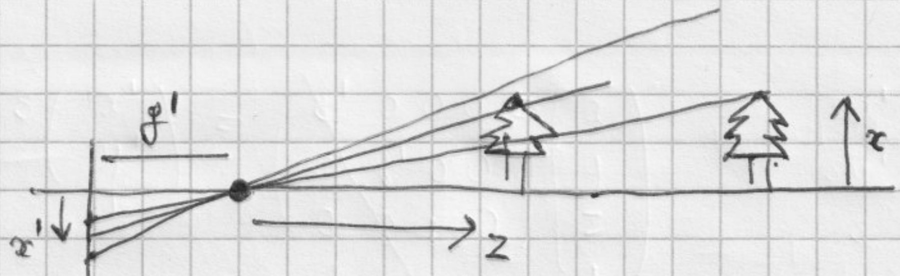
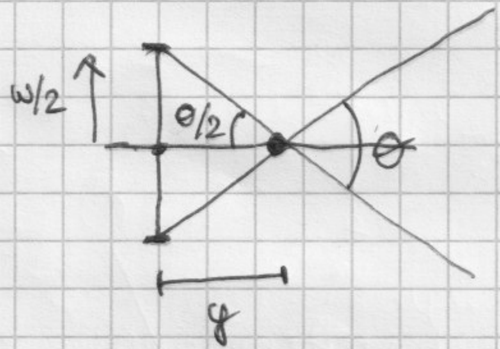



$$x' = f'\frac{x}{Z}$$

2·5



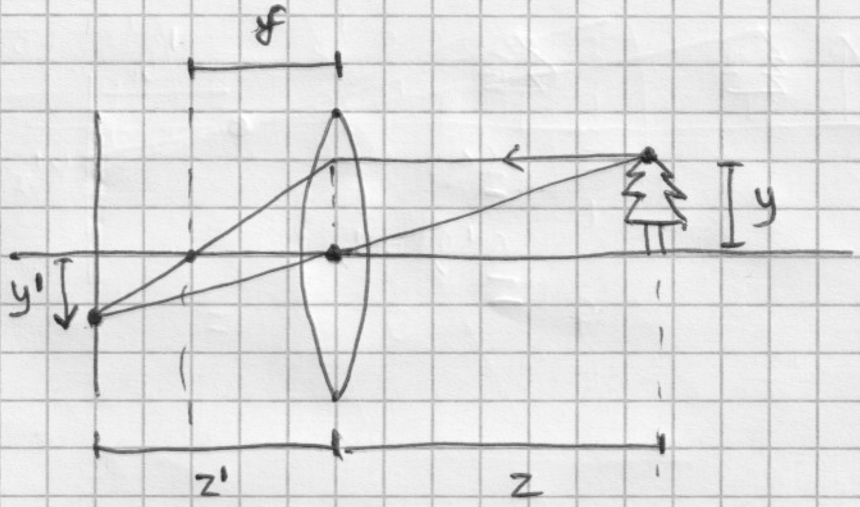

$$\tan \theta/2 = \frac{w}{2f}$$

$$\theta = 2\,\mathrm{atan}\frac{w}{2f}$$

$w = 35\,\mathrm{mm}$, $f = 50\,\mathrm{mm}$ $\rightarrow$ $\underline{\theta} = 2\,\mathrm{atan}\,0.35 = \underline{38.6^\circ}$

$100\,\mathrm{mm}$ $\underline{19.9^\circ}$

2.6



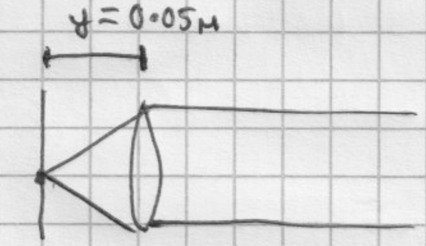

$$\frac{y'}{z'} = \frac{y}{z} \quad \text{(pinhole)}$$

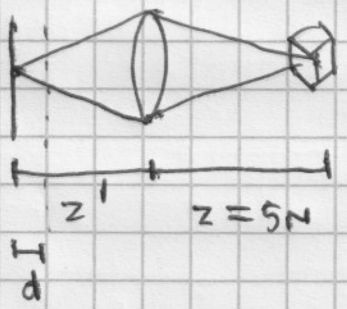
$$\frac{y'}{y} = \frac{z'}{z} = \frac{z'-f}{f}$$

