

CPSC 425: Computer Vision

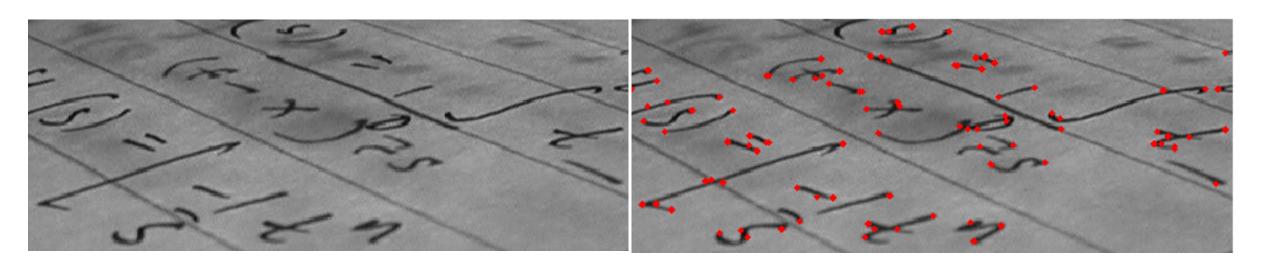


Image Credit: https://en.wikipedia.org/wiki/Corner_detection

Lecture 11: Corner Detection (cont.)

(unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung**)

Menu for Today (October 10, 2024)

Topics:

- Harris Corner Detector (review)
- Blob Detection

- Searching over Scale
- Texture Synthesis & Analysis

Readings:

— Today's Lecture: Forsyth & Ponce (2nd ed.) 5.3, 6.1, 6.3, 3.1-3.3

Reminders:

- Assignment 2: Face Detection in a Scaled Representation is due today
- Assignment 3: Texture Synthesis is out next Wednesday
- (practice) Quiz 1 and Quiz 2 are out; Quiz 3 will be out Monday

Menu for Today (October 10, 2024)

Topics:

- Harris Corner Detector (review)
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Readings:

— Today's Lecture: Forsyth & Ponce (2nd ed.) 5.3, 6.1, 6.3, 3.1-3.3

Reminders:

- Study questions for Midterm will be up on Canvas over the weekend
- Extra office hours next week (Friday)
- Review lecture next Thursday

Today's "fun" Example: Texture Camouflage



https://en.wikipedia.org/wiki/File:Camouflage.jpg

Today's "fun" Example: Texture Camouflage

Cuttlefish on gravel seabed



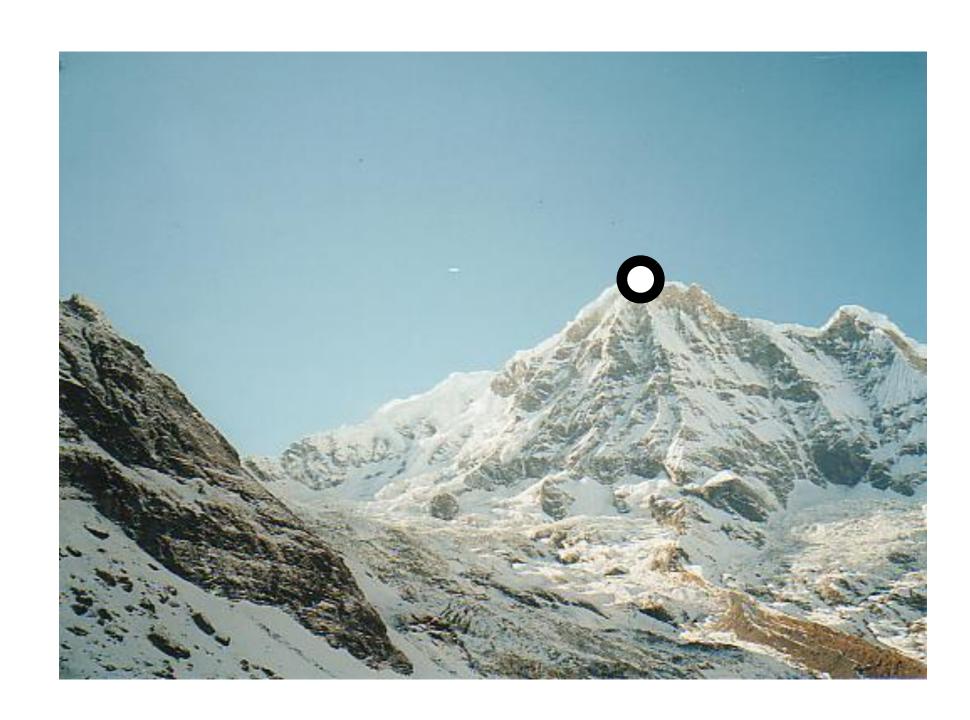
Seconds later...



Lecture 10: Re-cap (Correspondence Problem)

A basic problem in Computer Vision is to establish matches (correspondences) between images

This has **many** applications: rigid/non-rigid tracking, object recognition, image registration, structure from motion, stereo...





Lecture 10: Re-cap (Feature Detectors [last time and today])

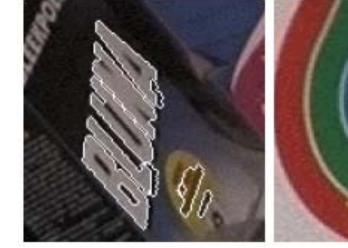


Corners/Blobs



Edges





Regions



Straight Lines

Lecture 10: Re-cap (Feature Descriptors [later — after midterm])

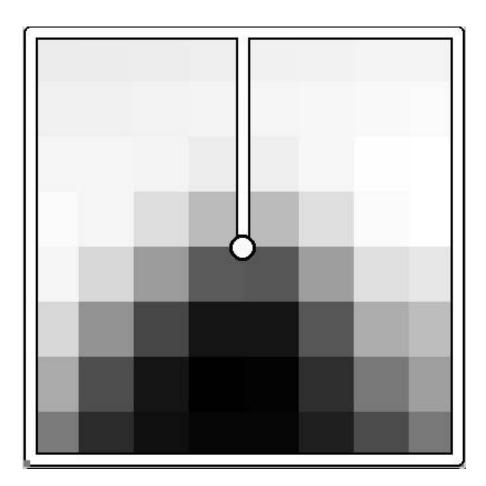
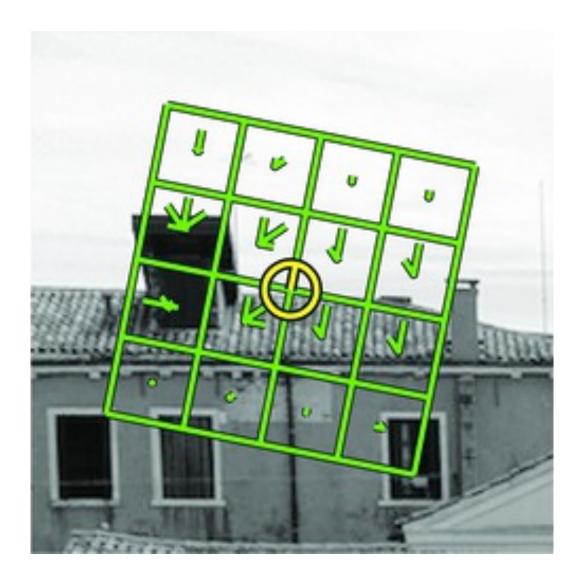
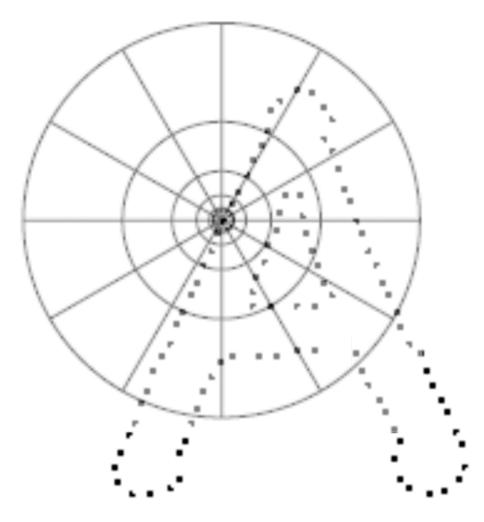


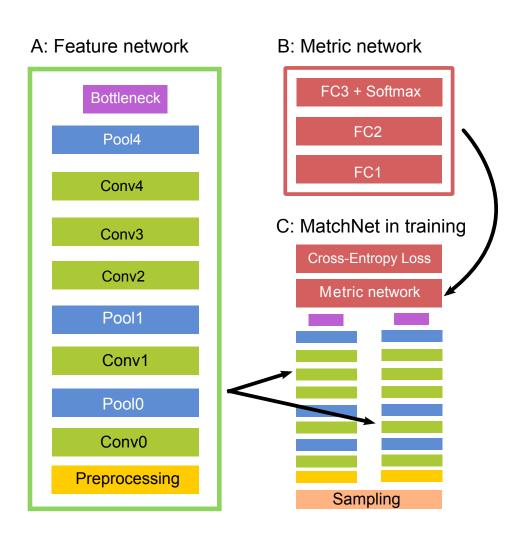
Image Patch



SIFT

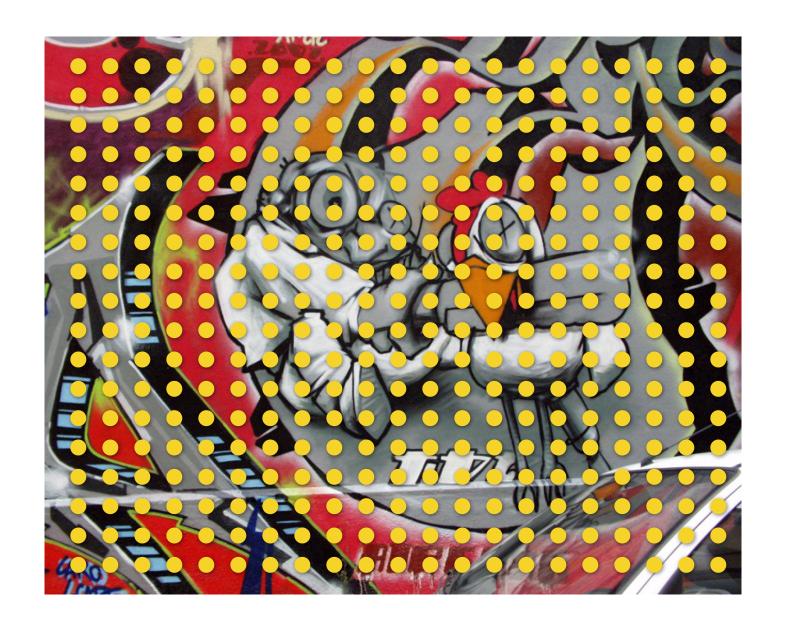


Shape Context

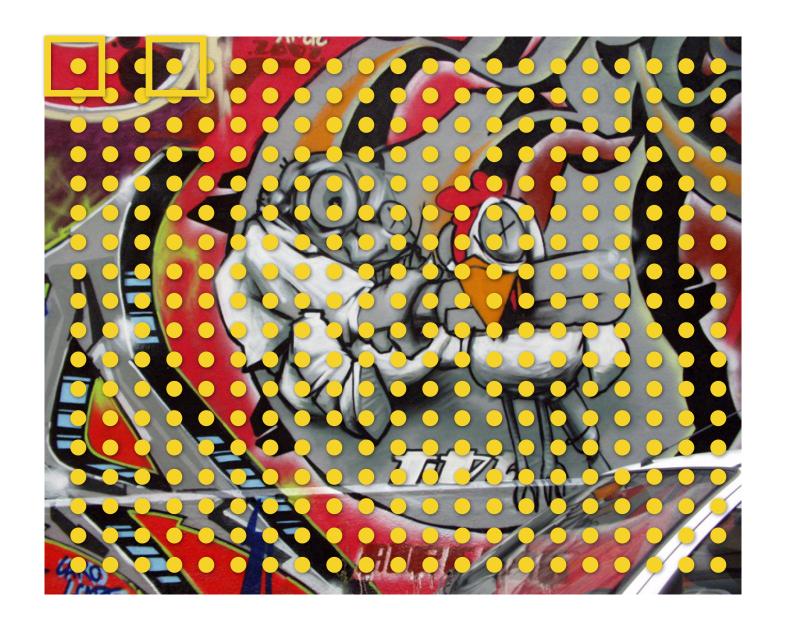


Learned Descriptors

Use **small neighborhoods** of pixels to do **feature detection** — find locations in image that we MAY be able to match (sometimes this will also come with an estimate of the scale or canonical orientation of the feature)



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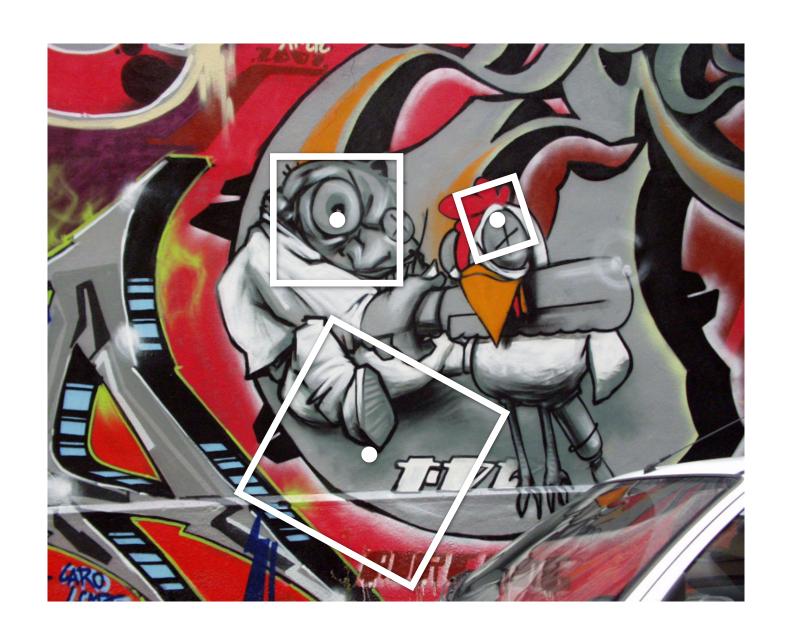
Use **small neighborhoods** of pixels to do **feature detection** — find locations in image that we MAY be able to match (sometimes this will also come with an estimate of the scale or canonical orientation of the feature)

Use (typically larger neighborhoods) around the feature detections to characterize the region well, using a **feature descriptor**, in order to do matching (the scale and orientation, if available, will impact the region of descriptor)



Use **small neighborhoods** of pixels to do **feature detection** — find locations in image that we MAY be able to match (sometimes this will also come with an estimate of the scale or canonical orientation of the feature)

Use (typically larger neighborhoods) around the feature detections to characterize the region well, using a **feature descriptor**, in order to do matching (the scale and orientation, if available, will impact the region of descriptor)



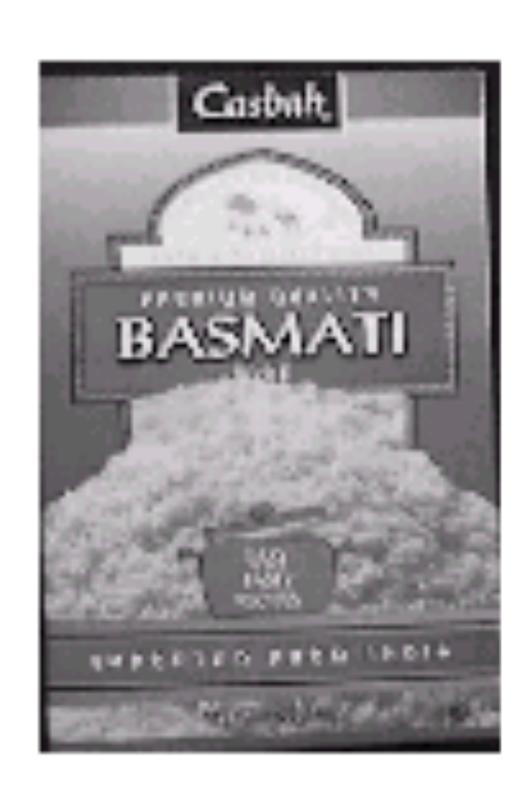
Local: features are local, robust to occlusion and clutter

Accurate: precise localization

Robust: noise, blur, compression, etc. do not have a big impact on the feature.

Distinctive: individual features can be easily matched

Efficient: close to real-time performance



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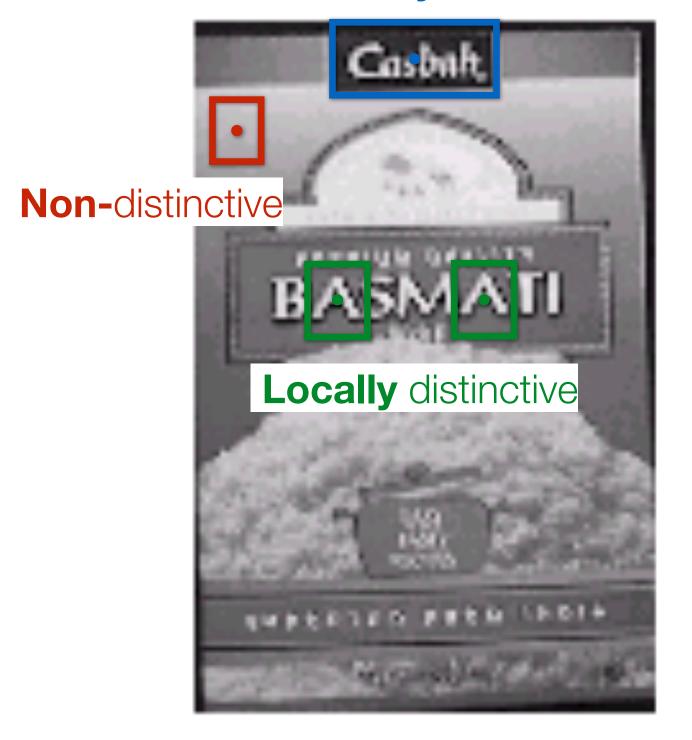
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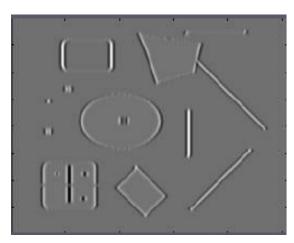
Globally distinctive



Lecture 10: Re-cap (Harris Corner Detection)

- 1.Compute image gradients over small region
- 2. Compute the covariance matrix
- 3.Compute eigenvectors and eigenvalues
- 4. Use threshold on eigenvalues to detect corners

$$I_x = \frac{\partial I}{\partial x}$$



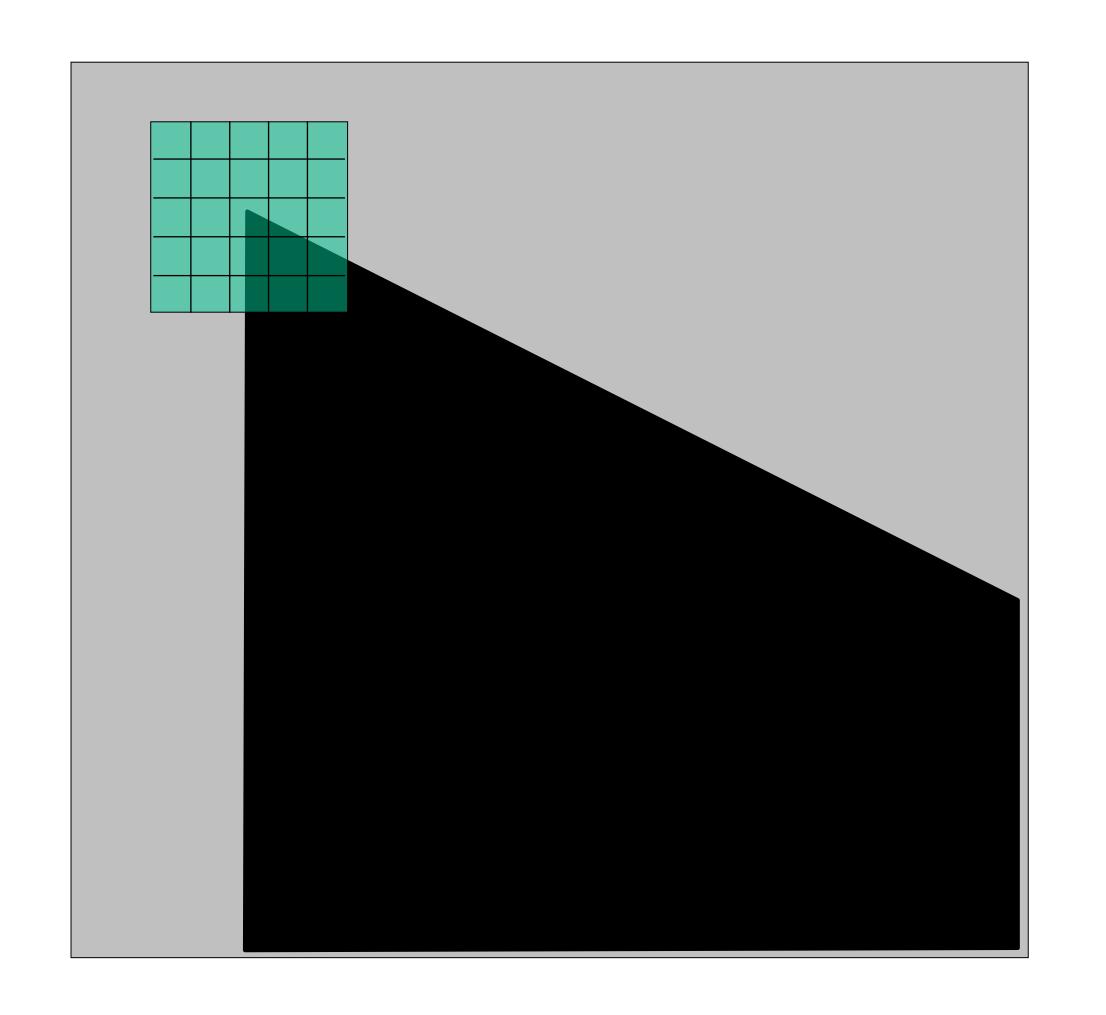
$$I_y = \frac{\partial I}{\partial y}$$



$$\left[egin{array}{ccc} \sum\limits_{p\in P}I_xI_x & \sum\limits_{p\in P}I_xI_y \ \sum\limits_{p\in P}I_yI_x & \sum\limits_{p\in P}I_yI_y \ \end{array}
ight]$$

Lecture 10: Re-cap (compute image gradients at patch)

(not just a single pixel)



array of x gradients

$$I_x = \frac{\partial I}{\partial x}$$

array of y gradients

$$I_y = \frac{\partial I}{\partial y}$$

Lecture 10: Re-cap (compute the covariance matrix)

Sum over small region around the corner

Gradient with respect to x, times gradient with respect to y

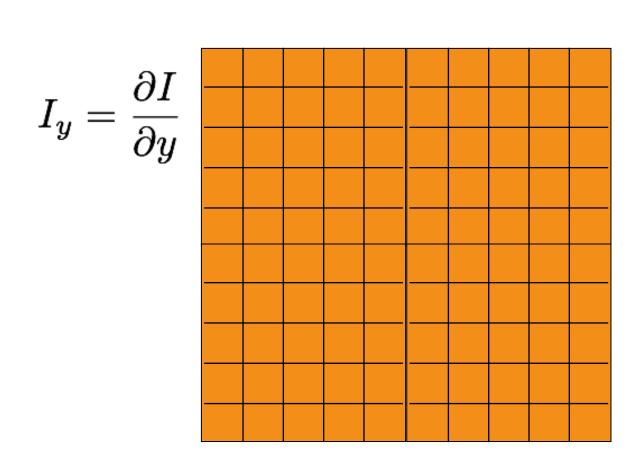
$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

