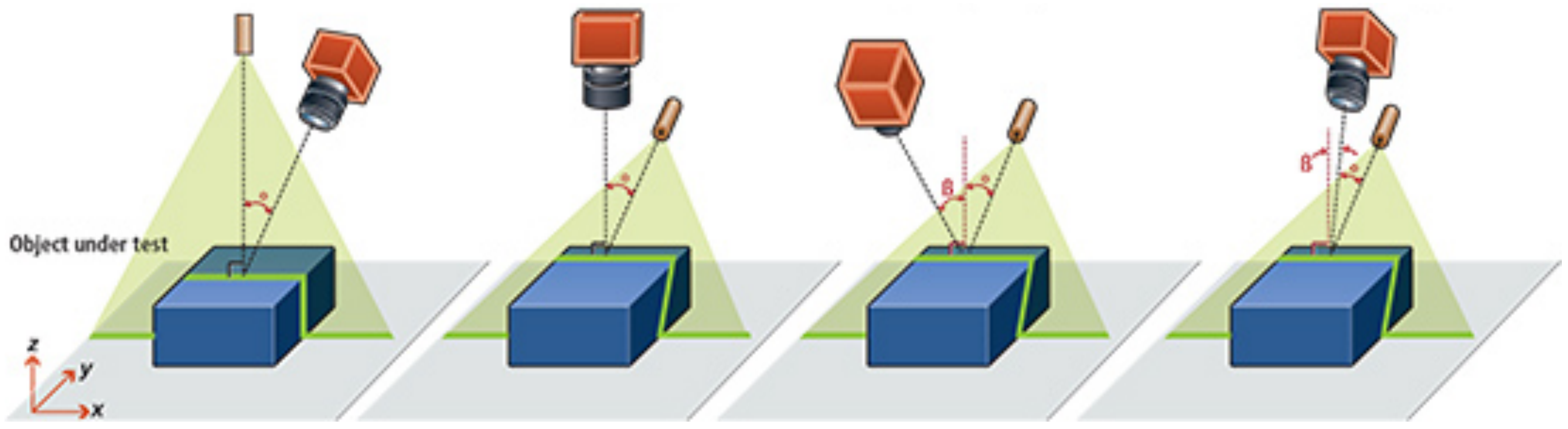
THE UNIVERSITY OF BRITISH COLUMBIA

# CPSC 425: Computer Vision




**Lecture 2:** Image Formation

( unless otherwise stated slides are taken or adopted from **Bob Woodham, Jim Little** and **Fred Tung** )

# **Menu** for Today (**September 9, 2024**)

## Topics:

— Image Formation

— Cameras and Lenses

— Projection

— Human Eye ??

## Redings:

— **Today's** Lecture: Szeliski Chapter 2, Forsyth & Ponce (2nd ed.) 1.1.1 — 1.1.3

— **Next** Lecture: Forsyth & Ponce (2nd ed.) 4.1, 4.5

## Reminders:

— Complete **Assignment 0** (ungraded) by Wednsday, **September 11**

— **Assignment 1** (graded) will be out Wednsday, **September 11**

— Please sign up for Piazza (140 students signed up so far)

# Today's "**fun**" