20.1 $\quad \dfrac{\partial L}{\partial x} = \dfrac{\partial i_1}{\partial x}\dfrac{\partial a}{\partial i_1}\dfrac{\partial y}{\partial a}\dfrac{\partial i_2}{\partial y}\dfrac{\partial L}{\partial i_2}$

$\longleftarrow$ reverse

forward $\longrightarrow$

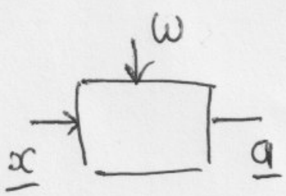
suppose input $\underline{x}$ is vector length n

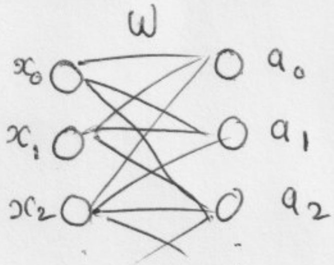
forward autodiff requires n passes to compute each $\dfrac{\partial L}{\partial x_i}$

reverse autodiff still 1 pass (since L is scalar)

reverse mode (backprop) is better if $n_{out} < n_{input}$

20.2 Linear layer / fully connected backpass





$\underline{a} = w\underline{x} \qquad a_i = \sum_j w_{ij} x_j$

$$\frac{\partial L}{\partial x_i} = \sum_j \frac{\partial L}{\partial a_j}\underbrace{\frac{\partial a_j}{\partial x_i}}_{= w_{ji}}$$

$$\frac{\partial L}{\partial w_{ij}} = \frac{\partial L}{\partial a_i}\underbrace{\frac{\partial a_i}{\partial w_{ij}}}_{= x_j}$$

$$\frac{\partial L}{\partial \underline{x}} = w^T \frac{\partial L}{\partial \underline{a}}$$

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial \underline{a}}\,\underline{x}^T$$

$$\frac{\partial L}{\partial \underline{x}} = \begin{pmatrix} \frac{\partial L}{\partial x_0} \\ \frac{\partial L}{\partial x_1} \\ \vdots \end{pmatrix}$$

same size as $\underline{x}$

same size as w

20.3

correlation

$$\overset{k}{\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}} * \overset{I_i}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} = \overset{I_o}{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & i & h & g & 0 \\ 0 & f & e & d & 0 \\ 0 & c & b & a & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$$

(x,y)

$$\frac{\partial L}{\partial I_o}(x-1, y-1) \times i + \frac{\partial L}{\partial I}(x, y-1) \times h + \ldots$$

$$\frac{\partial L}{\partial I_i}(x,y) = \sum_{dx} \sum_{dy} \frac{\partial L}{\partial I_o}(x+dx, y+dy)\, k(-dx, -dy)$$

$$\frac{\partial L}{\partial I_i} = \text{conv}\left(\frac{\partial L}{\partial I_o}, k\right)$$

backward pass of correlation is convolution

(and vice versa)