Matrix is **symmetric**

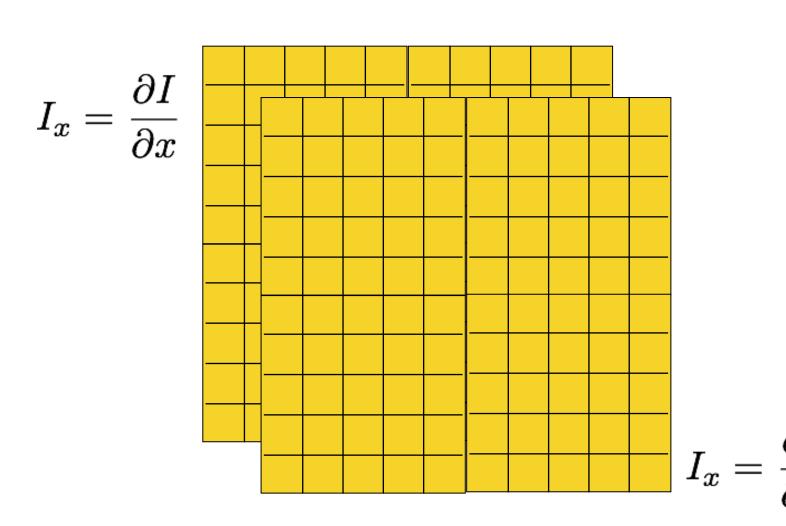
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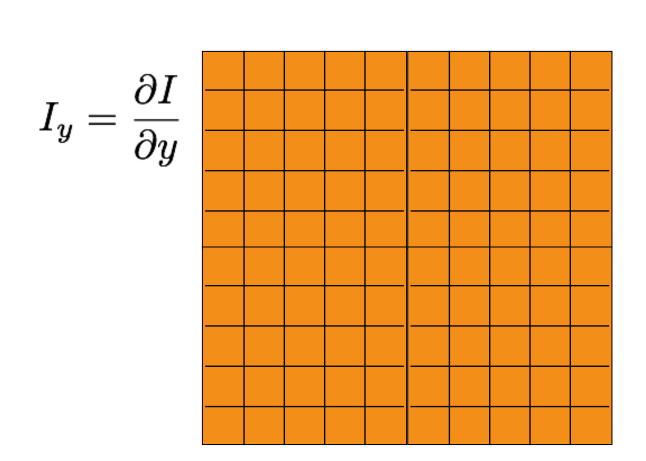
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$$I_x = rac{\partial I}{\partial x}$$

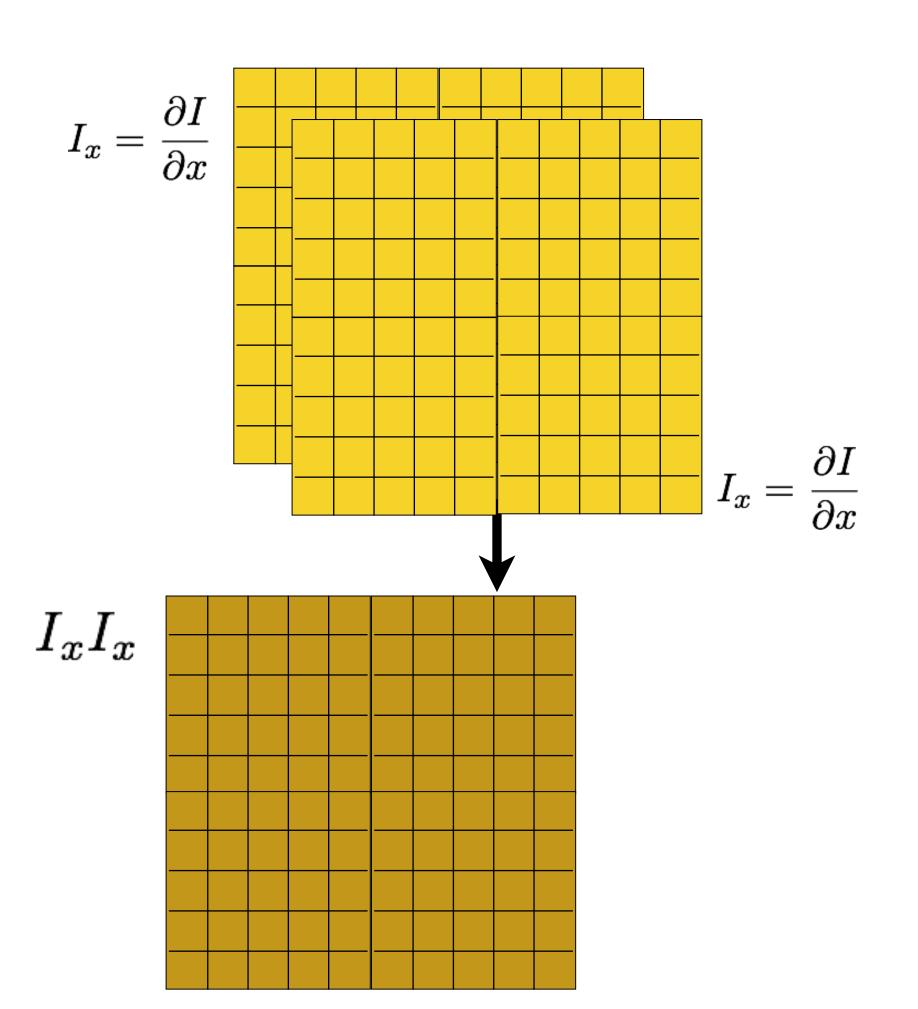


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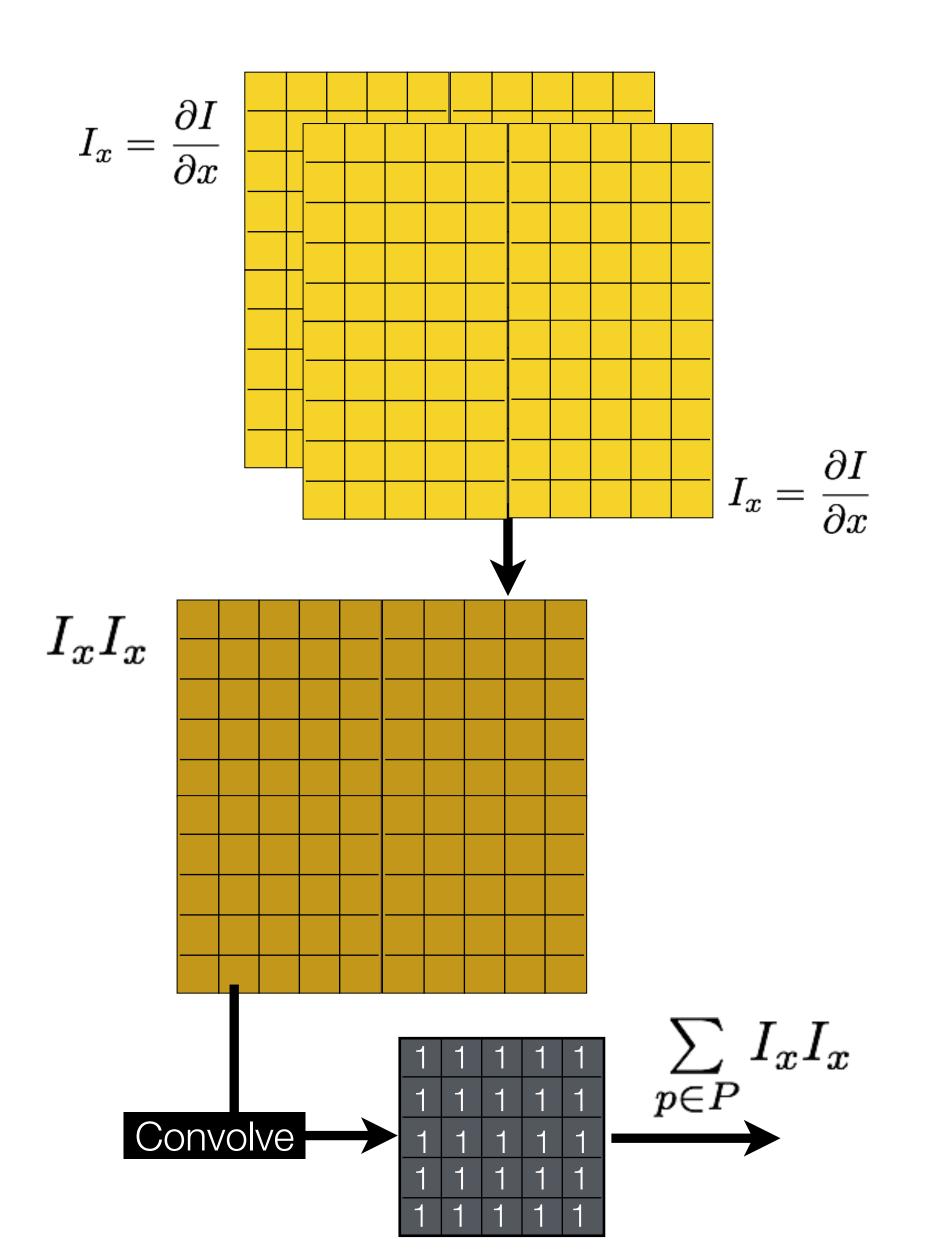


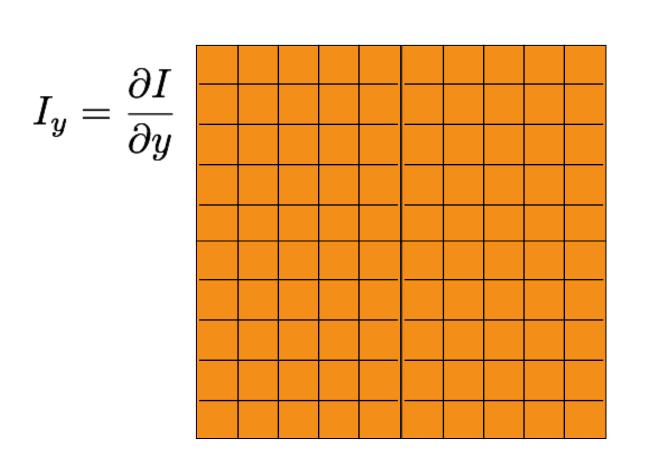
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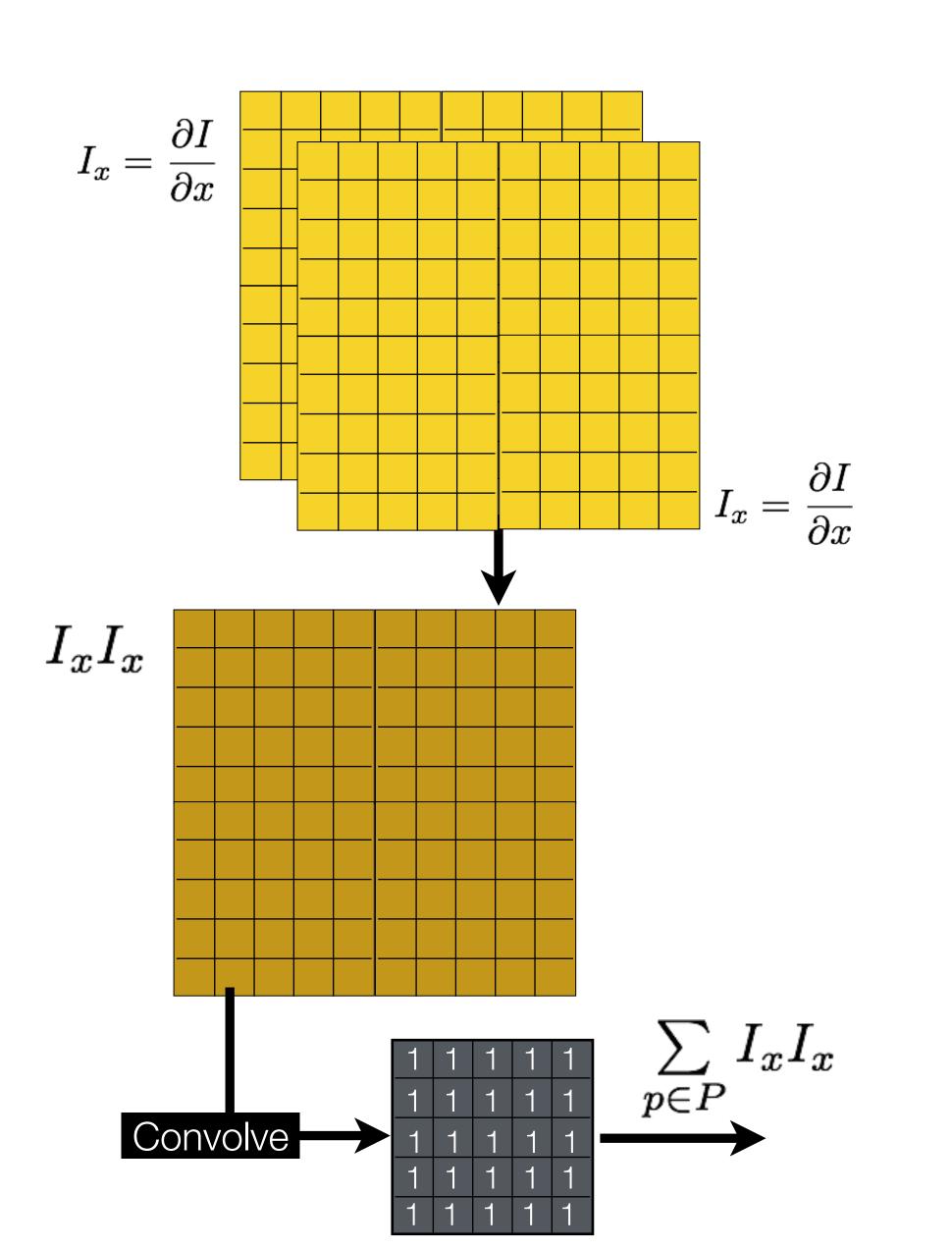
$$I_y = rac{\partial I}{\partial y}$$

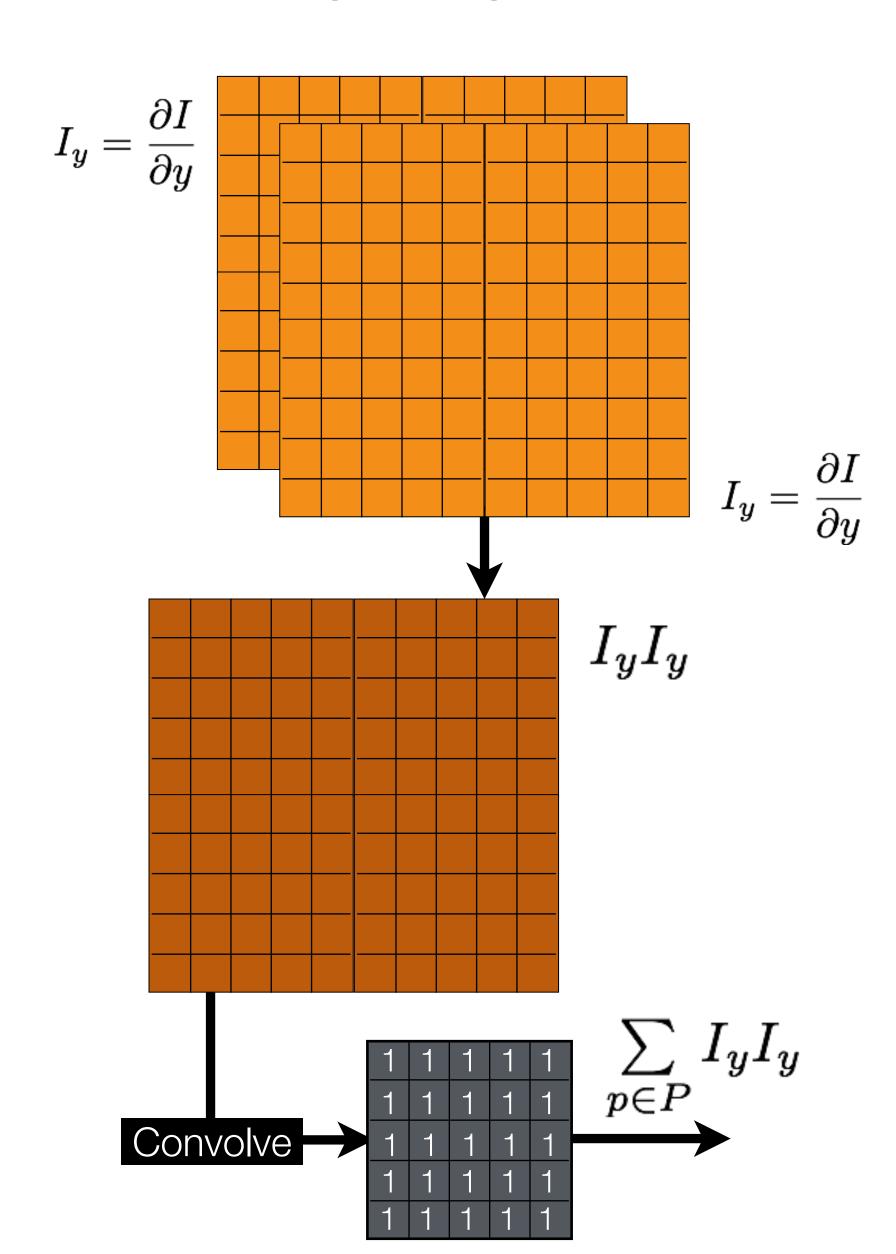
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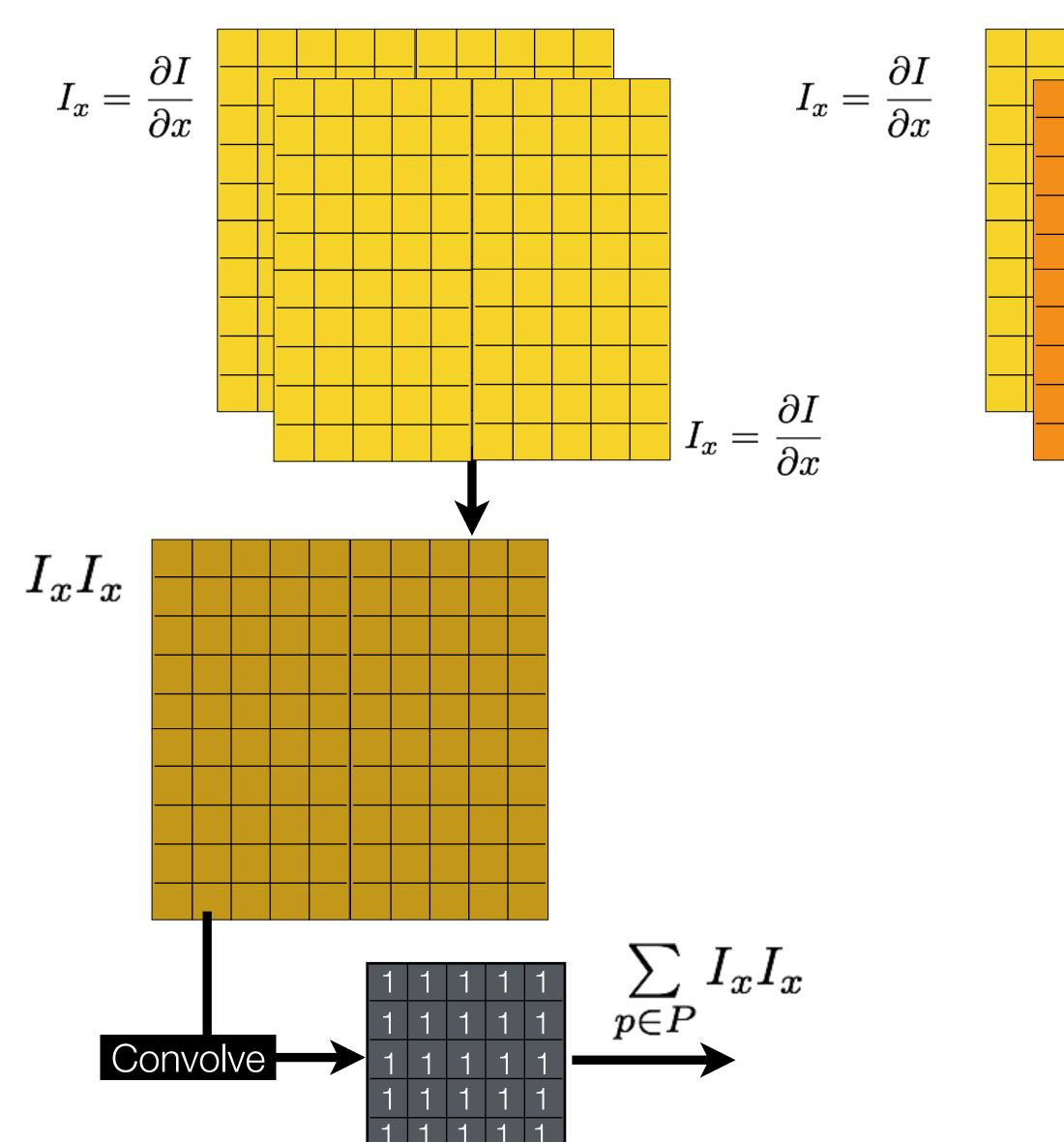


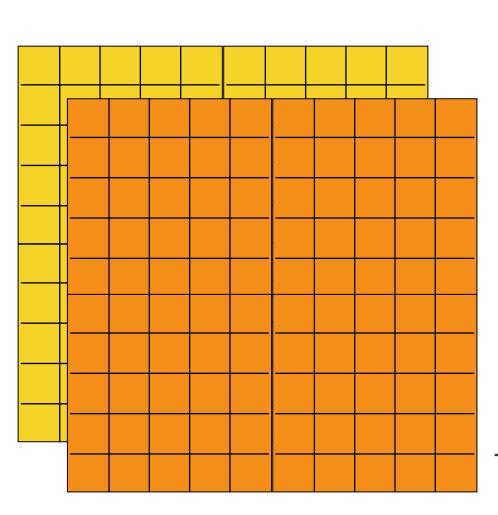
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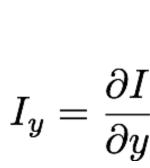


