Scale Invariant Feature Transform (SIFT)



- SIFT describes both a **detector** and **descriptor**
 - 1. Multi-scale extrema detection
 - 2. Keypoint localization
 - 3. Orientation assignment
 - 4. Keypoint descriptor

Where do points lead?



lmage 1

With COTR, we find dense correspondences, which we can reconstruct a dense 3D model from just two calibrated views.



1. Multi-scale Extrema Detection

$$\begin{array}{c} G(k^{4}\sigma) \\ G(k^{3}\sigma) \\ G(k^{2}\sigma) \\ G(k\sigma) \\ G(k^{3}\sigma) \\ G(k^{3}\sigma) \\ g(k^{2}\sigma) \\ G(k$$





Half the size



Difference of Gaussian (DoG)

1. Multi-scale Extrema Detection Detect maxima and minima of Difference of Gaussian in scale space



Selected if larger or smaller than all 26 neighbors

Difference of Gaussian (DoG)



2. Keypoint Localization

- After keypoints are detected, we reare **poorly localized** along an edge

How do we decide whether a keypoint is poorly localized, say along an edge, vs. well-localized?

 $C = \begin{bmatrix} \sum_{p \in P} I \\ \sum_{p \in P} I \\ \sum_{p \in P} I \end{bmatrix}$

- After keypoints are detected, we remove those that have low contrast or

$$\left[egin{array}{ccc} I_x I_x & \sum\limits_{p \in P} I_x I_y \ P & p \in P \end{array}
ight] \left[egin{array}{ccc} I_y I_x & \sum\limits_{p \in P} I_y I_y \ P & p \in P \end{array}
ight]$$

3. Orientation Assignment

- Create **histogram** of local gradient directions computed at selected scale
- Assign canonical orientation at peak of smoothed histogram
- Each key specifies stable 2D coordinates (x, y, scale, orientation)





4. SIFT Descriptor

(weighted by a Gaussian with sigma half the size of the window) - Create array of orientation histograms - 8 orientations \times 4 \times 4 histogram array



- Image gradients are sampled over 16 \times 16 array of locations in scale space



SIFT Matching

- Each SIFT feature is represented by 128-D vector (numbers)
- Feature matching becomes the task of finding the closest 128-D vector
- Nearest-neighbor matching:
 - $NN(j) = \arg$
- This is expensive (linear time), but good approximation algorithms exist

$$\min_{i} |\mathbf{x}_{i} - \mathbf{x}_{j}|, \ i \neq j$$

e.g., Best Bin First K-d Tree [Beis Lowe 1997], FLANN (Fast Library for Approximate Nearest Neighbours) [Muja Lowe 2009]

Match Ratio Test

Compare ratio of distance of **nearest** neighbour (1NN) to **second** nearest (2NN) neighbour — this will be a non-matching point

Rule of thumb: d(1NN) < 0.8 * d(2NN) for good match



Histogram of Oriented Gradients (HOG) Features

Dalal, Triggs. Histograms of Oriented Gradients for Human Detection. CVPR, 2005



Single scale, no dominant orientation





Histogram of Oriented Gradients (HOG) Features

Pedestrian detection

128 pixels 16 cells 15 blocks

1 cell step size



64 pixels 8 cells 7 blocks

Redundant representation due to overlapping blocks

visualization



 $15 \times 7 \times 4 \times 9 =$ 3780





'Speeded' Up Robust Features (SURF)

4 x 4 cell grid



Each cell is represented by 4 values: $\left[\sum d_x, \sum d_y, \sum |d_x|, \sum |d_y|\right]$

Haar wavelets filters



How big is the SURF descriptor? 64 dimensions



'Speeded' Up Robust Features (SURF)















Keypoint **Detectors** vs. **Descriptors**

Harris Blob (Laplacian) SIFT

- SIFT
- HoG
- SURF

Failure Case: Repetitive Structures

- Repetitive structures cause problems for feature matching
- Multiple locations in an image provide good matches and have similar matching scores
- They are particularly common in man-made environments

Window detail



Brick pattern

Learning Descriptors

Descriptor design as a learning (embedding) problem



[Winder Brown 2007]

DeepDesc [ICCV 2015] Learning an "embedding"



Minimize the distance for corresponding matches. Maximize it for non-corresponding patches.

