamics of Black Holes, http://www.living reviews.org/Articles/Volume4/2001-6wald

[4] Stephen Hawking, 1988, A Brief Hisotry of Time, Chapter 7.

[5] Hartle, Basic General Relativity, 1999, Chapter XI.