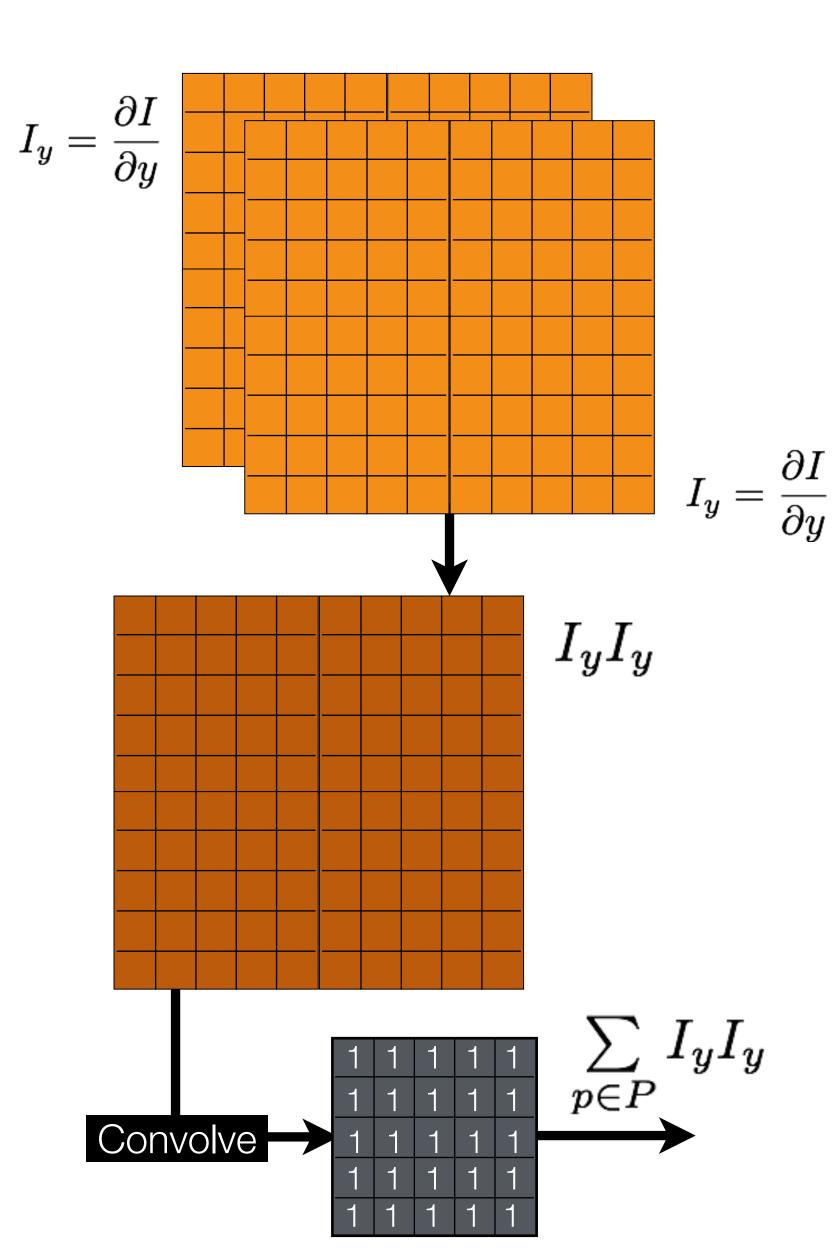


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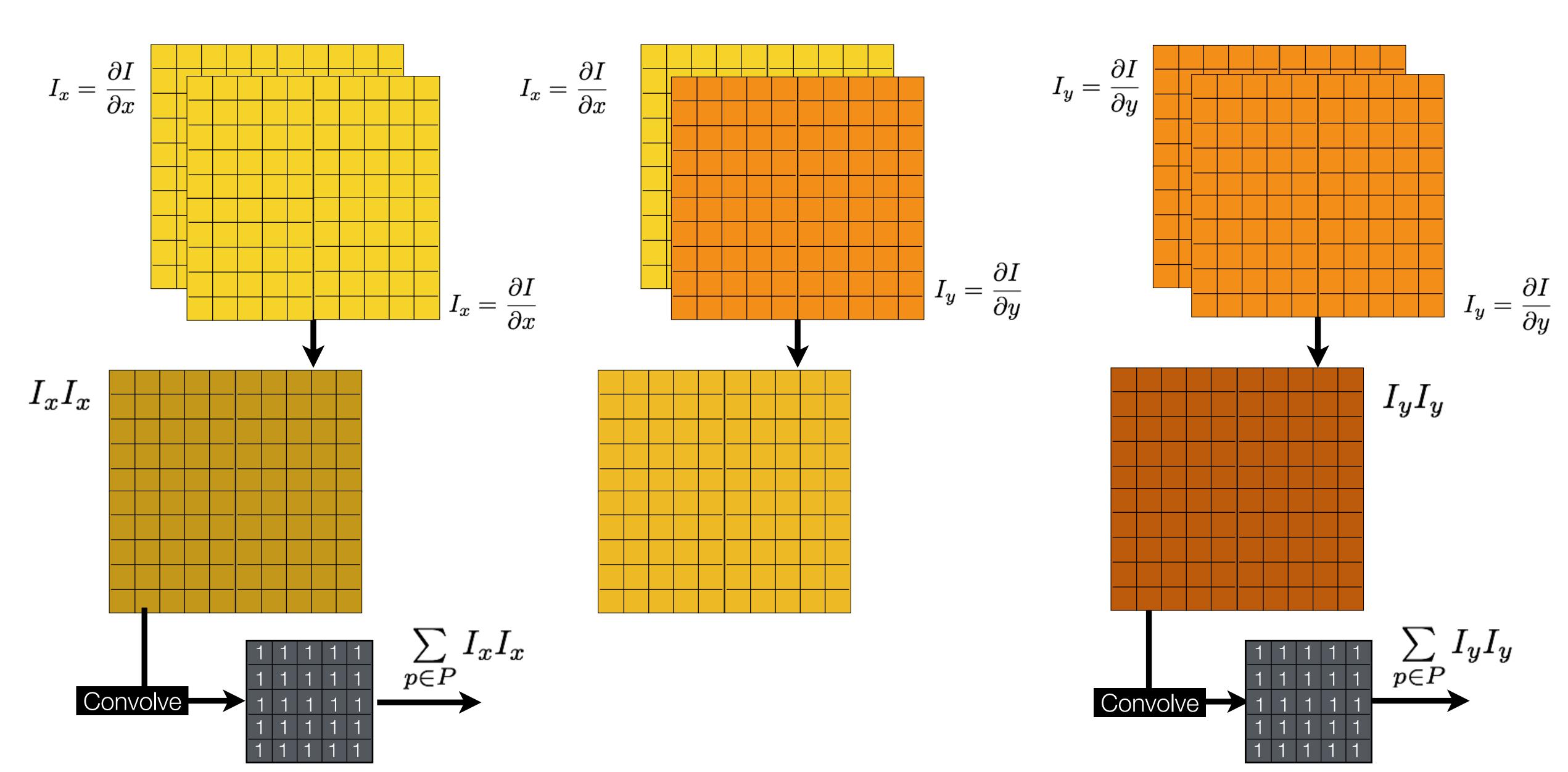






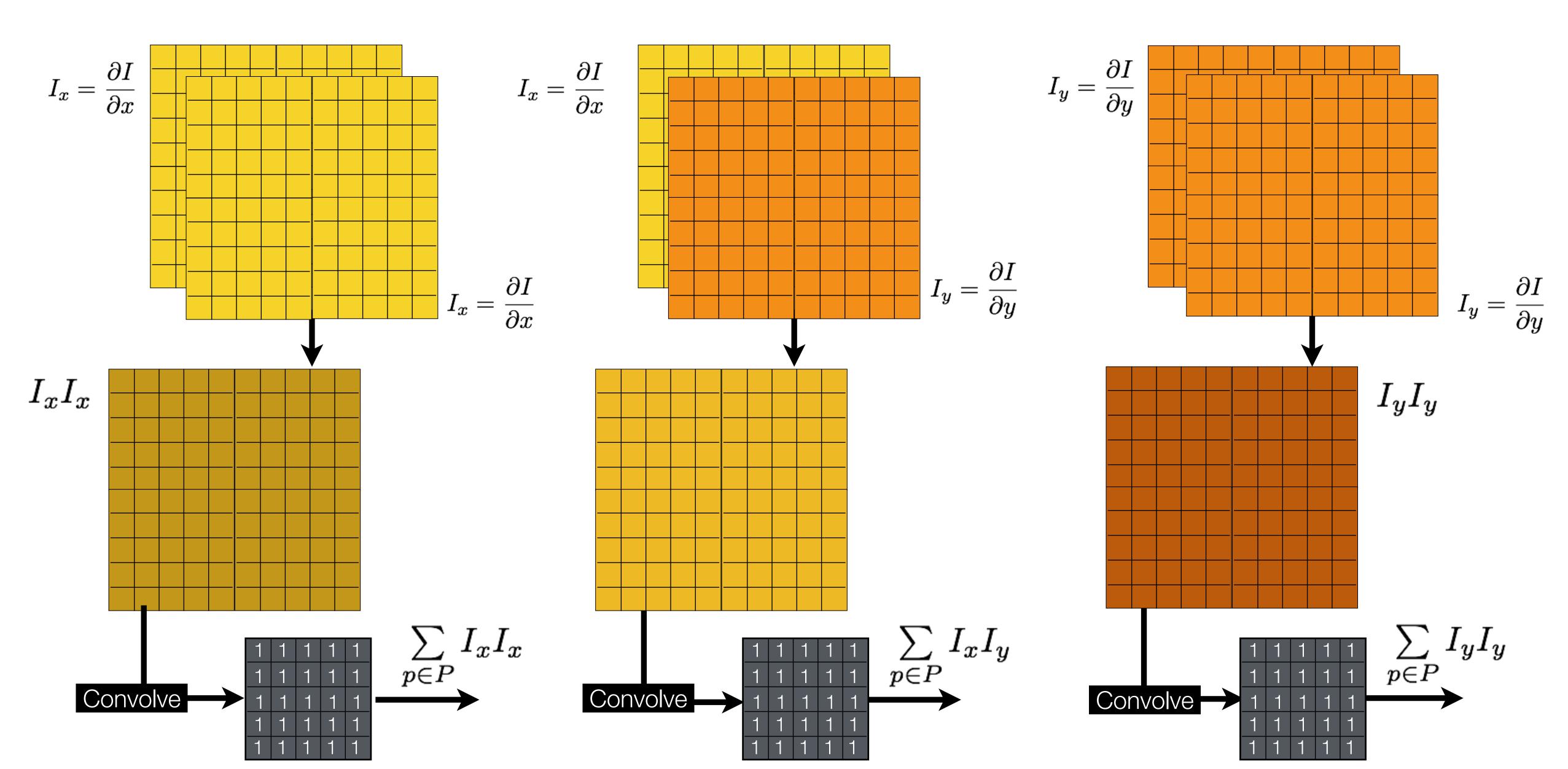


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Computing Covariance Matrix Efficiently

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$



Lecture 10: Re-cap

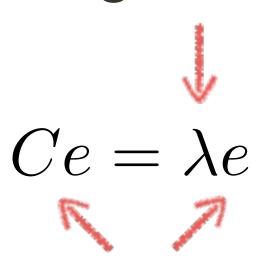
It can be shown that since every C is symmetric:



$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

Lecture 10: Re-cap (computing eigenvalues and eigenvectors)

eigenvalue



$$(C - \lambda I)e = 0$$

1. Compute the determinant of (returns a polynomial)

$$C - \lambda I$$

2. Find the roots of polynomial (returns eigenvalues)

$$\det(C - \lambda I) = 0$$

3. For each eigenvalue, solve (returns eigenvectors)

$$(C - \lambda I)e = 0$$

Lecture 10: Re-cap (interpreting eigenvalues)

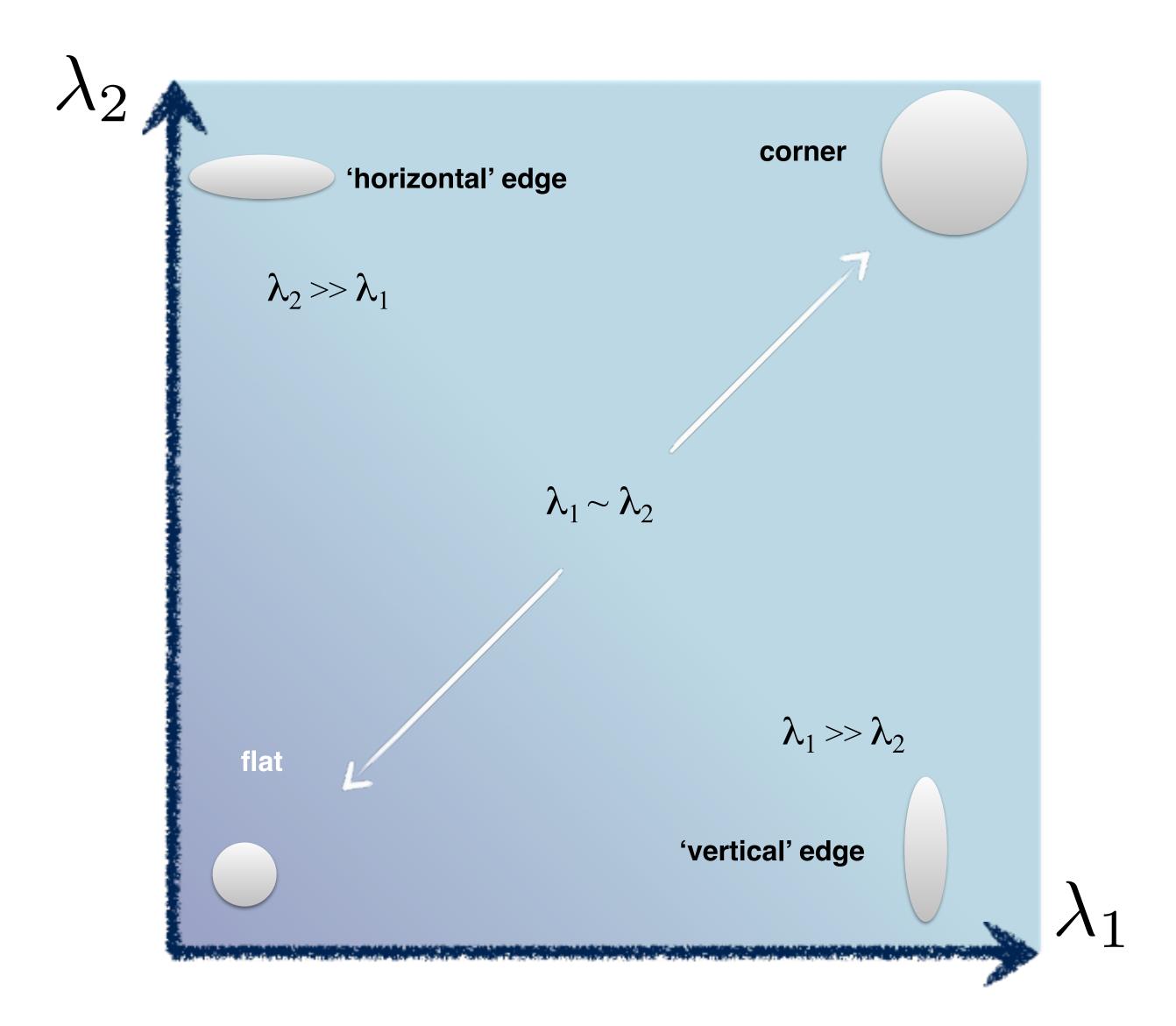
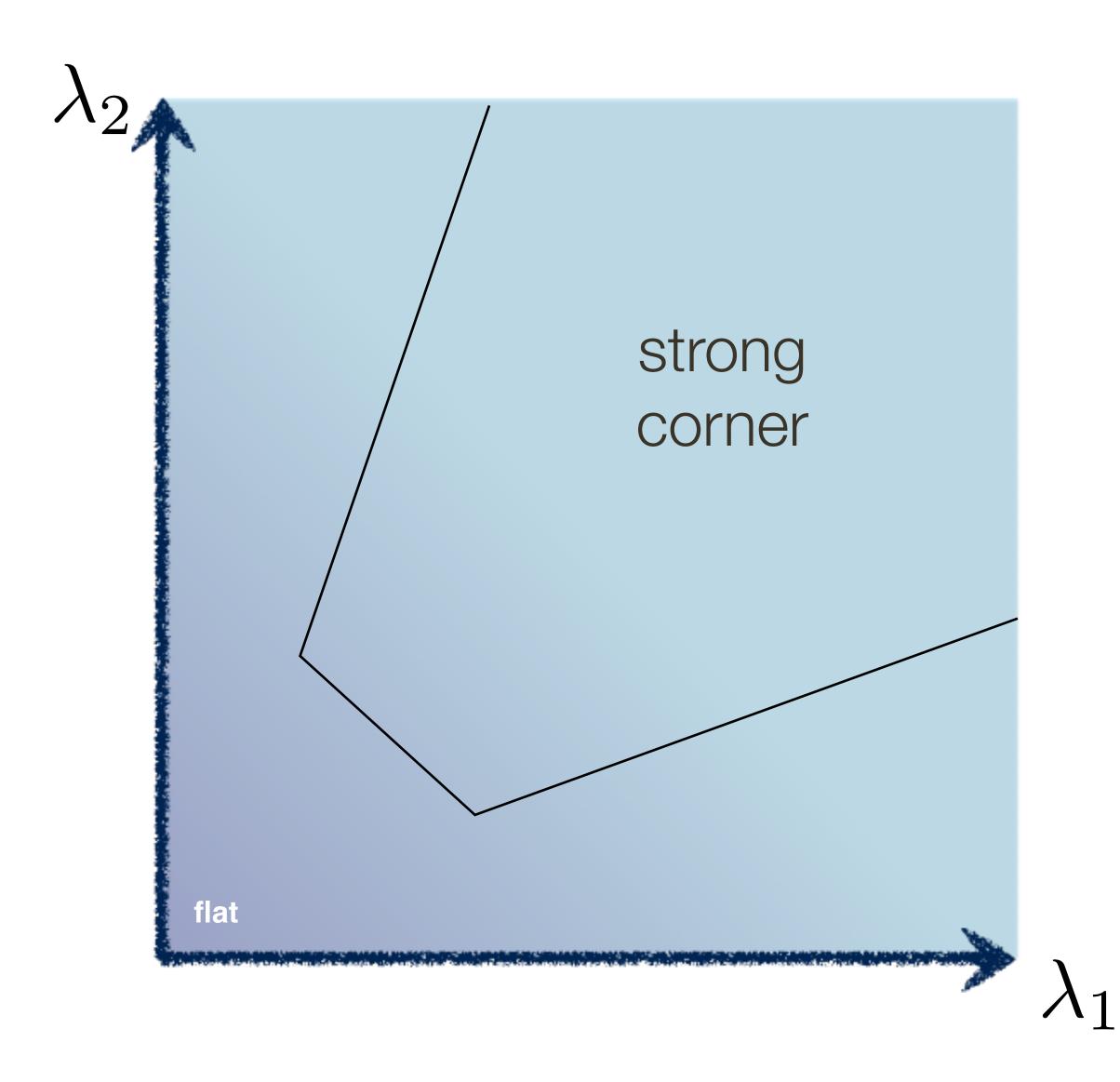


Image Credit: Ioannis (Yannis) Gkioulekas (CMU)

Lecture 10: Re-cap (Threshold on Eigenvalues to Detect Corners)



Think of a function to score 'cornerness'

Lecture 10: Re-cap (Threshold on Eigenvalues to Detect Corners)

Harris & Stephens (1988)

$$\det(C) - \kappa \operatorname{trace}^2(C)$$

Kanade & Tomasi (1994)

$$\min(\lambda_1,\lambda_2)$$

Nobel (1998)

$$\frac{\det(C)}{\operatorname{trace}(C) + \epsilon}$$

Example: Harris Corner Detection

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

Example: Harris Corner Detection

Lets compute a measure of "corner-ness" for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

Lets compute a measure of "corner-ness" for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

Lets compute a measure of "corner-ness" for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

			_	-		
0	0	0	0	0	0	
-1	1	0	0	-1	1	
-1	0	0	0	1	0	
-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

$$I_x = \frac{\partial I}{\partial x}$$

Lets compute a measure of "corner-ness" for the green pixel:

0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
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0	0	1	1	1	0	0
0	0	1	1	1	0	0

0	0	0	0	0	0	
-1	1	0	0	-1	1	
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-1	0	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	
0	-1	0	0	1	0	

0	-1	0	0	0	-1	0
0	0	-1	-1	-1	1	0
0	0	0	0	0	0	0
0	1	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$$I_x = \frac{\partial I}{\partial x}$$

$$I_y = \frac{\partial I}{\partial u}$$

Lets compute a measure of "corner-ness" for the green pixel:

5				6.04	<u> </u>	
0	0	0	0	0	0	0
0	1	0	0	0	1	0
0	1	1	1	1	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0
0	0	1	1	1	0	0

-0.36

Harris Corner Detection Review

- Filter image with Gaussian
- Compute magnitude of the x and y gradients at each pixel
- Construct C in a window around each pixel
 - Harris uses a Gaussian window
- Solve for product of the λ 's

Harris & Stephens (1988)

$$\det(C) - \kappa \operatorname{trace}^2(C)$$

- If λ 's both are big (product reaches local maximum above threshold) then we have a corner
 - Harris also checks that ratio of λs is not too high

Compute the Covariance Matrix

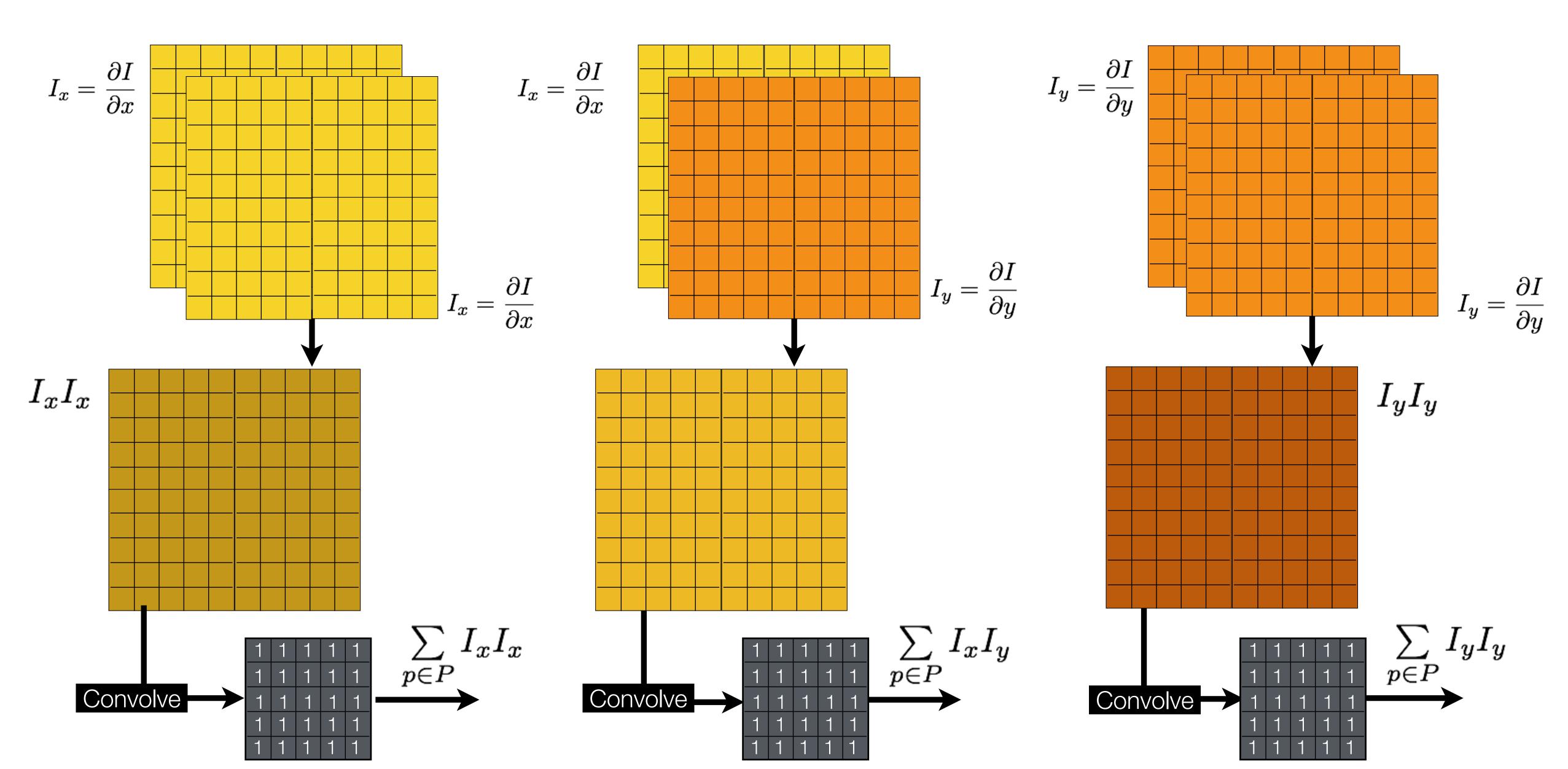
Sum can be implemented as an (unnormalized) box filter with

$$C = \begin{bmatrix} \sum_{p \in P} I_x I_x & \sum_{p \in P} I_x I_y \\ \sum_{p \in P} I_y I_x & \sum_{p \in P} I_y I_y \end{bmatrix}$$