Learning with SfM dataset











3k images, 59k unique points, 380k



Training set #1:





THE UNIVERSITY OF BRITISH COLUMBIA

CPSC 425: Computer Vision



Lecture 13: Planar Geometry and RANSAC

Menu for Today

Topics:

- **Planar** Geometry
- Image Alignment, Object Recognition

Readings:

- Today's Lecture: Szeliski 2.1, 8.1, Forsyth & Ponce 10.4.2

Reminders:

-Assignment 3: Due Wednesday!





Learning Goals

Linear (Projective) Transformations Good results don't happen by chance (or do they?)

3. Good == more support

Aim: warp our images together using a 2D transformation





Aim: warp our images together using a 2D transformation



Find corresponding (matching) points between the images



Compute the transformation to align the points



We can also use this transformation to reject outliers



We can also use this transformation to reject outliers



Planar Geometry

- 2D Linear + **Projective** transformations Euclidean, Similarity, Affine, Homography

Robust Estimation and RANSAC Estimating 2D transforms with noisy correspondences

2D Transformations

— We will look at a family that can be represented by 3x3 matrices



This group represents perspective projections of planar surfaces

Affine Transformation

- Transformed points are a linear function of the input points

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$

$$\begin{array}{c} a_{12} \\ a_{22} \end{array} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$$

Affine Transformation

- Transformed points are a linear function of the input points

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$$

This can be written as a single matrix multiplication using homogeneous coordinates

$$\begin{bmatrix} x'_1 \\ y'_1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \\ 0 \end{bmatrix}$$

$$\begin{array}{c} a_{12} \\ a_{22} \end{array} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a_{13} \\ a_{23} \end{bmatrix}$$



$$\begin{array}{ccc} a_{12} & a_{13} \\ a_{22} & a_{23} \\ 0 & 1 \end{array} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Linear Transformation

- Consider the action of the unit square under, sample transform $\begin{vmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$







Linear Transformation

- Consider the action of the unit square under, sample transform











Linear (or Affine) Transformations



Translation, rotation, scale, shear (parallel lines preserved)
Linear (or Affine) Transformations





Translation, rotation, scale, shear (parallel lines preserved)

These transforms are not affine (parallel lines not preserved)

Linear (or Affine) Transformations

Consider a single point correspondence

Y



$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{12} \\ a_{22} \\ 0 \end{bmatrix}$$

Linear (or Affine) Transformations

Consider a single point correspondence

Y



$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \\ 0 \end{bmatrix}$$

How many points are needed to solve for **a**?

Lets compute an affine transform from correspondences:

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \\ 0 \end{bmatrix}$$

Re-arrange unknowns into a vector

$$\begin{bmatrix} x_1' \\ y_1' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} 0 & x_1 \\ 0 & y_1 \\ 0 & 1 \\ x_1 & 0 \\ y_1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$\begin{array}{ccc} a_{12} & a_{13} \\ a_{22} & a_{23} \\ 0 & 1 \end{array} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Linear system in the unknown parameters **a**

x_1	y_1	1	0
0	0	0	x_1
x_2	y_2	1	0
0	0	0	x_2
x_3	y_3	1	0
0	0	0	x_3

Of the form

Ma = y

Solve for a using Gaussian Elimination

Once we solve for a transform, we can now map any <u>other points</u> between the two images ... or resample one image in the coordinate system of the other



$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} \\ a_{21} & a_{21} \\ 0 \end{bmatrix}$$

$$\begin{array}{ccc} a_{12} & a_{13} \\ a_{22} & a_{23} \\ 0 & 1 \end{array} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

This allows us to "stitch" the two images

Once we solve for a transform, we can now map any other points between the two images ... or resample one image in the coordinate system of the other



Other linear transforms are special cases of **affine** transform:

 $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$

Other linear transforms are special cases of **affine** transform:



 $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}$

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Other linear transforms are special cases of **affine** transform:

 $\begin{vmatrix} a_{11} \\ a_{21} \\ 0 \end{vmatrix}$



$$\begin{array}{ccc} a_{12} & a_{13} \\ a_{22} & a_{23} \\ 0 & 1 \end{array}$$

Face Alignment





Face Alignment





Face Alignment





2D Transformations

Transformation	Matrix	# DoF	Preserves	Icon
translation	$\left[egin{array}{c c} oldsymbol{I} & t \end{array} ight]_{2 imes 3}$	2	orientation	
rigid (Euclidean)	$\left[egin{array}{c c} m{R} & t \end{array} ight]_{2 imes 3}$	3	lengths	
similarity	$\left[\begin{array}{c c} s oldsymbol{R} & t \end{array} ight]_{2 imes 3}$	4	angles	
affine	$\left[\begin{array}{c} oldsymbol{A} \end{array} ight]_{2 imes 3}$	6	parallelism	
projective	$\left[egin{array}{c} ilde{oldsymbol{H}} \end{array} ight]_{3 imes 3}$	8	straight lines	

Example: Warping with Different Transformations Projective Translation Affine (homography)







Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)



Aside: We can use homographies when ...

1.... the scene is planar; or

2.... the scene is very far or has small (relative) depth variation → scene is approximately planar





Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Aside: We can use homographies when ...

3.... the scene is captured under camera rotation only (no translation) or pose change)



Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Projective Transformation

General 3x3 matrix transformation

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Projective Transformation

General 3x3 matrix transformation

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Lets try an example:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{H} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Transformation

$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$



Points

Transformed Points

Projective Transformation

General 3x3 matrix transformation

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

Lets try an example:

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Transformation

$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$



Points

Transformed Points

Divide by the last row: $\begin{bmatrix} 0 & 0 & 1 & 0.5 \\ 0 & 0.5 & 0 & 0.5 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Compute H from Correspondences

Each match gives 2 equations to solve for 8 parameters

$$\begin{bmatrix} x_1' \\ y_1' \\ 1 \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$



 \rightarrow 4 correspondences to solve for **H** matrix Solution uses Singular Value Decomposition (SVD) In Assignment 4 you can compute this using cv2.findHomography

a_{11}	a_{12}	a_{13}	x_1
 a_{21}	a_{22}	a_{23}	y_1
a_{31}	a_{32}	a_{33}	1

Image Alignment

Find corresponding (matching) points between the image



$\mathbf{u} = \mathbf{H}\mathbf{x}$

2 points for Similarity3 for Affine4 for Homography

Image Alignment

In practice we have many noisy correspondences + outliers





Image Alignment

In practice we have many noisy correspondences + outliers

e.g., for an affine transform we have a linear system in the parameters a:

$\int x$		y_1	1	0
	С	0	0	x_1
X	$\dot{2}$	y_2	1	0
	С	0	0	x_2
x	3	y_3	1	0
	С	0	0	x_3
-			•	

It is overconstrained (more equations than unknowns) and subject to outliers (some rows are completely wrong)

Let's deal with these problems in a simpler context ...



Fitting a Model to Noisy Data

We can fit a line using two points

If we draw pairs of points uniformly at random, what fraction of pairs will consist entirely of 'good' data points (inliers)?

will consist entirely of 'good' data points (inliers)

points lie close to the line fitted to the pair

that lie close to the line

Suppose we are **fitting a line** to a dataset that consists of 50% outliers

- If we draw pairs of points uniformly at random, then about 1/4 of these pairs
- We can identify these good pairs by noticing that a large collection of other
- A better estimate of the line can be obtained by refitting the line to the points

RANSAC (**RAN**dom **SA**mple **C**onsensus)

- sample)
- Size of consensus set is model's **support**
- 3. Repeat for N samples; model with biggest support is most robust fit
 - Points within distance t of best model are inliers
 - Fit final model to all inliers

1. Randomly choose minimal subset of data points necessary to fit model (a

2. Points within some distance threshold, t, of model are a **consensus set**.

Slide Credit: Christopher Rasmussen

RANSAC (**RAN**dom **SA**mple **C**onsensus)

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 - Points within distance t of best model are inliers
 - Fit final model to all inliers

RANSAC is very useful for variety of applications

1. Randomly choose minimal subset of data points necessary to fit model (a

2. Points within some distance threshold, t, of model are a **consensus set**.

Slide Credit: Christopher Rasmussen

RANSAC (**RAN**dom **SA**mple **C**onsensus)

sample) Fitting a Line: 2 points

2. Points within some distance threshold, t, of model are a **consensus set**. Size of consensus set is model's support

3. Repeat for N samples; model with biggest support is most robust fit Points within distance t of best model are inliers