$$\frac{y'}{z'-f} = \frac{y}{f}$$

$$\frac{1}{z} = \frac{1}{f} - \frac{1}{z'}$$

$$\frac{1}{f} = \frac{1}{z'} + \frac{1}{z} \quad \text{lens eqn.}$$



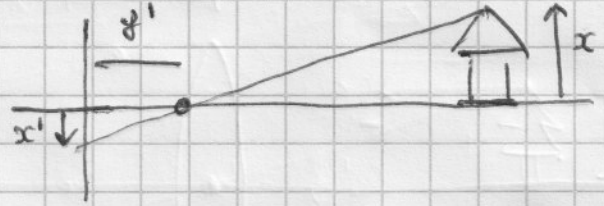

$$\frac{1}{z'} + \frac{1}{z} = \frac{1}{f}$$

$$\frac{1}{z'} = \frac{1}{0.05} - \frac{1}{5}$$

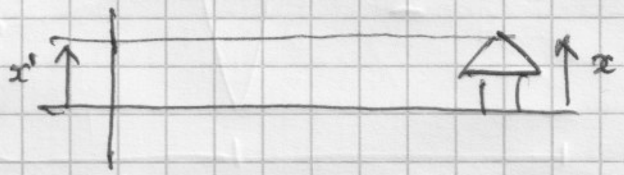
$$= 19.8$$

$$d = z' - f = \frac{1}{19.8} - 0.05 \simeq 0.5\text{mm}$$

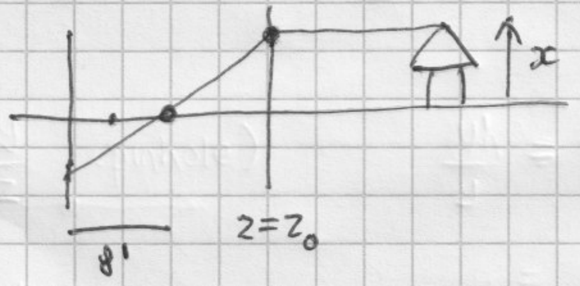
## 3.1




pinhole $x' = f' \frac{x}{z}$




orthographic $x' = x$




weak perspective

$$x' = \frac{f'}{z_0} x$$

$$x' = Mx$$

3.2

$$I_{corr}(x,y) = \int_t \int_s \overset{\text{image}}{I(x+s,\, y+t)}\, \overset{\text{kernel}}{k(s,t)}\, ds\, dt$$

1D $\quad I_{corr}(x) = \int_s I(x+s)\,k(s)\,ds = \sum_s I(x+s)\,k(s)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $I$ | 9 | 5 | 2 | 1 | 3 | 4 | 6 | 2 | 4 |
| $k$ | 1 | 2 | 1 | | | | | | |
| $I_{corr}$ | | 21 | 10 | 7 | ... | | | | |

3.3

$$\frac{1}{4}[1\ 2\ 1] * [0\ 72\ 88\ 62\ 52\ 37\ 0]$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 72 \\ 88 \\ 62 \\ 52 \\ 37 \\ 0 \end{bmatrix}$$

convolution/correlation can be written as a matrix multiply $\therefore$ linear e.g. $(\alpha_1 F_1 + \alpha_2 F_2) * F_3 = \alpha_1 F_1 * F_3 + \alpha_2 F_2 * F_3$

associative $(a * b) * c = a * (b * c)$

commutativity $A * B = B * A$ ?

$$\text{conv}(A, B) = \text{conv}(B, A)$$

but $\text{corr}(A, B) \neq \text{corr}(B, A)$

$$= \text{corr}(B', A')$$

$$A'(x, y) = A(-x, -y)$$

$$B'(\ \text{"}\ ) = B'(\ \text{"}\ )$$

4.1 1D Gaussian $g(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$

$$g(x)\,g(y) = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}\left(\frac{x^2+y^2}{\sigma^2}\right)} = g(x,y) \quad \text{2D Gaussian}$$

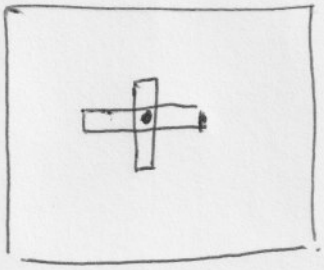
4.2

$$\mathrm{corr}(I,F) = \sum_i \sum_j I(x+i,y+j)\,\underbrace{F(i,j)}_{=F(i)F(j)} \qquad F(i,j) = F(i)F(j)$$

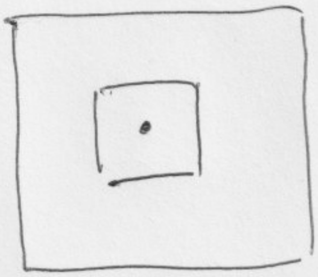
$$= \sum_i F(i) \underbrace{\sum_j I(x+i,y+j)F(j)}_{=\ \mathrm{corr}(I(x+i,y),F(y)) = I'(x+i,y)}$$

$$= (I(x,y) * F(y)) * F(x)$$

4.3



2M */+



$M^2$ */+

e.g.

$$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

4.4

$$[1\ -1\ 1\ -1\ 1\ -1\ 1\ -1] * [1\ 1]^{1/2} \rightarrow [0\ 0\ 0\ \cdots\ ]$$

$$[1\ 2\ 3\ 2\ 1\ 0\ -1\ -2\ -3] * [1\ 1]^{1/2}$$

$$\rightarrow [1.5\ 2.5\ 2.5\ 1.5\ 0.5\ -0.5\ -1.5\ -2.5]$$