Harris uses a Gaussian weighting instead

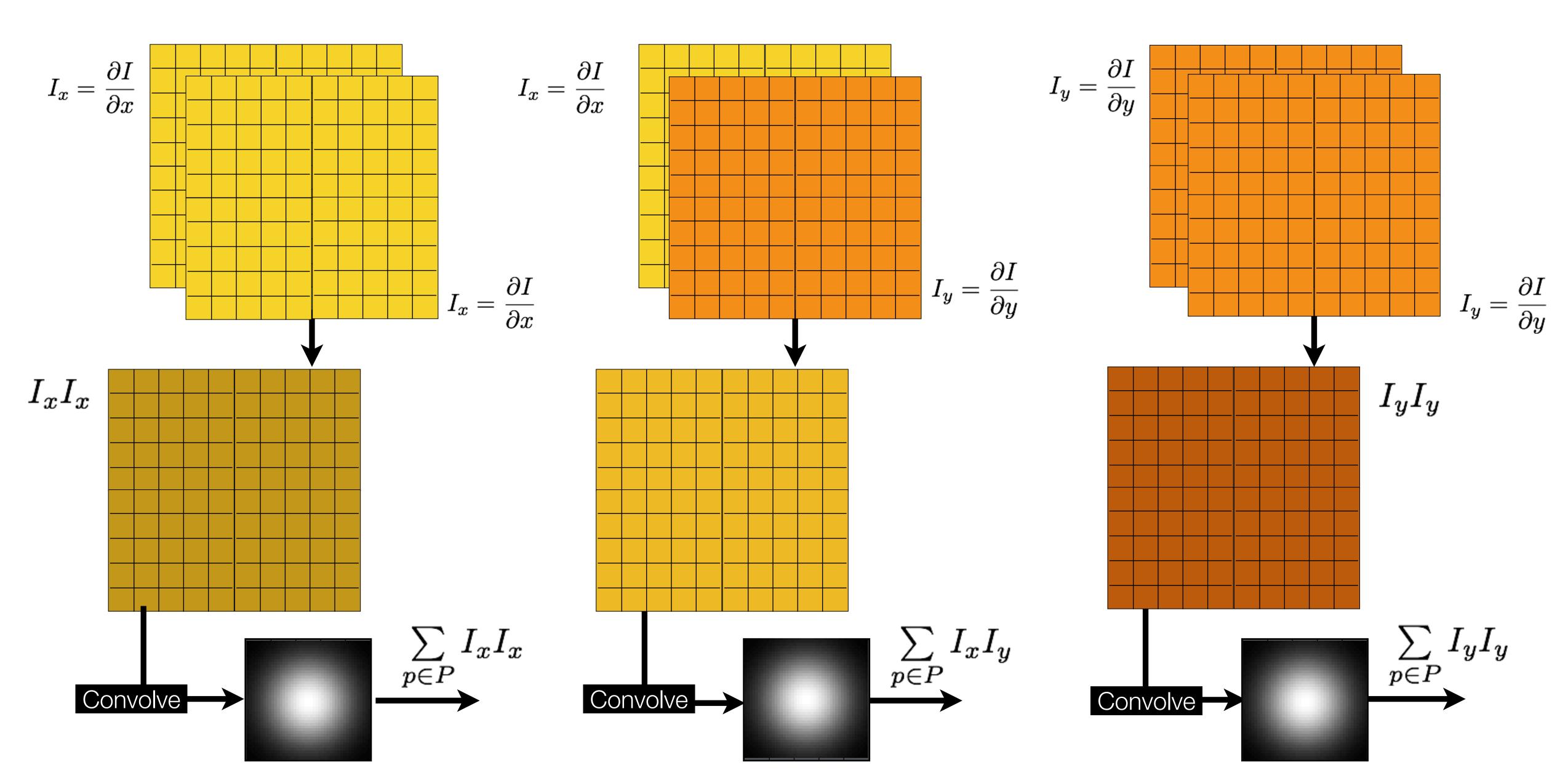
Computing Covariance Matrix Efficiently

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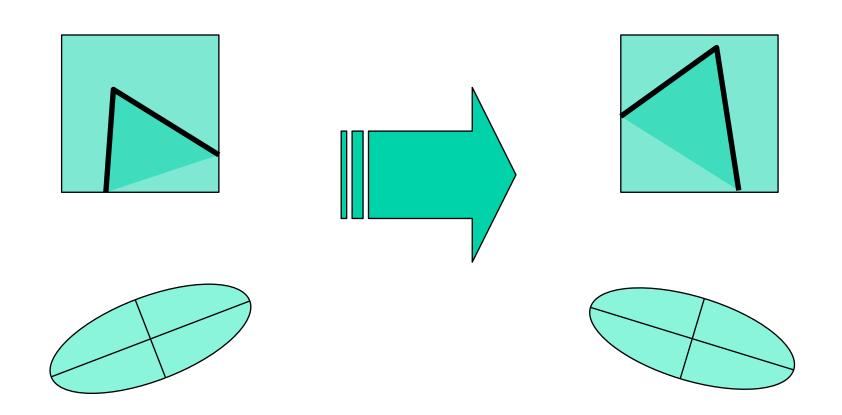


Computing Covariance Matrix Efficiently

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Properties: Rotational Invariance



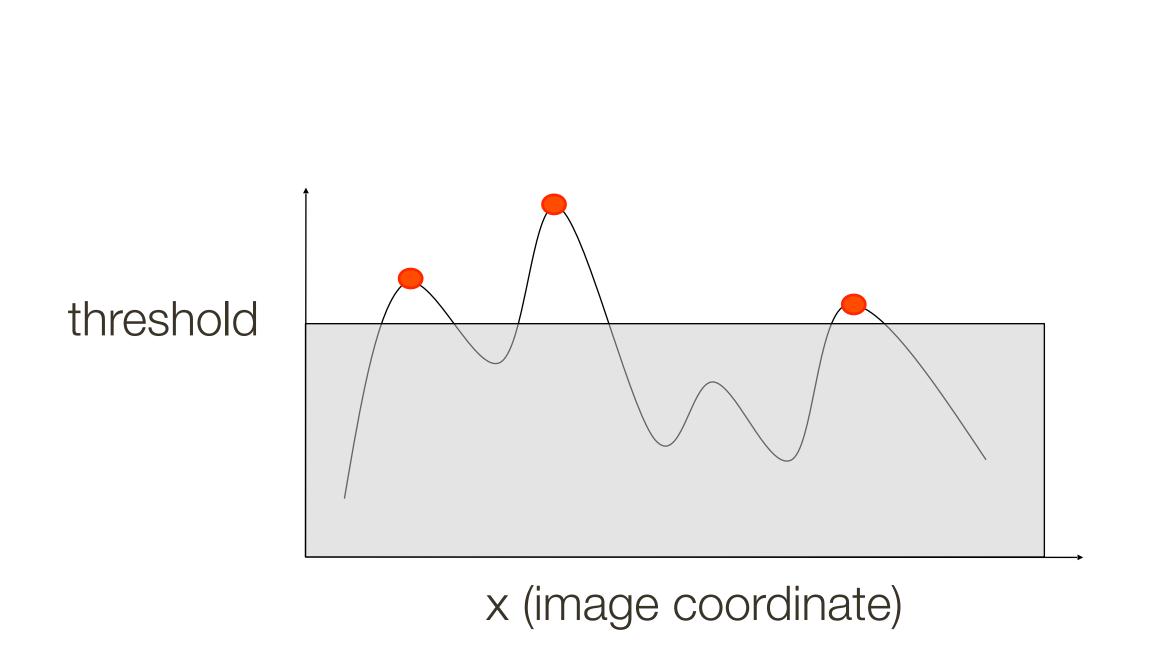
Ellipse rotates but its shape (eigenvalues) remains the same

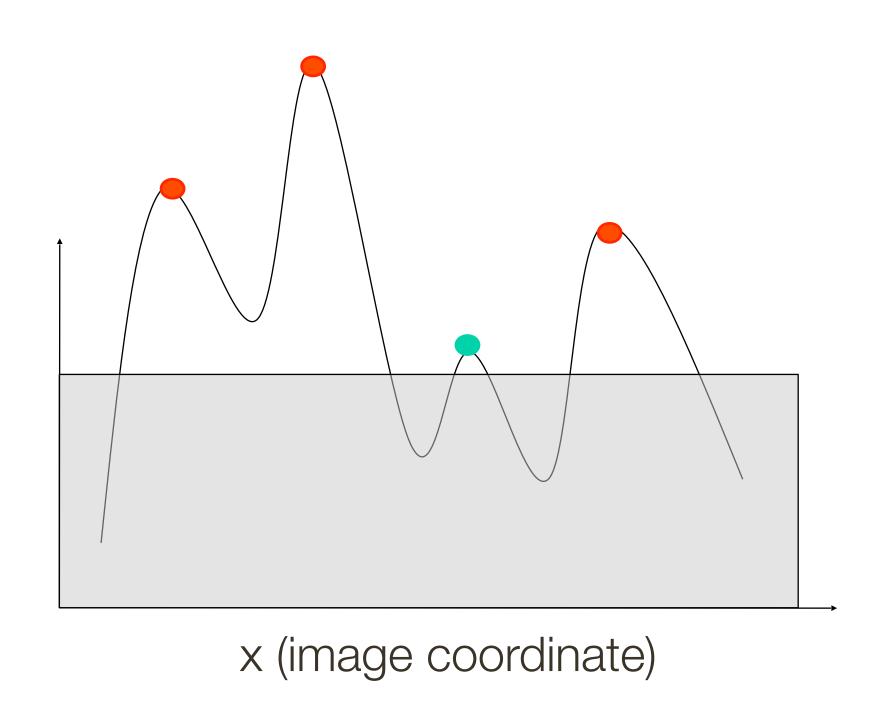
Corner response is invariant to image rotation

Properties: (partial) Invariance to Intensity Shifts and Scaling

Only derivatives are used -> Invariance to intensity shifts

Intensity scale could effect performance





Slide Credit: Ioannis (Yannis) Gkioulekas (CMU)

Properties: NOT Invariant to Scale Changes



Example 2: Wagon Wheel (Harris Results)



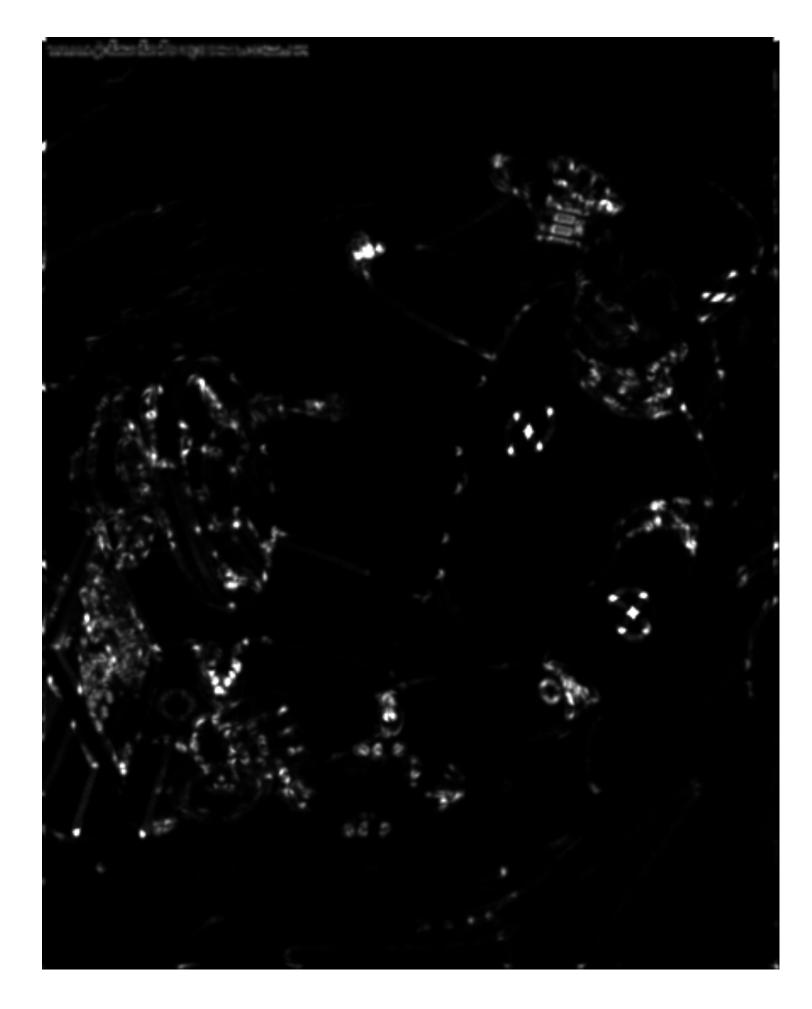




 $\sigma=1$ (219 points) $\sigma=2$ (155 points) $\sigma=3$ (110 points) $\sigma=4$ (87 points)



Example 3: Crash Test Dummy (Harris Result)



corner response image



 $\sigma = 1$ (175 points)

Original Image Credit: John Shakespeare, Sydney Morning Herald

Example 2: Wagon Wheel (Harris Results)



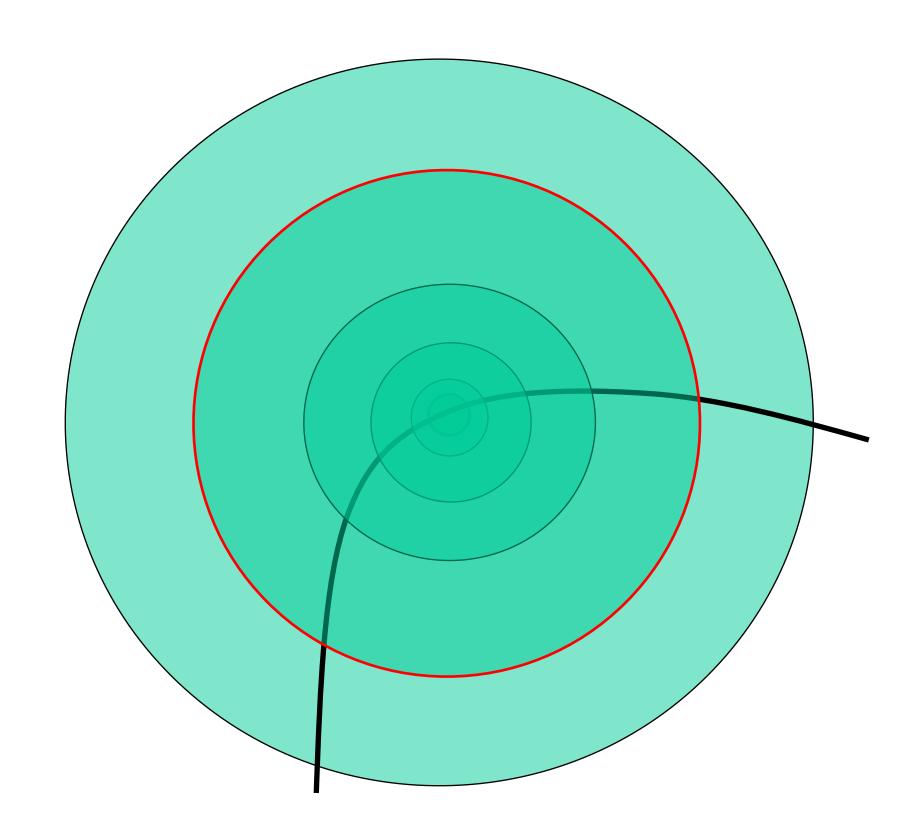
 $\sigma=1$ (219 points) $\sigma=2$ (155 points) $\sigma=3$ (110 points) $\sigma=4$ (87 points)

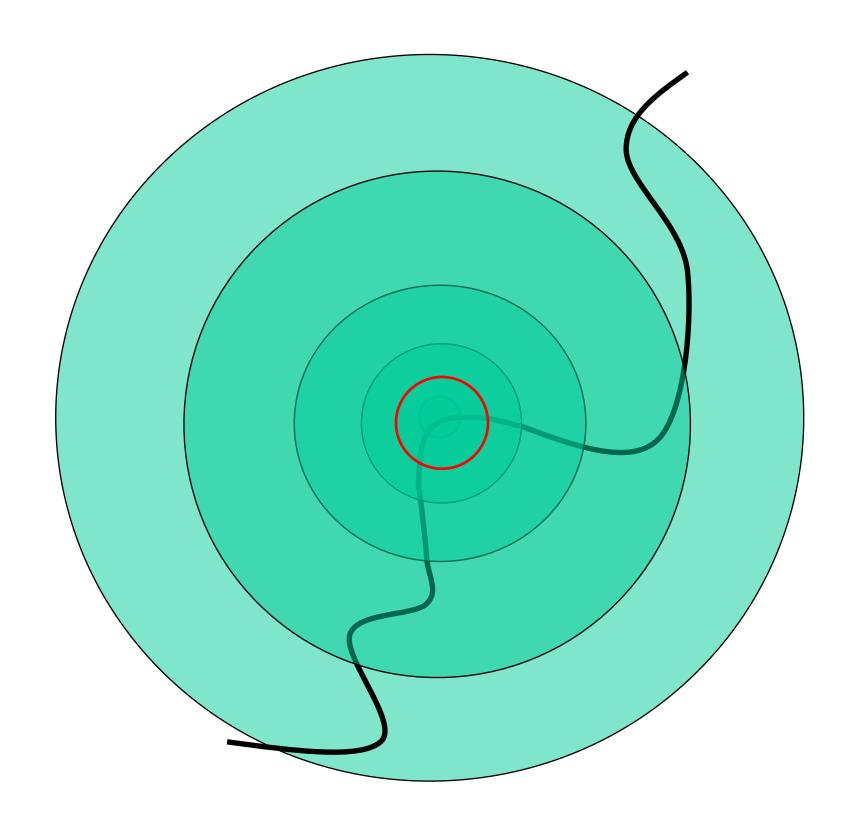






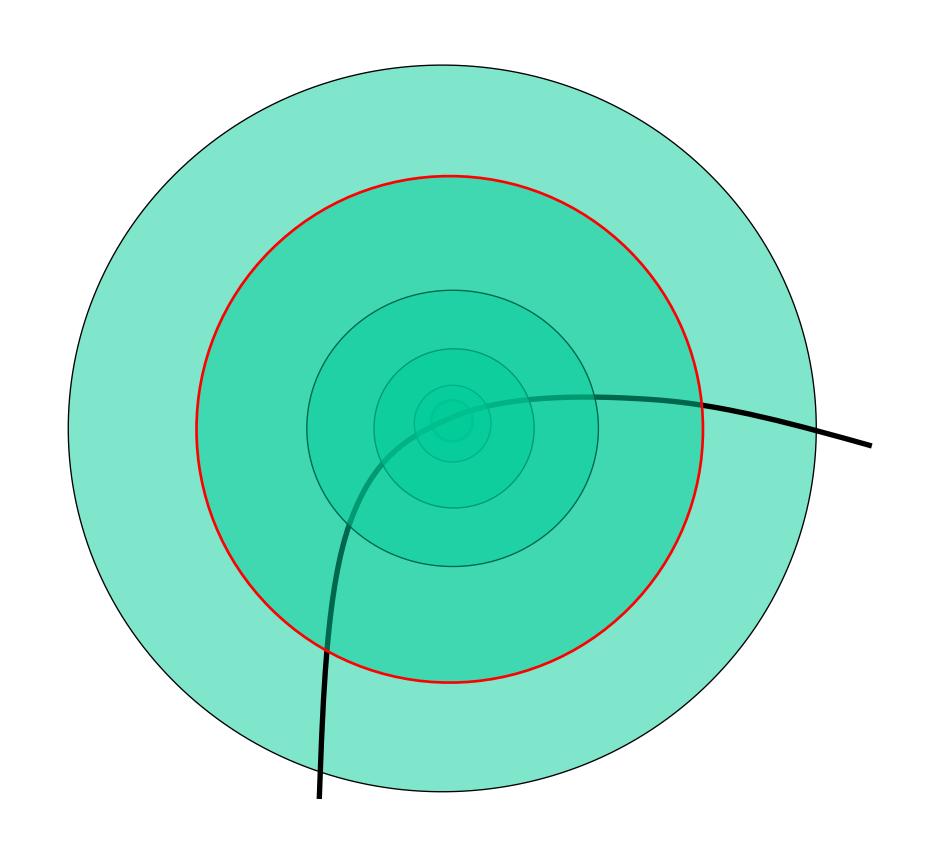
Intuitively ...

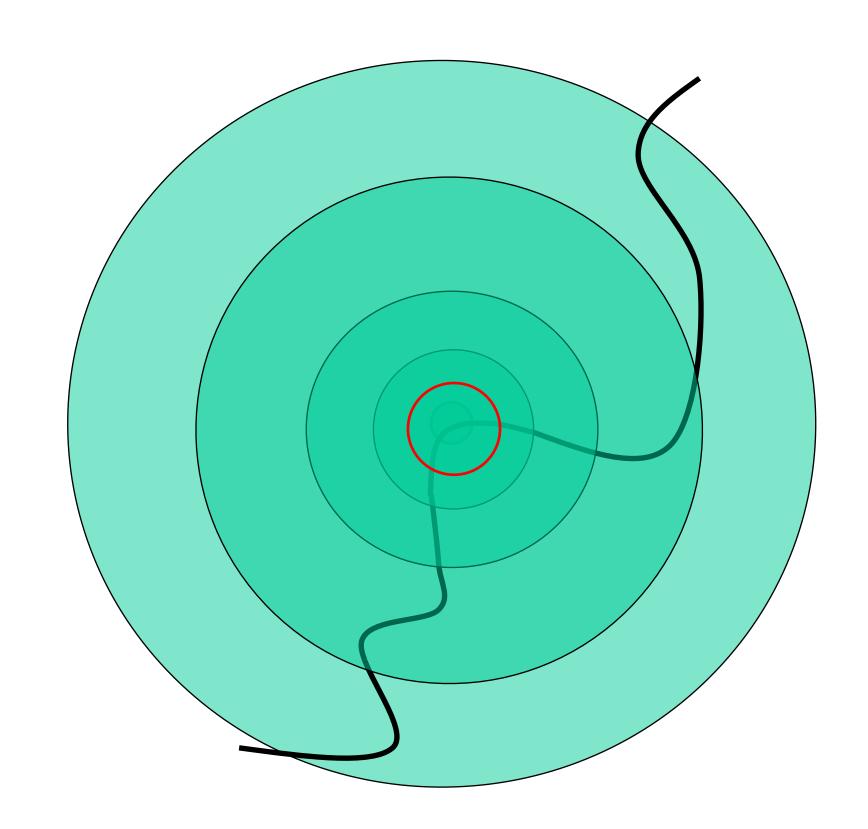


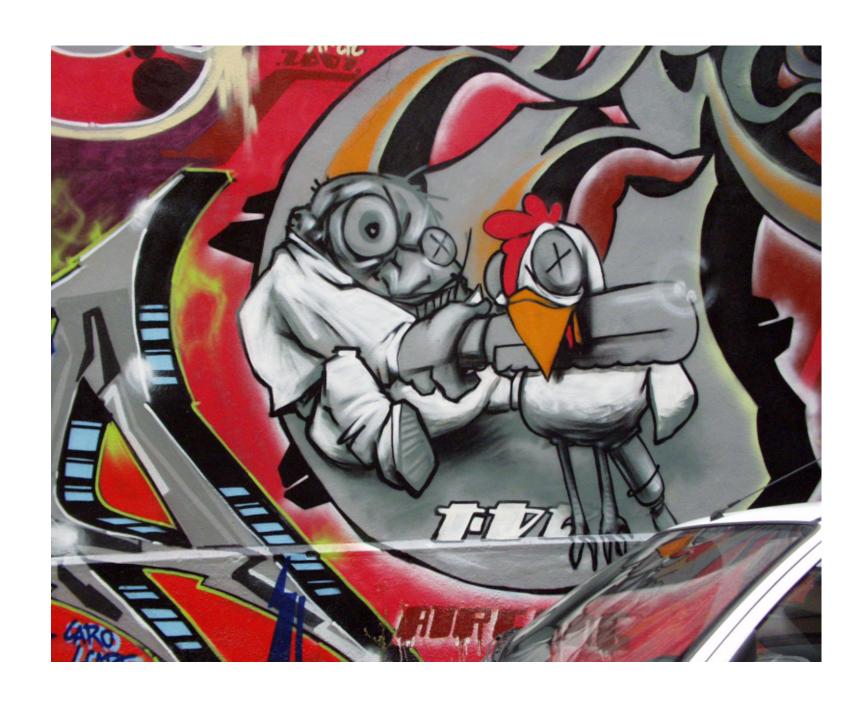


Intuitively ...