- Fit final model to all inliers

1. Randomly choose minimal subset of data points necessary to fit model (a

Slide Credit: Christopher Rasmussen

Example 1: Fitting a Line



\bigcirc

Example 1: Fitting a Line



Example 1: Fitting a Line



After RANSAC

from minimal set of inliers

Improve this initial estimate with estimation over all inliers (e.g., with standard least-squares minimization)

But this may change inliers, so alternate fitting with re-classification as inlier/ outlier

RANSAC divides data into inliers and outliers and yields estimate computed

Example 2: Fitting a Line







Example 2: Fitting a Line



Image Alignment + RANSAC

In practice we have many noisy correspondences + outliers








RANSAC solution for Similarity Transform (2 points)



4 inliers (red, yellow, orange, brown),

RANSAC solution for Similarity Transform (2 points)



4 outliers (blue, light blue, purple, pink)

RANSAC solution for Similarity Transform (2 points)



4 inliers (red, yellow, orange, brown), 4 outliers (blue, light blue, purple, pink)

RANSAC solution for Similarity Transform (2 points)



cbbeskvingtcimdiggancese #inliers = 2





RANSAC solution for Similarity Transform (2 points)



chebkwaeppimkigearces #inliers = 2



RANSAC solution for Similarity Transform (2 points)



#inliers = 4

checkossapein, ageargees



- **1.** Match feature points between 2 views
- **2.** Select minimal subset of matches^{*}
- **3.** Compute transformation T using minimal subset
- count #inliers with distance < threshold
- **5.** Repeat steps 2-4 to maximize #inliers

* Similarity transform = 2 points, Affine = 3, Homography = 4



Assignment 4

4. Check consistency of all points with T - compute projected position and



2-view Rotation Estimation

Find features + raw matches, use RANSAC to find Similarity







2-view Rotation Estimation

Remove outliers, can now solve for R using least squares







2-view Rotation Estimation

Final rotation estimation





Object Instance Recognition

Database of planar objects











Instance recognition





Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Object Instance Recognition with SIFT

- Match SIFT descriptors between query image and a database of known keypoints extracted from training examples
- use fast (approximate) nearest neighbour matching
- threshold based on ratio of distances between 1NN and 2NN
- Use **RANSAC** to find a **subset of matches** that all agree on an object and geometric transform (e.g., **affine transform**)
- Optionally **refine pose estimate** by recomputing the transformation using all the RANSAC inliers

Fitting a Model to Noisy Data Suppose we are **fitting a line** to a dataset that consists of 50% outliers

We can fit a line using two points

If we draw pairs of points uniformly at random, what fraction of pairs will consist entirely of 'good' data points (inliers)?



RANSAC: How many samples?

Let p_0 be the fraction of outliers (i.e., points on line)

- Let *n* be the number of points needed to define hypothesis (n = 2 for a line in the plane)
- Suppose k samples are chosen
- How many samples do we need to find a good solution?



RANSAC: How many samples? (p = 0.99)

Sample size	Proportion of outliers						
n	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

Figure Credit: Hartley & Zisserman

In practice...



Fig. 9 Validation – Performance vs. cost for RANSAC. We evaluate six RANSAC variants, using 8k SIFT features with "both" matching and a ratio test threshold of r=0.8. The inlier threshold η and iterations limit Γ are variables – we plot only the best η for each method, for clarity, and set a budget of 0.5 seconds per image pair (dotted red line). For each RANSAC variant, we pick the largest Γ under this time "limit" and use it for all validation experiments. Computed on 'n1standard-2' VMs on Google Compute (2 vCPUs, 7.5 GB).



Re-cap: RANSAC

RANSAC is a technique to fit data to a model

- divide data into inliers and outliers
- estimate model from minimal set of inliers
- improve model estimate using all inliers
- alternate fitting with re-classification as inlier/outlier

- easy to implement
- easy to estimate/control failure rate

RANSAC only handles a moderate percentage of outliers without cost blowing up

RANSAC is a general method suited for a wide range of model fitting problems

Menu for Today

Topics:

- **Planar** Geometry
- Image Alignment, Object Recognition

Readings:

- Today's Lecture: Szeliski 2.1, 8.1, Forsyth & Ponce 10.4.2

Reminders:

-Assignment 3: Due Wednesday!