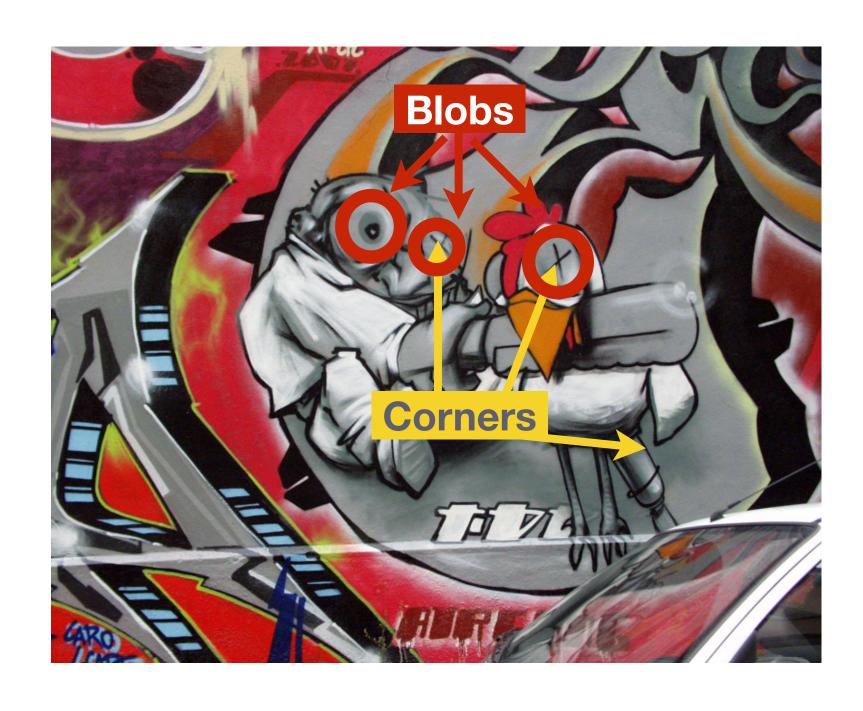
Find local maxima in both position and scale

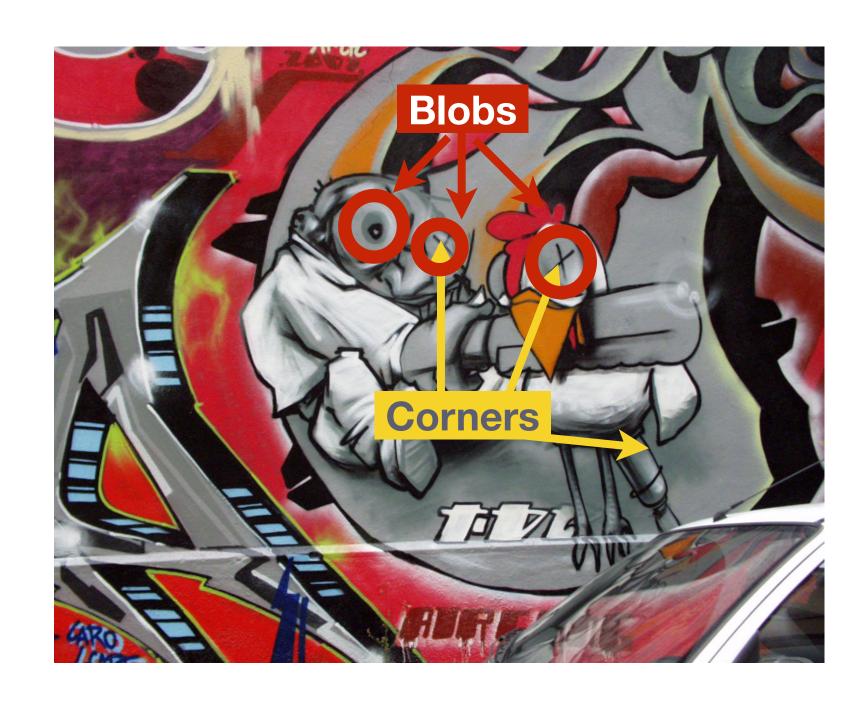




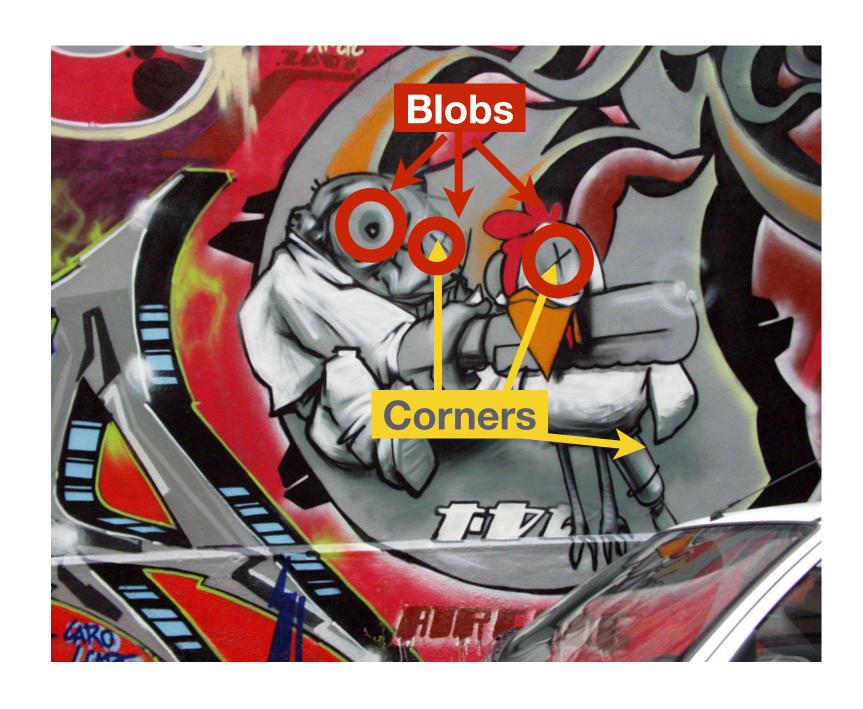








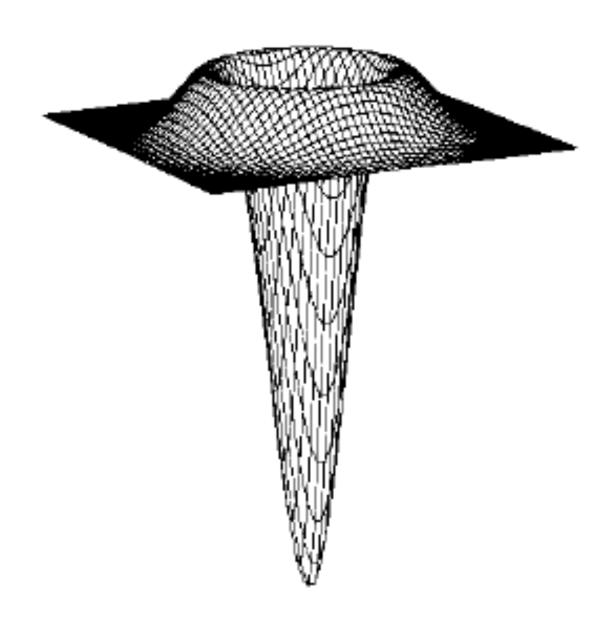






Recall: Marr / Hildreth Laplacian of Gaussian

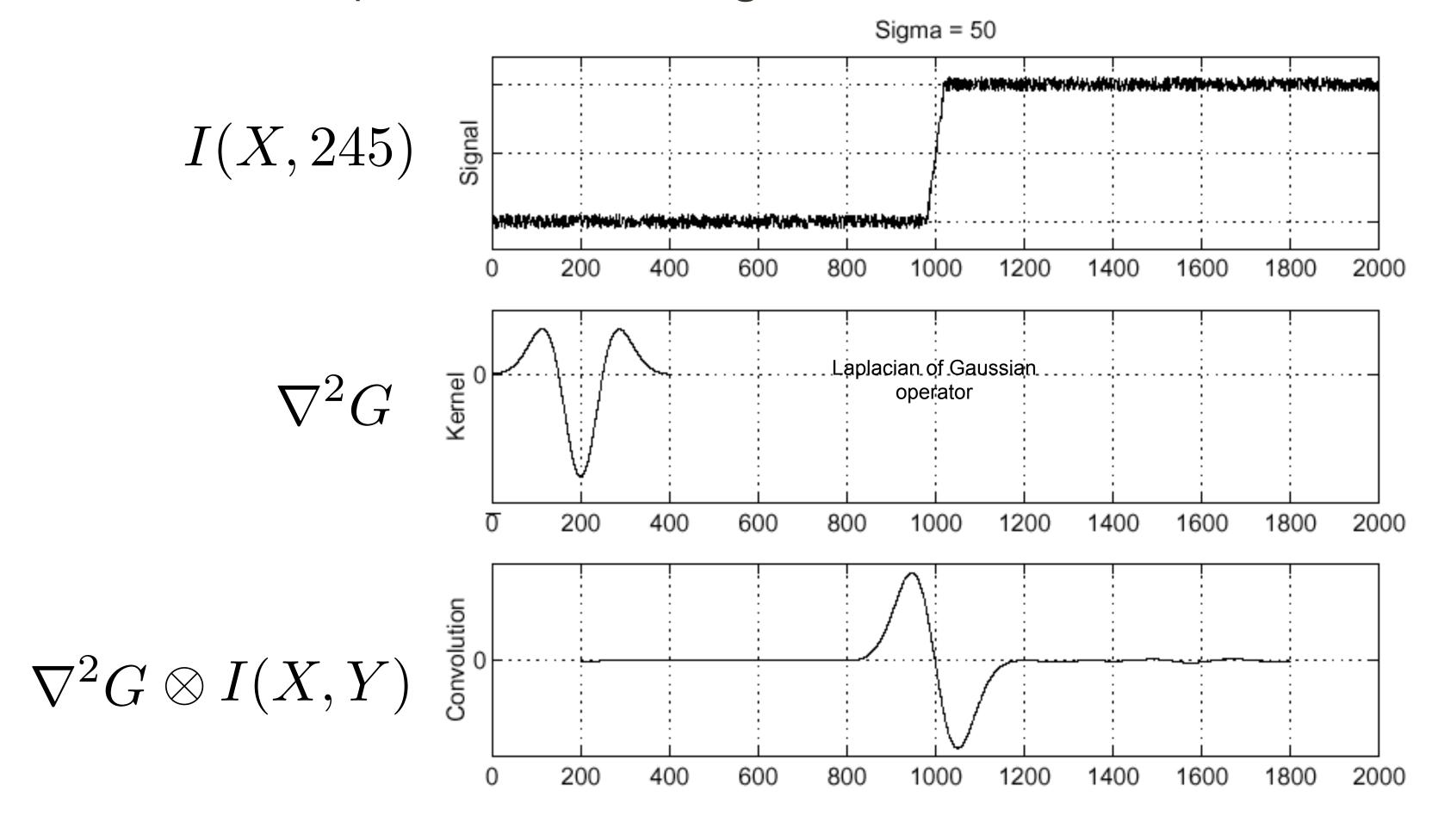
Here's a 3D plot of the Laplacian of the Gaussian ($abla^2G$)



... with its characteristic "Mexican hat" shape

Recall: Marr / Hildreth Laplacian of Gaussian

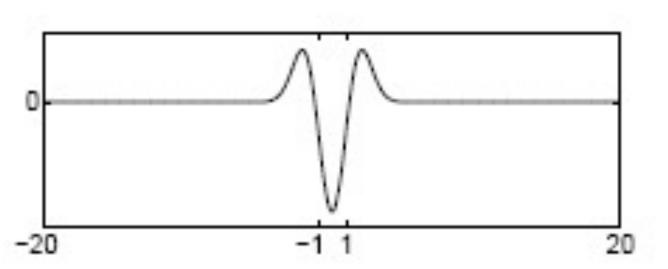
Lets consider a row of pixels in an image:

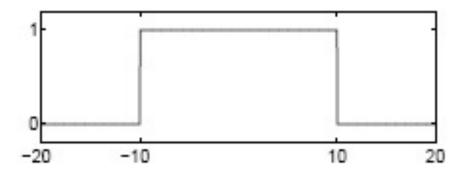


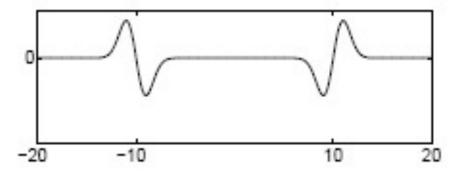
Where is the edge?

Zero-crossings of bottom graph

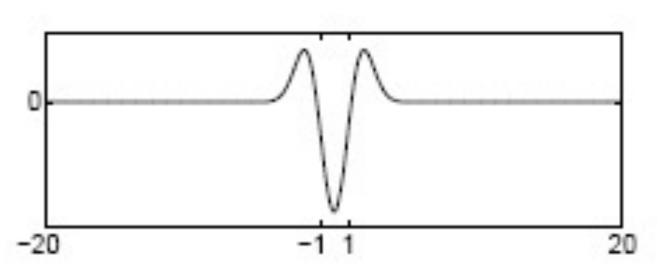


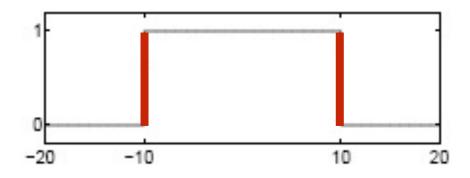


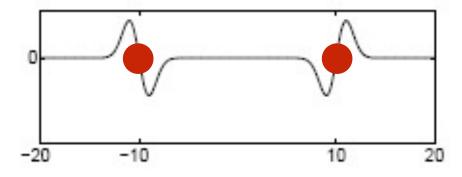


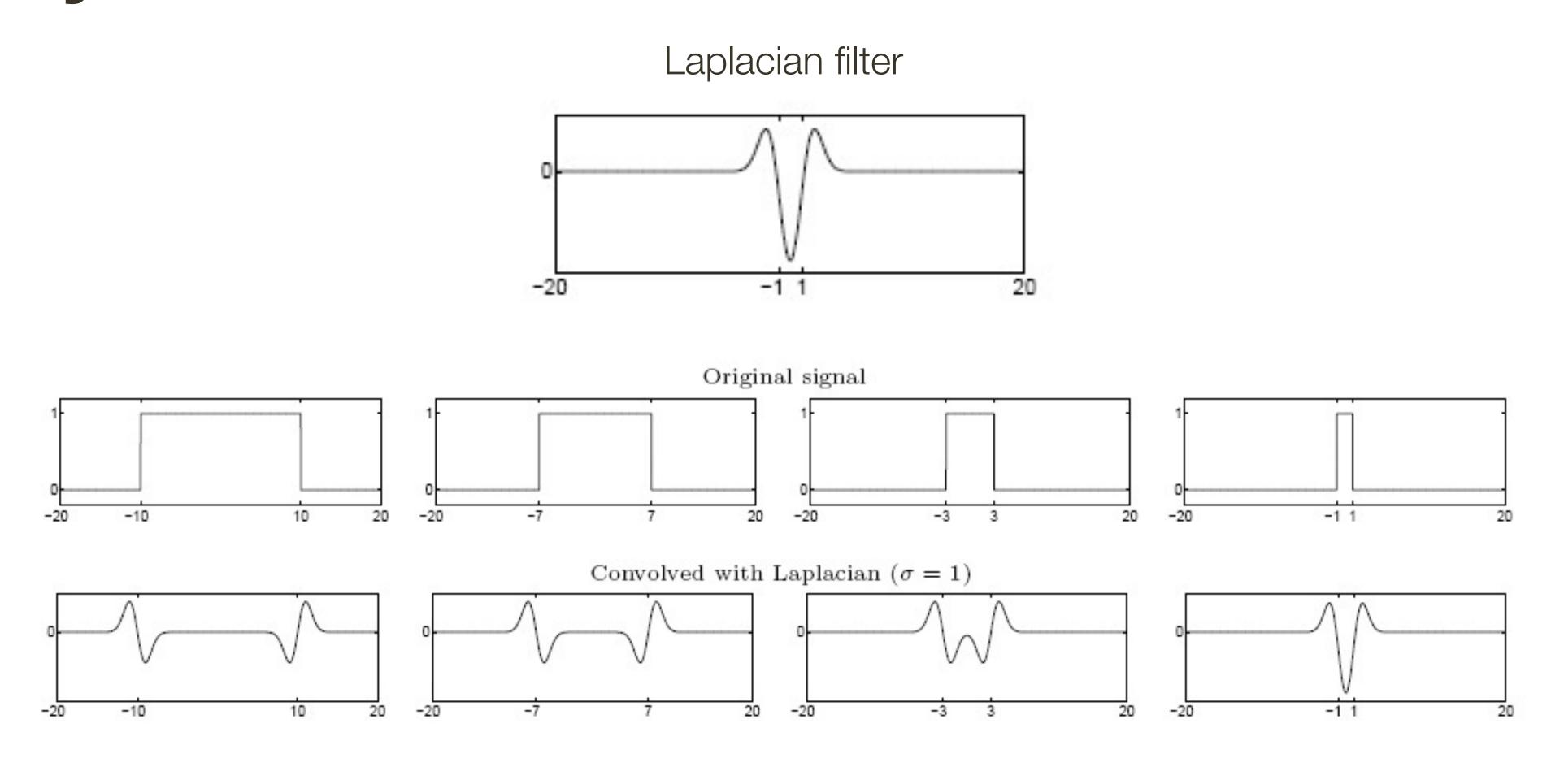




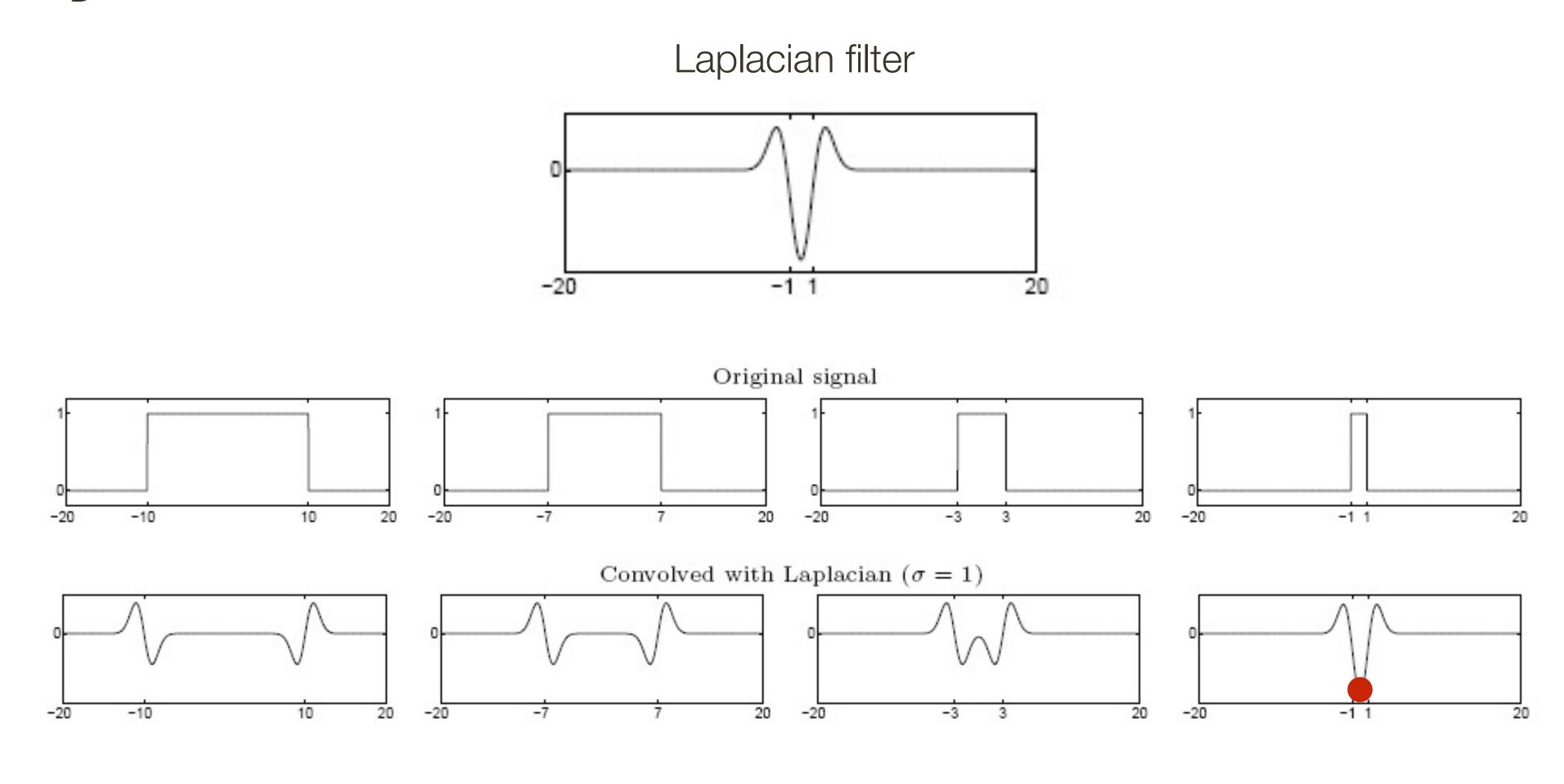








Highest response when the signal has the same **characteristic scale** as the filter

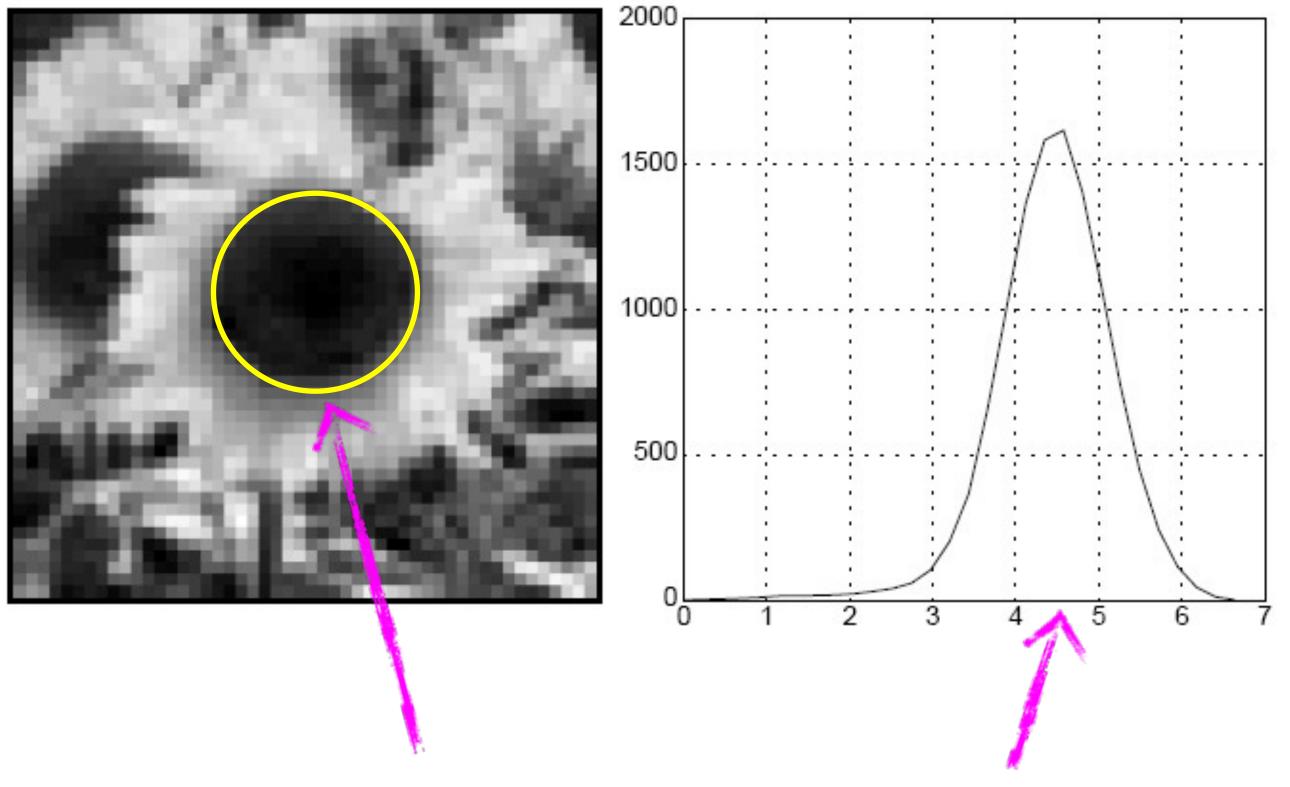


Highest response when the signal has the same **characteristic scale** as the filter



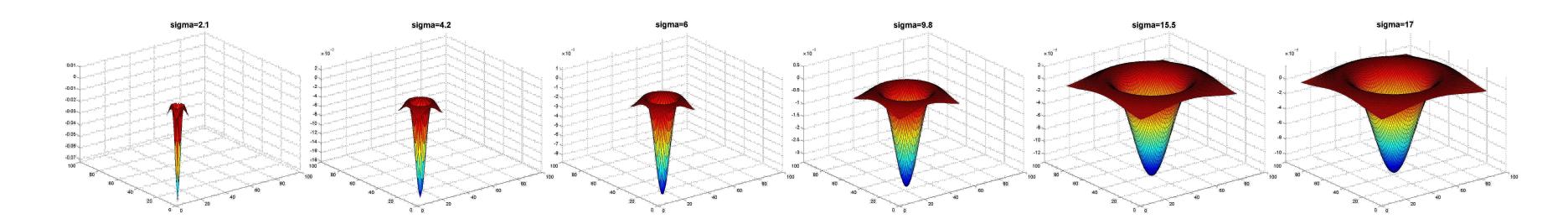
Characteristic Scale

characteristic scale - the scale that produces peak filter response



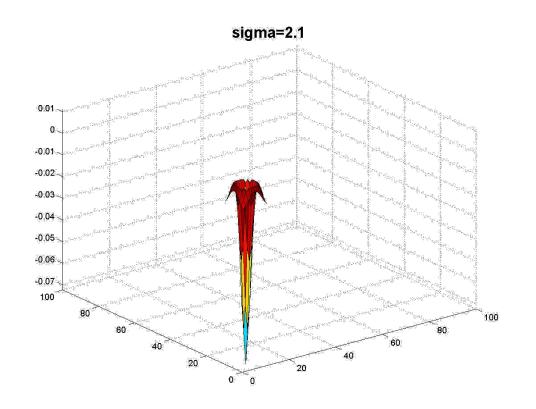
characteristic scale

we need to search for characteristic scales

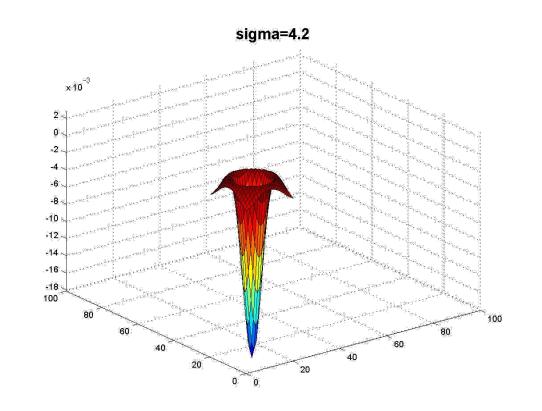


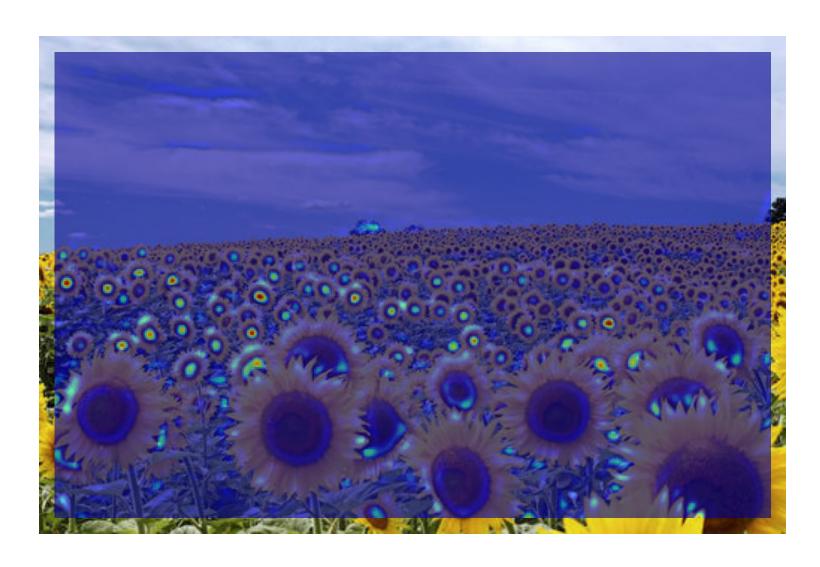
Full size

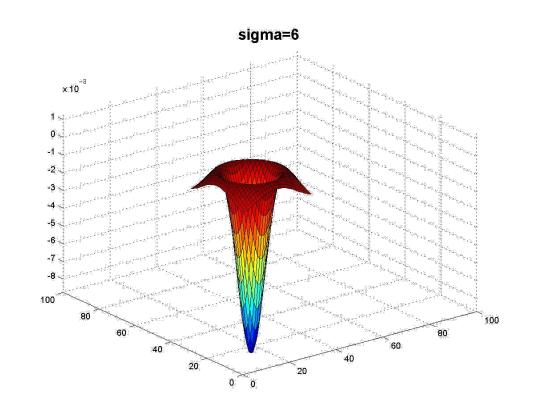




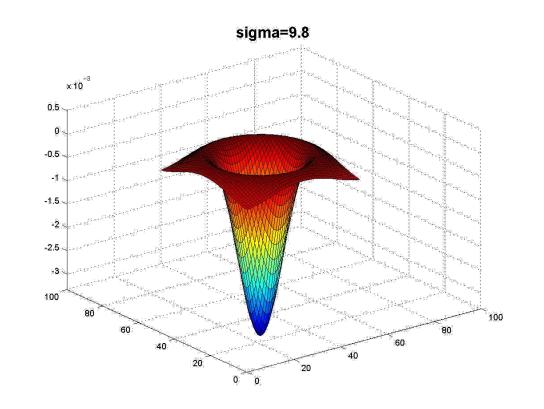




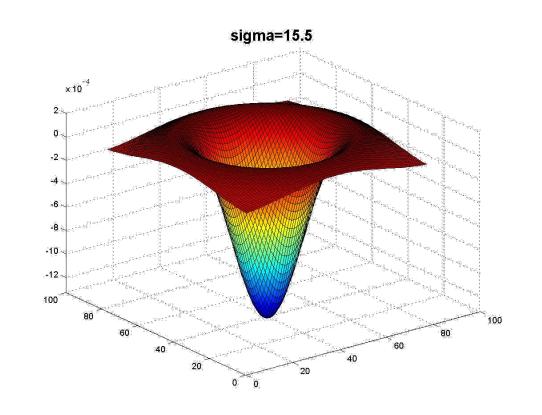




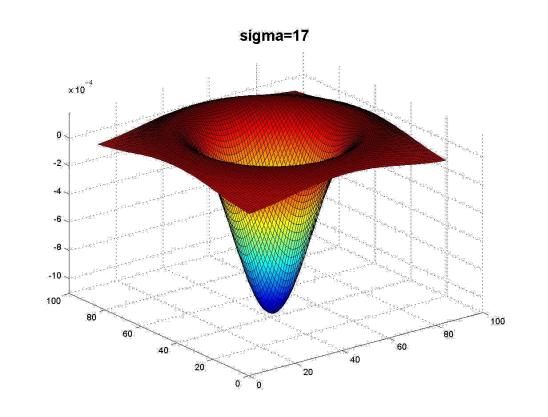












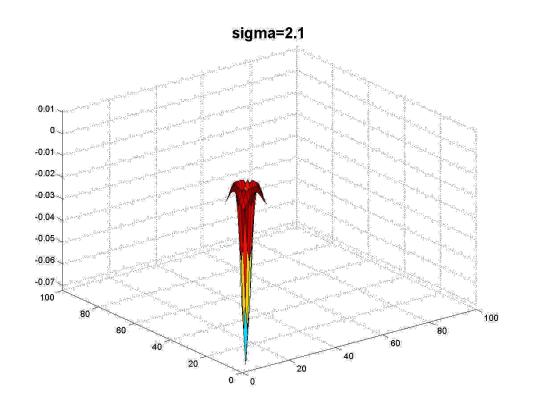


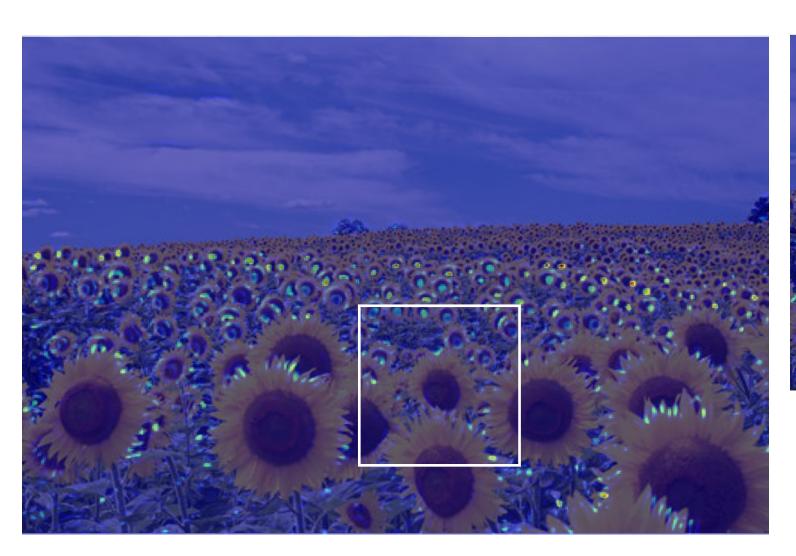
Full size



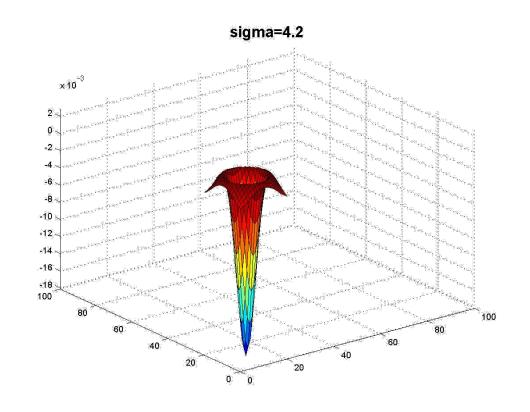
3/4 size



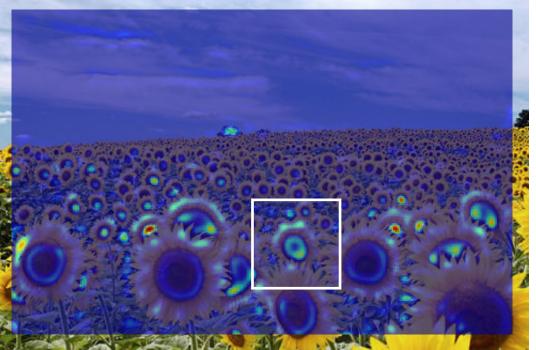


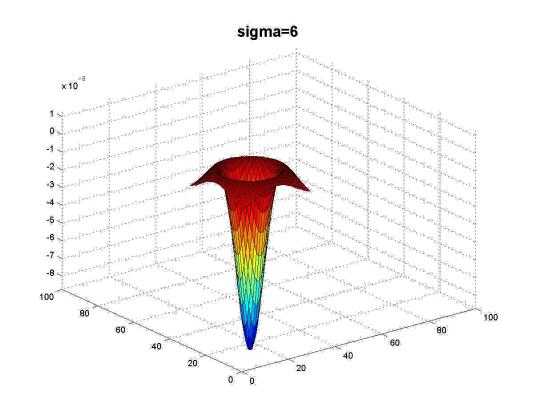


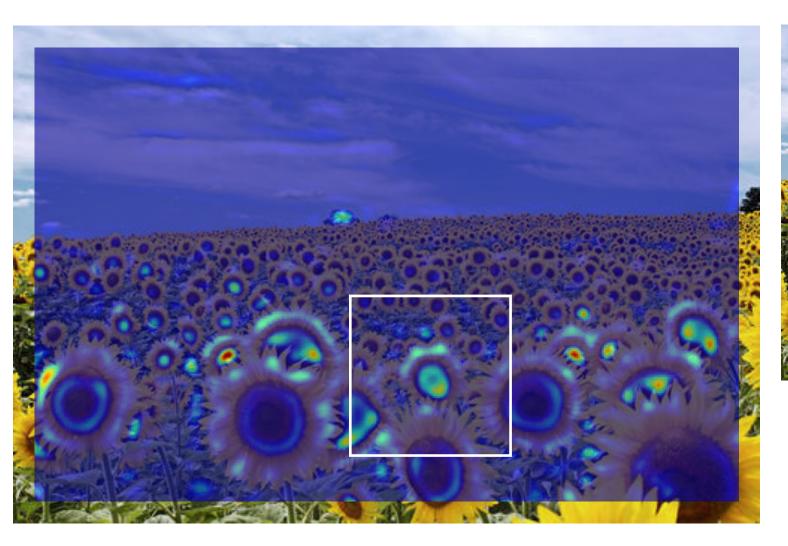




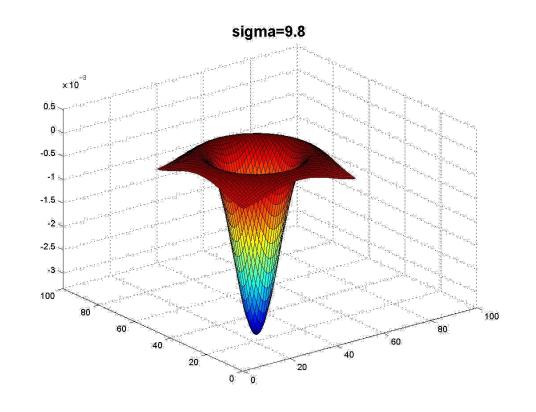


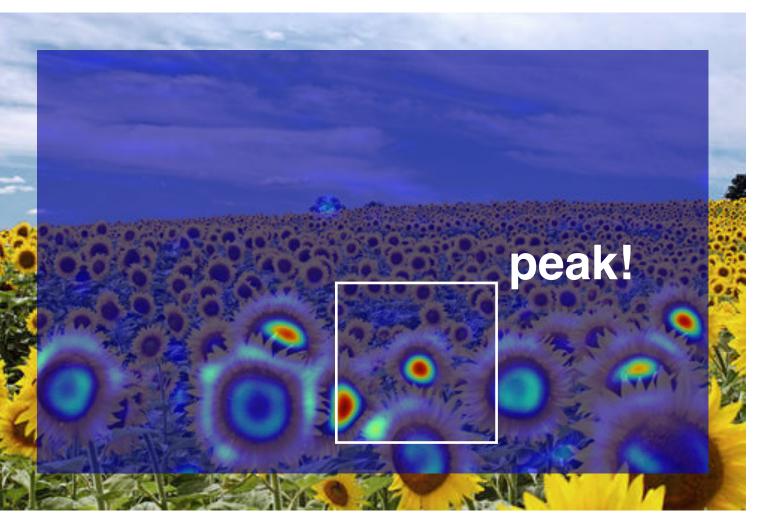


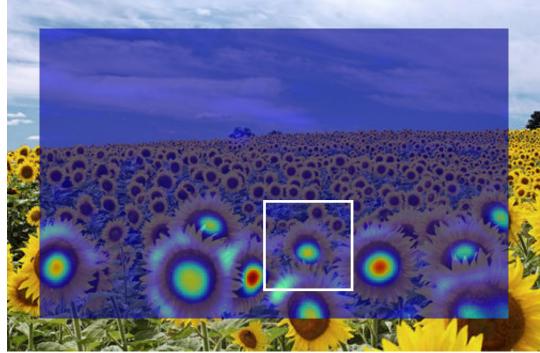


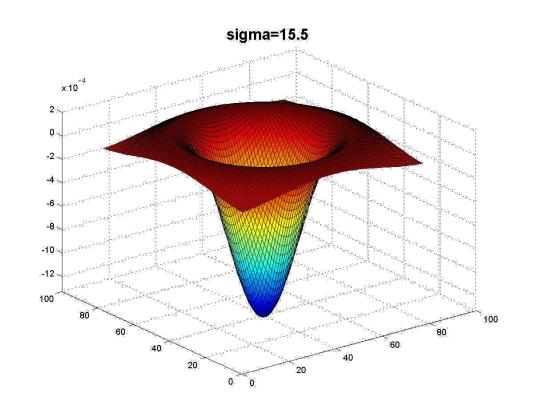


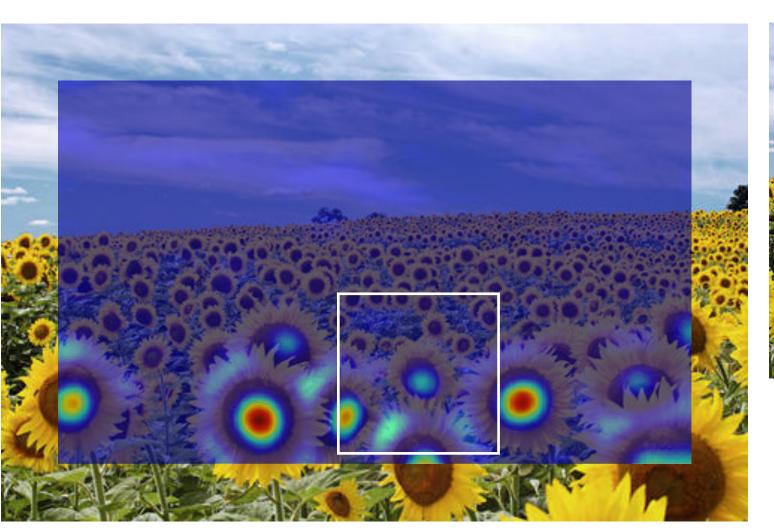


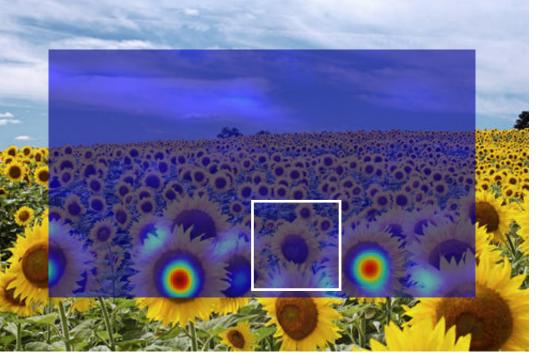


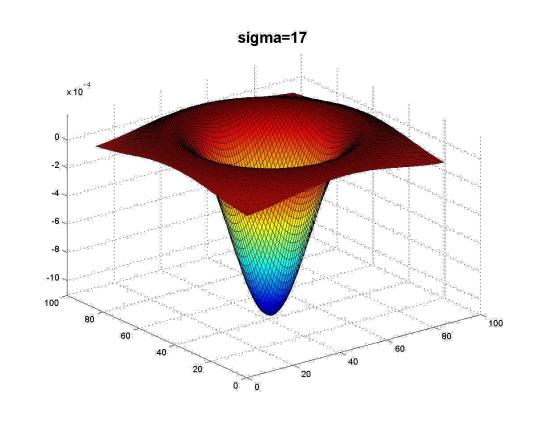


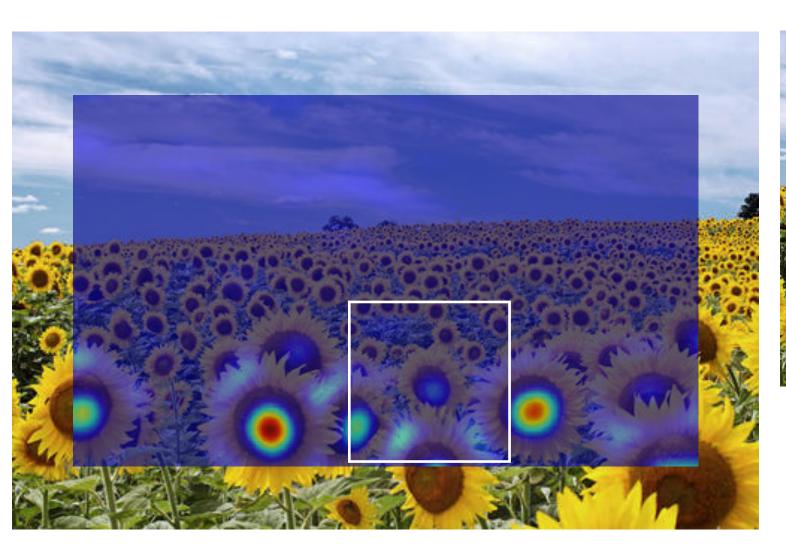


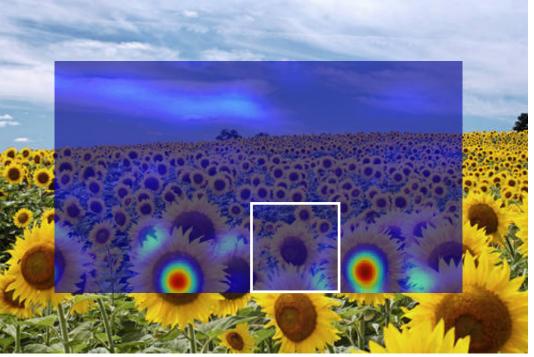












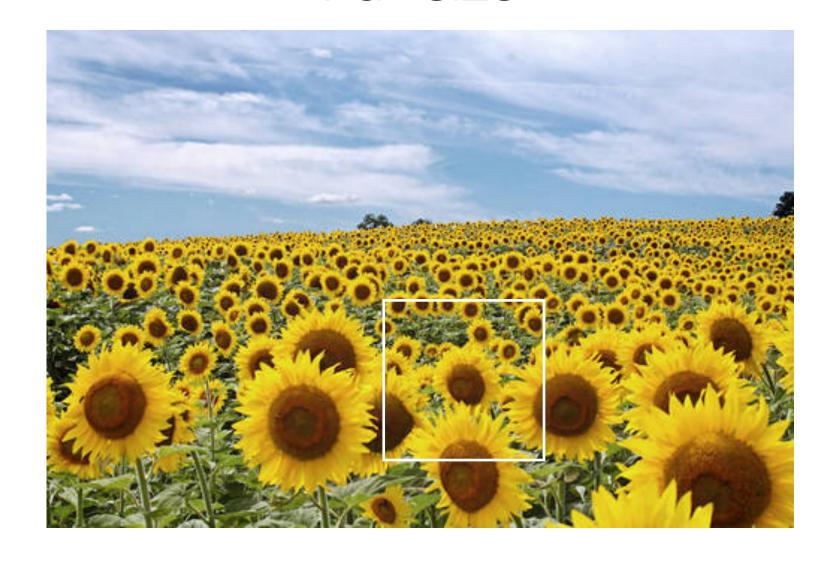
Full size

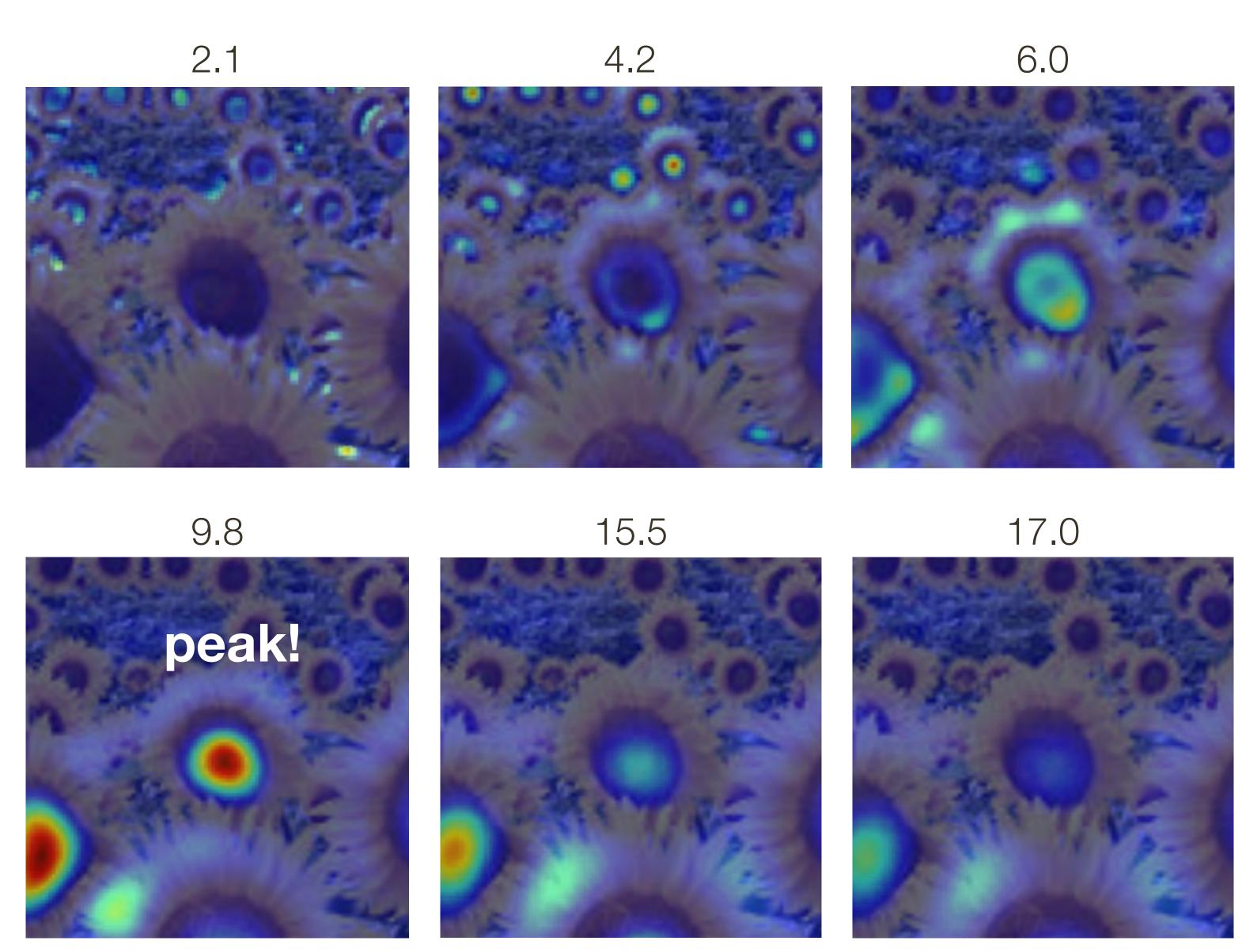


3/4 size



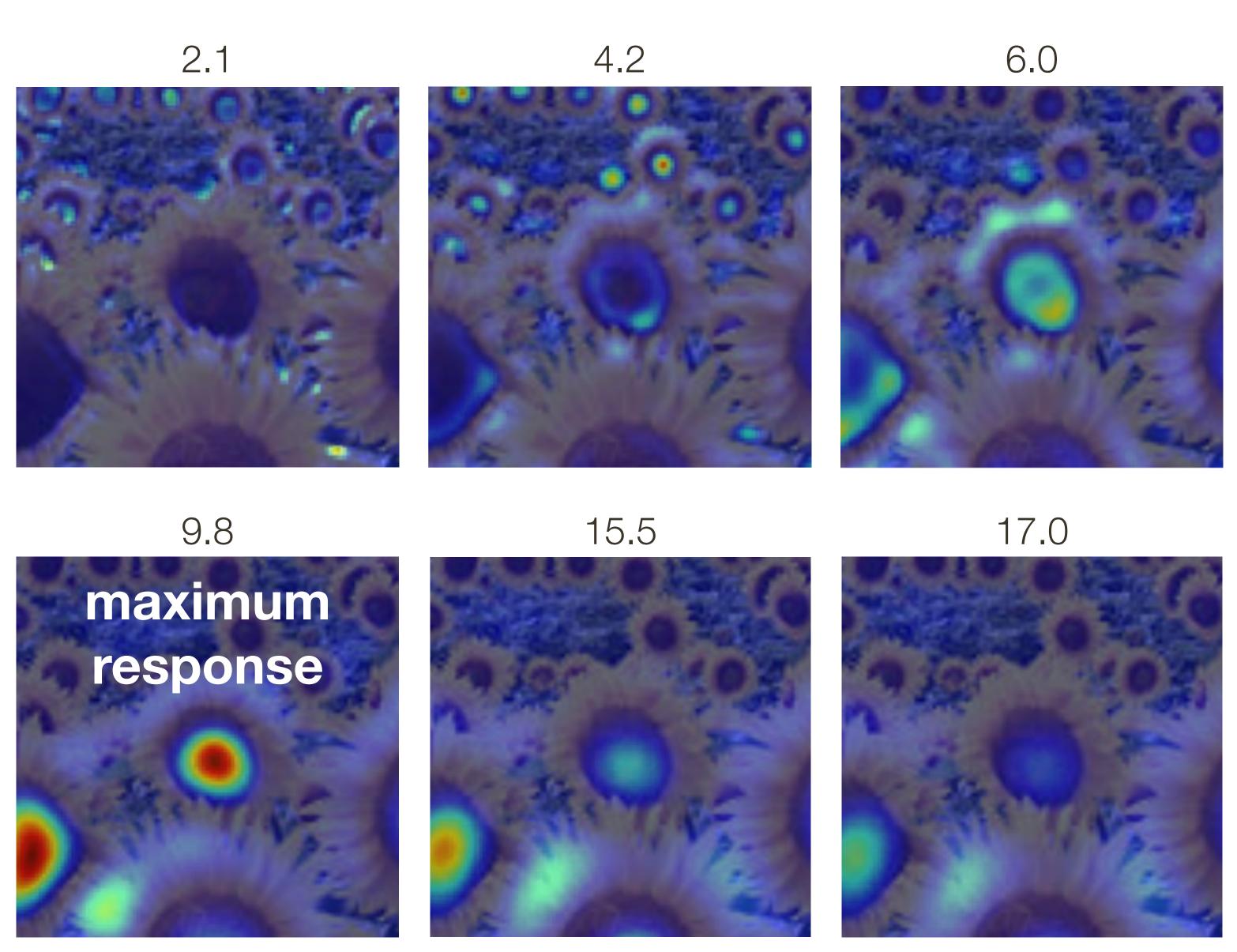
Full size

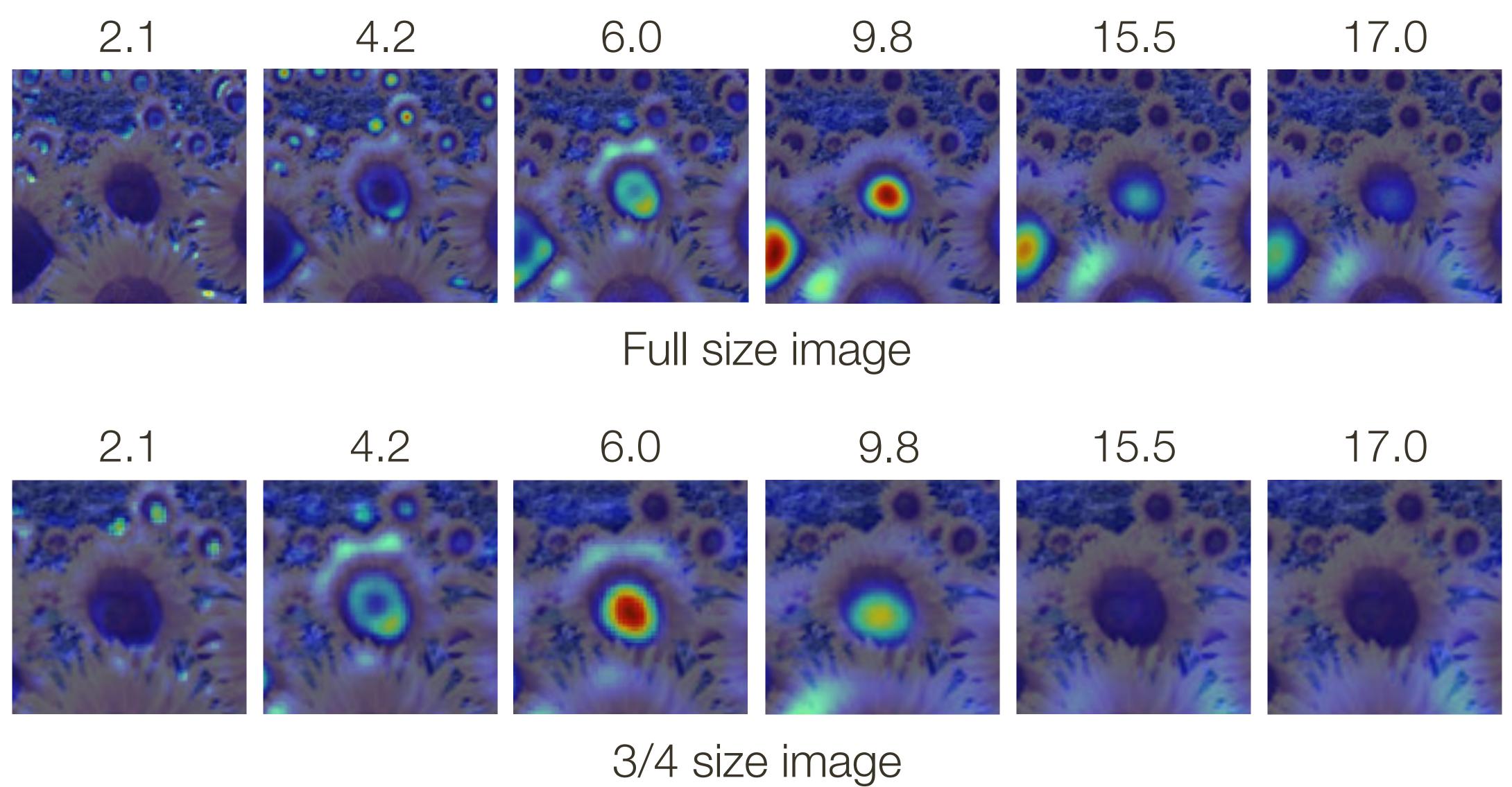


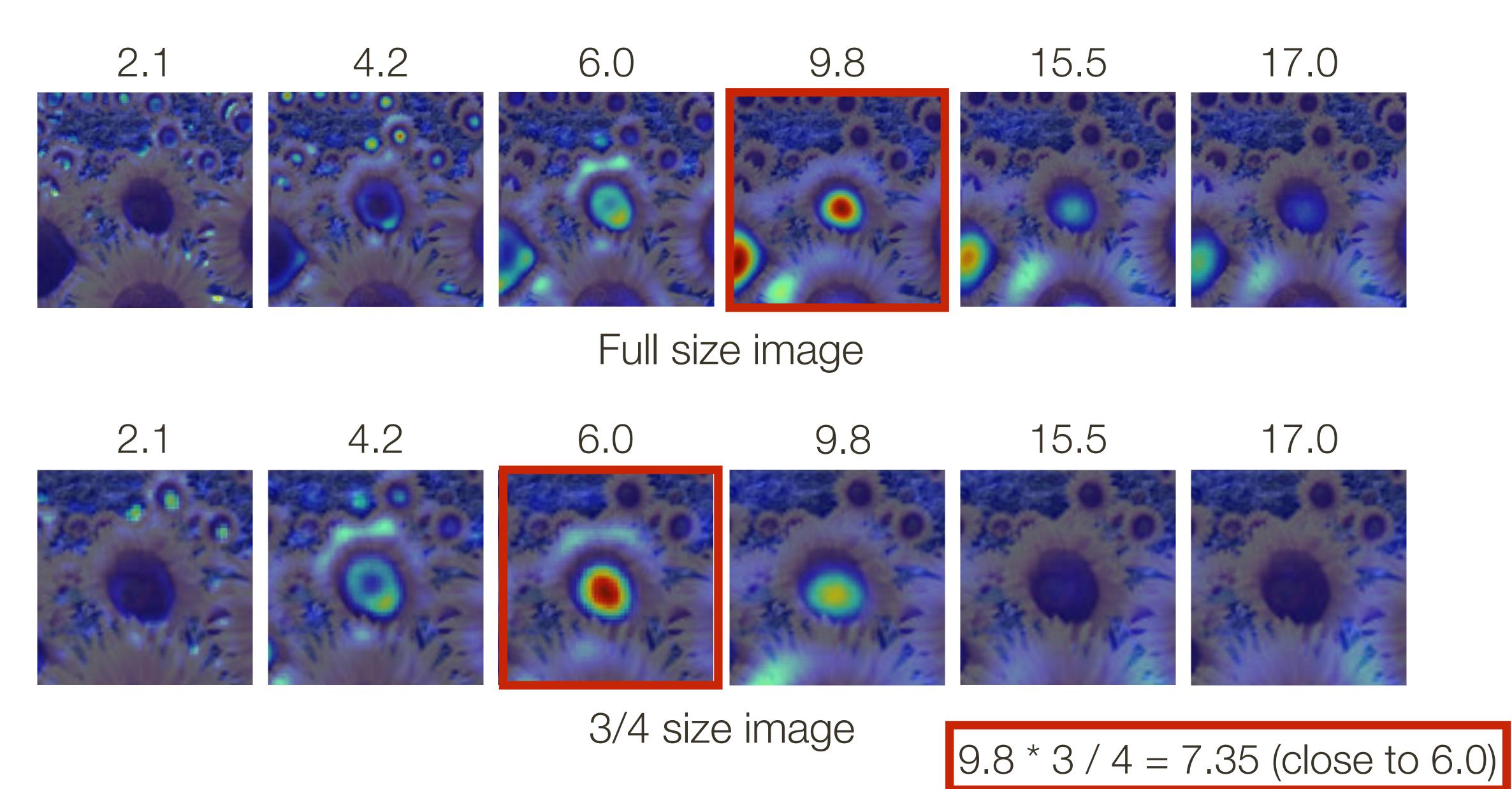


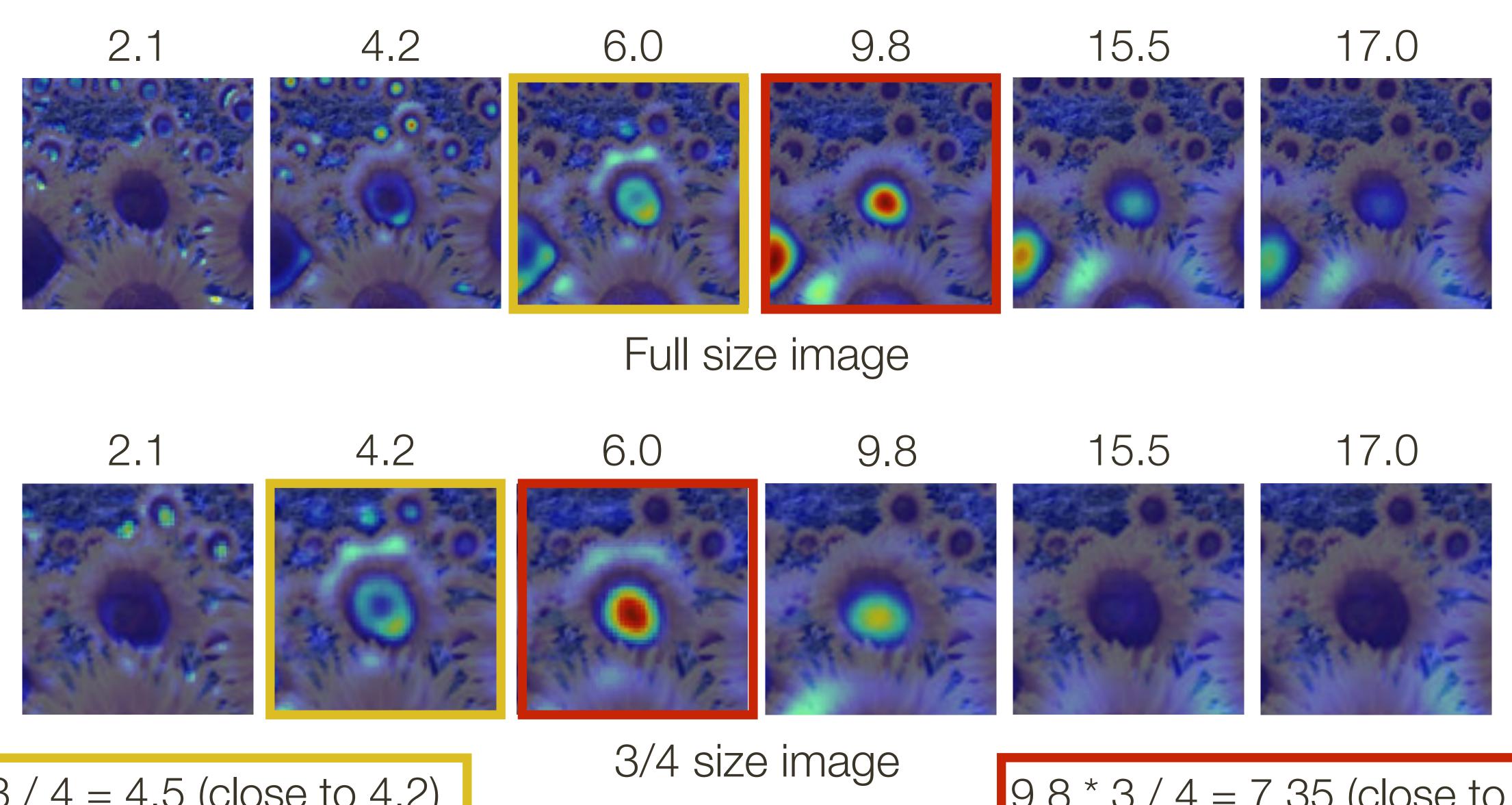
Full size





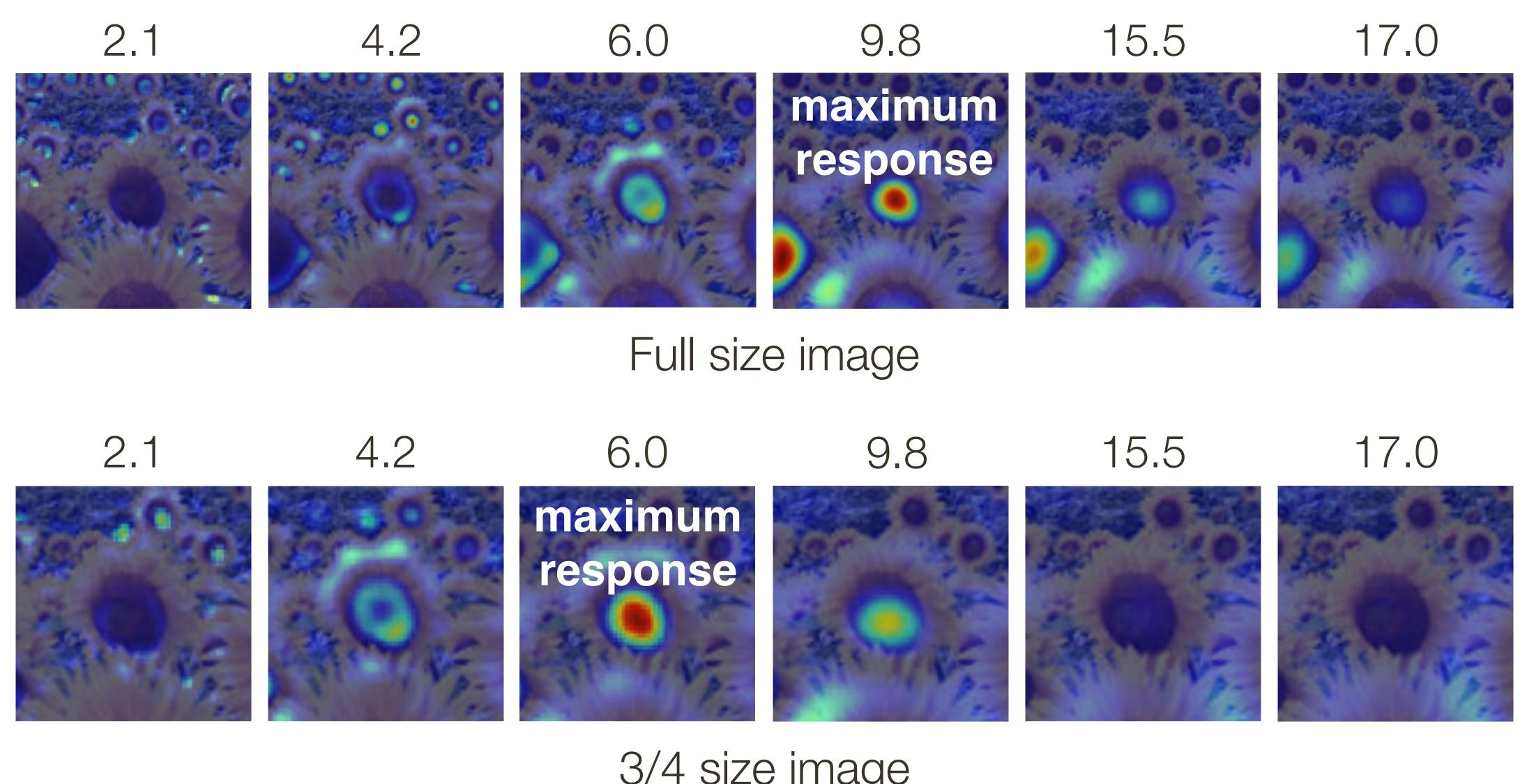






6*3/4 = 4.5 (close to 4.2)

9.8 * 3 / 4 = 7.35 (close to 6.0)



3/4 size image

Recall: Template matching

Level

Image Pyramid (s)

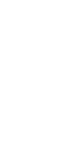
JUDYBATS

JUDYBATS

Template

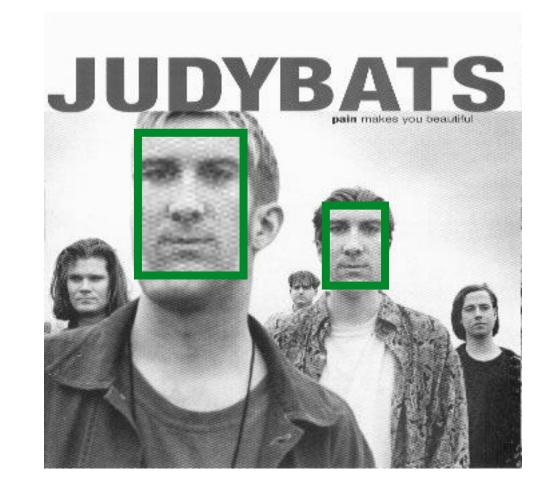
Template Pyramid (1/s)

Image















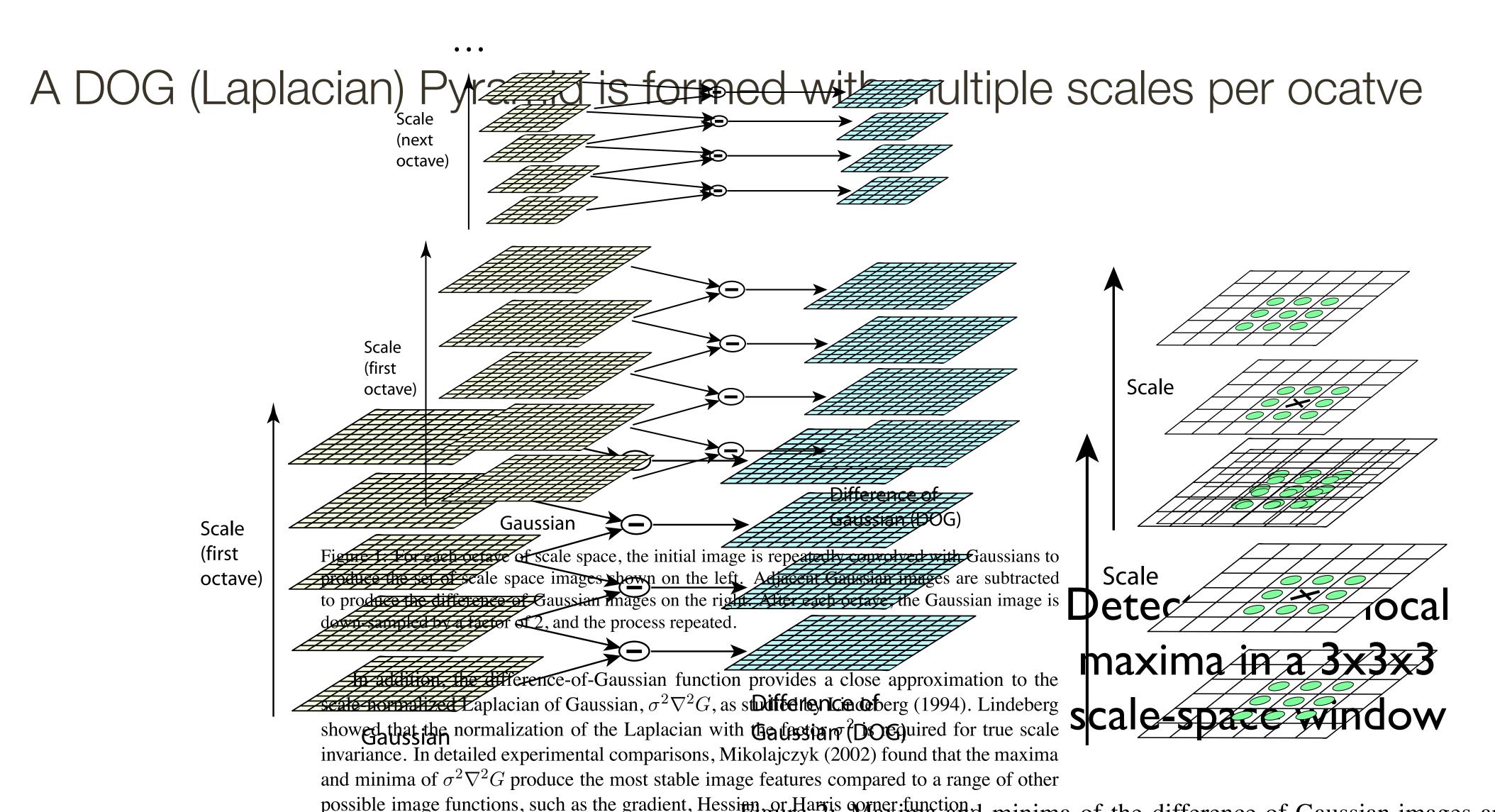




. . .

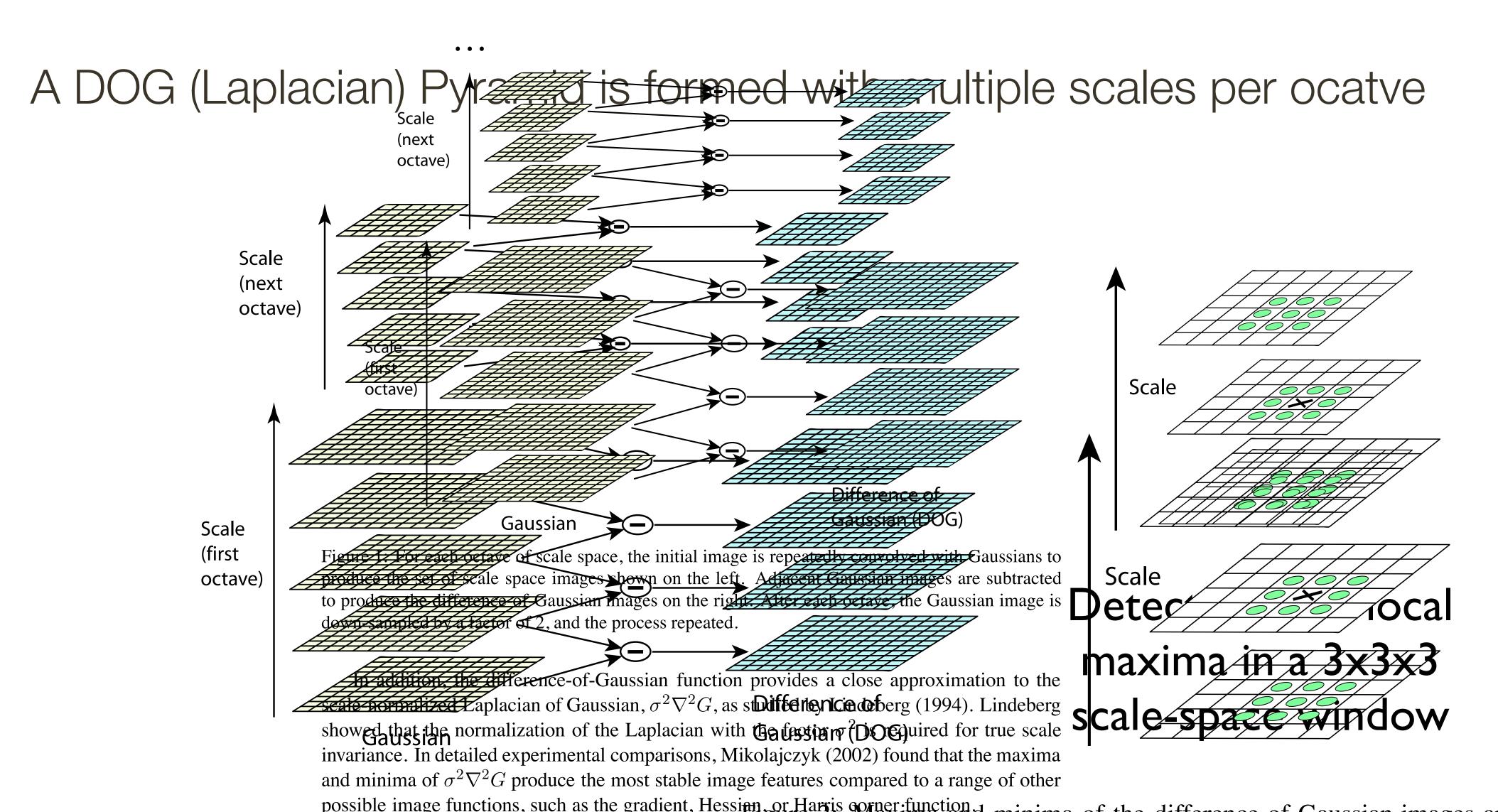
Both allow search over scale

Scale Selection



possible image functions, such as the gradient, Hessian or Harris corner function. The relationship between D and $\sigma^2 \nabla^2 G$ can be understood from the heat diffusion equation (parameterized in terms of σ rather than the more usual $t = \sigma$). With t = 0 in the diffusion equation (parameterized in terms of σ rather than the more usual t = 0). with circles).

Scale Selection

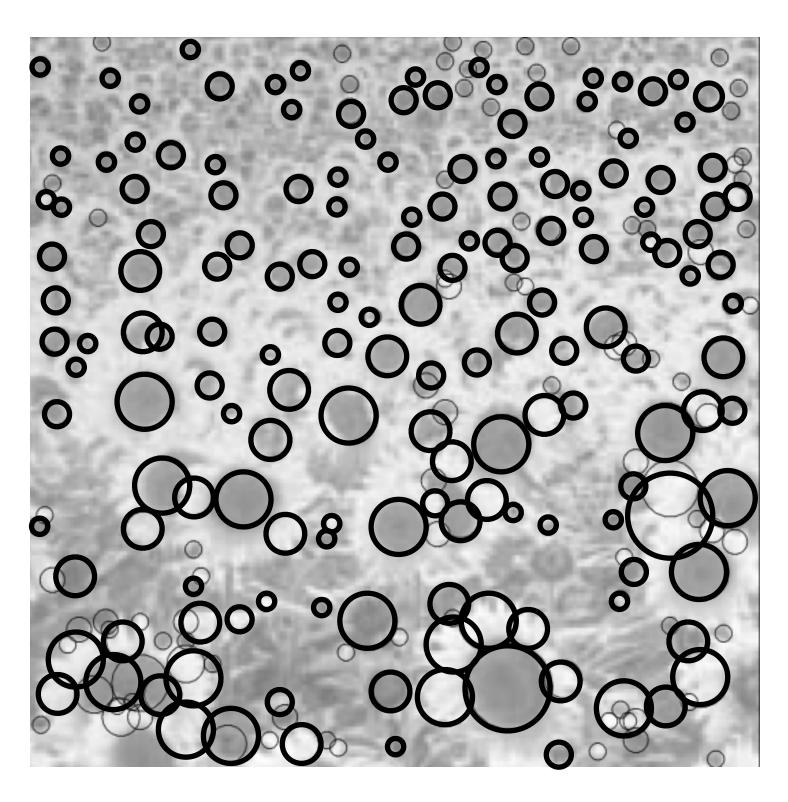


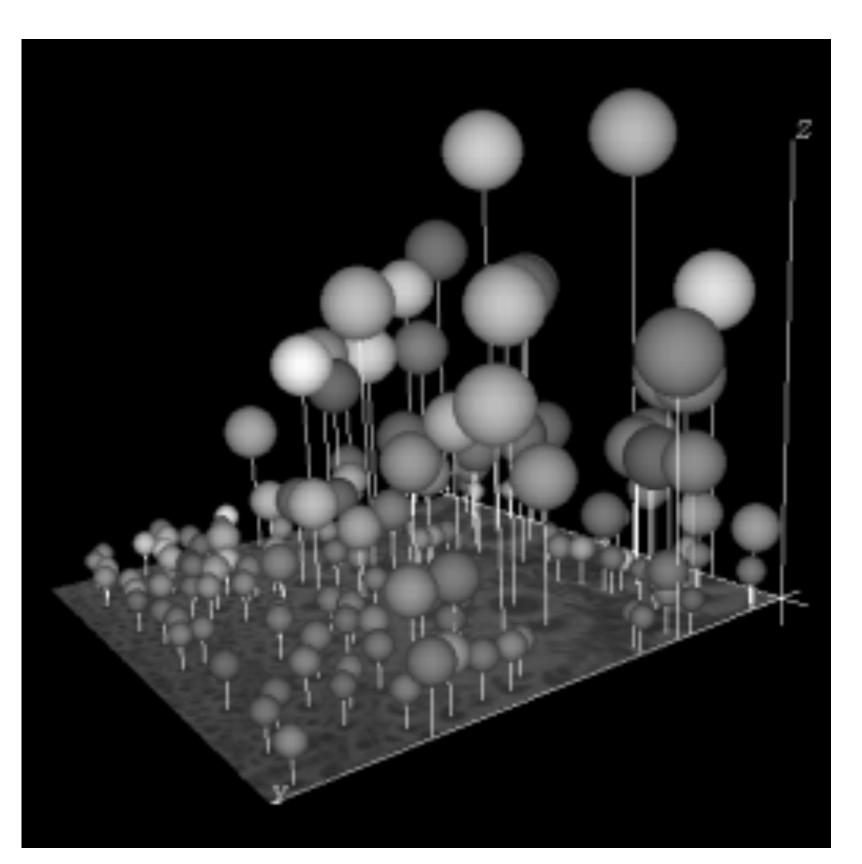
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Scale Selection

Maximising the DOG function in scale as well as space performs scale selection







[T. Lindeberg]

Difference of Gaussian blobs in 2020

Harris & Stephens (1988)

$$\det(C) - \kappa \operatorname{trace}^2(C)$$

Kanade & Tomasi (1994)

$$\min(\lambda_1,\lambda_2)$$

Nobel (1998)

$$\det(C)$$

$$\operatorname{trace}(C) + \epsilon$$

Difference of Gaussian blobs in 2020

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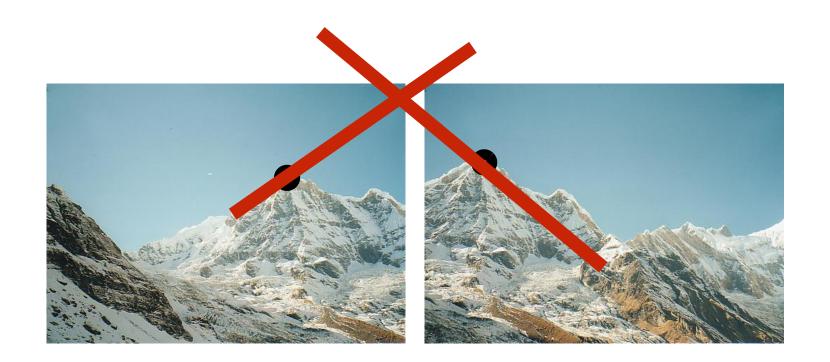
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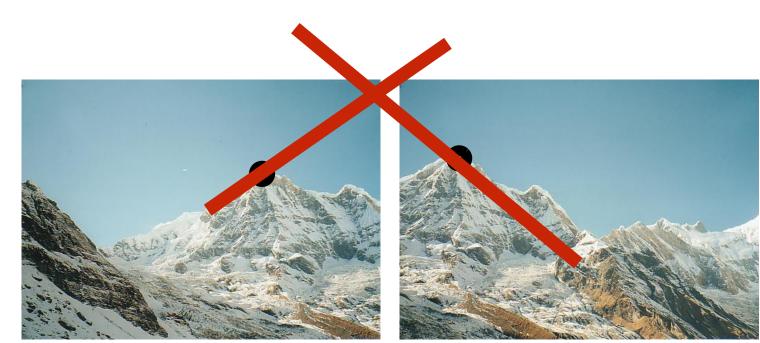
Kanade & Tomasi (1994)

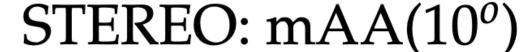
$$\min(\lambda_1,\lambda_2)$$

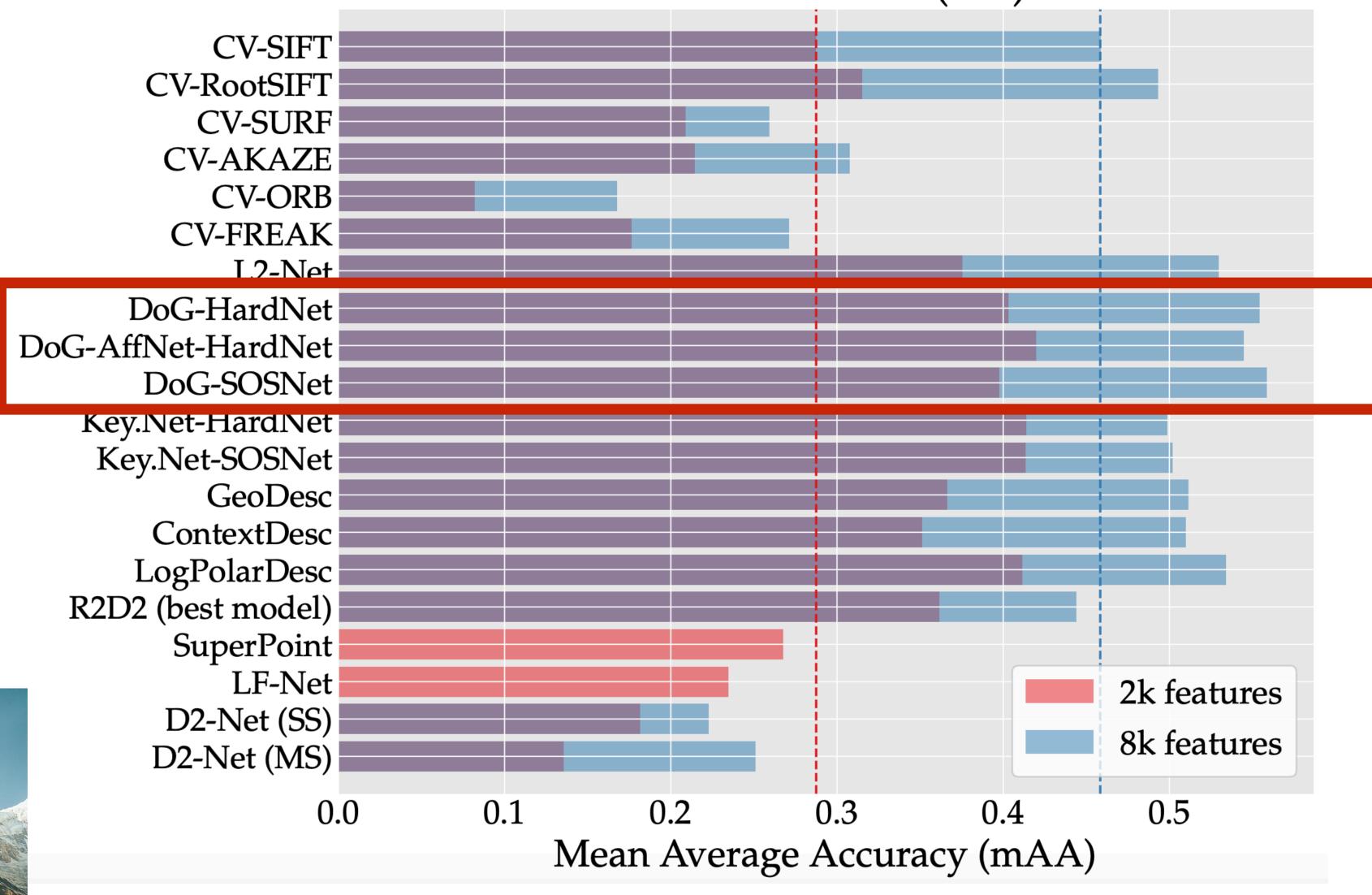
Nobel (1998)

$$\det(C)$$

$$\operatorname{trace}(C) + \epsilon$$







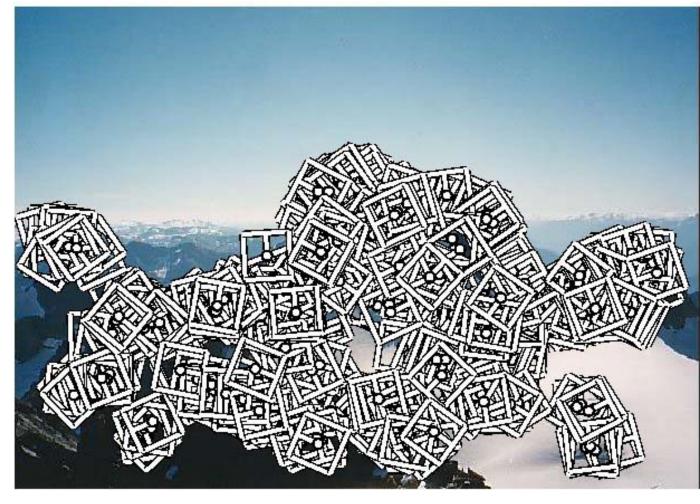
Implementation

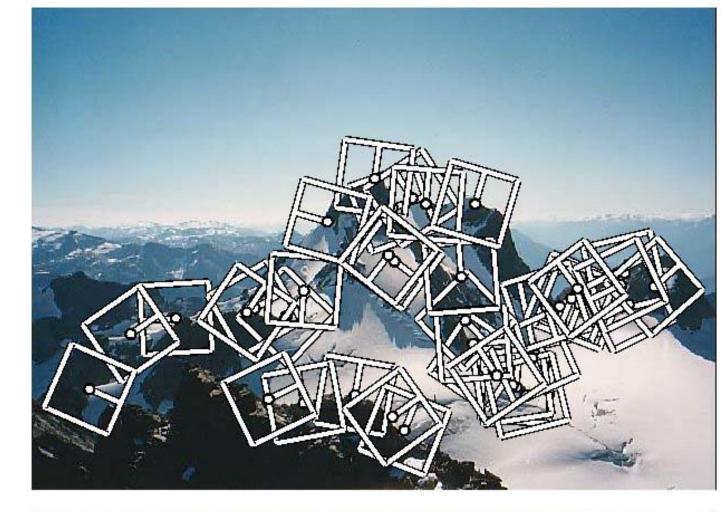
```
For each level of the Gaussian pyramid compute feature response (e.g. Harris, Laplacian)
```

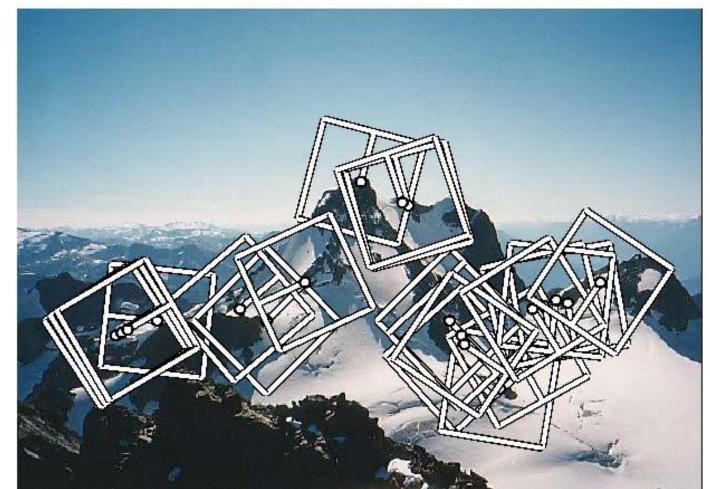
For each level of the Gaussian pyramid if local maximum and cross-scale save scale and location of feature (x,y,s)

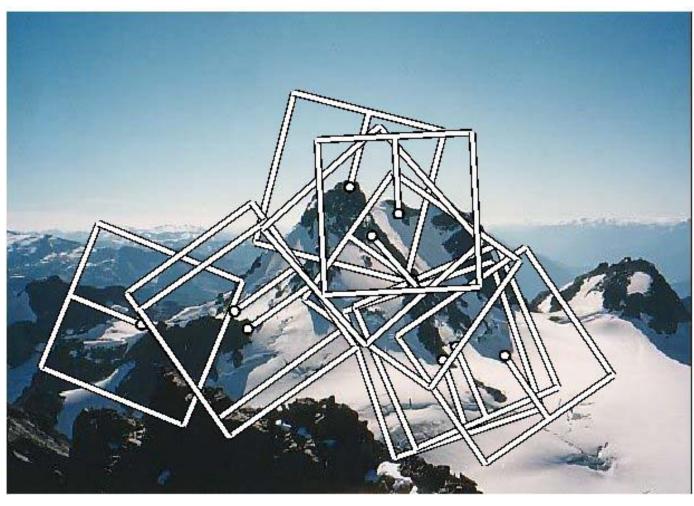
Multi-Scale Harris Corners

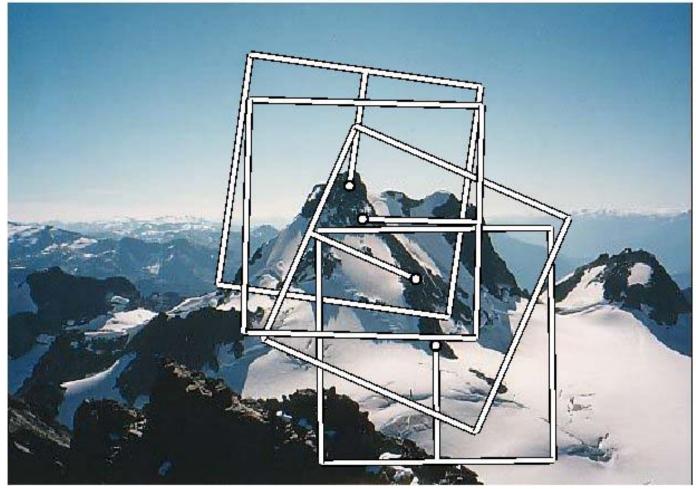












Representation	Results in	Approach	Technique
intensity	dense	template matching	(normalized) correlation
edge	relatively sparse	derivatives	Sobel, LoG, Canny
corner	sparse	locally distinct features	Harris (and variants)
blob	sparse	locally distinct features	LoG

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Course Re-cap

Course Beginning

Course Re-cap

Brittle

(failure in many conditions)

Robust (works with noise, complex images, clutter),

Robustness

Brittle (failure in m

(failure in many conditions)

Robust (works with noise, complex images, clutter).

Robustness

Global (templates)

Local (edges, corners, blobs, patches)

Compositional

(local + flexible global)

Image Representations



Brittle (failure in many conditions) Robust Robust Robust Robustness

Global (templates)

Local

(edges, corners, blobs, patches)

Image Representations

Compositional

(local + flexible global)

Hand defined (filters, thresholds)

Statistical

(means, covariances, histograms)

Learned

(SVMs, Neural Networks)

Method of Obtaining Image Representations

Summary

Edges are useful image features for many applications, but suffer from the aperture problem

Canny Edge detector combines edge filtering with linking and hysteresis steps

Corners / Interest Points have 2D structure and are useful for correspondence

Harris corners are minima of a local SSD function

DoG maxima can be reliably located in scale-space and are useful as interest points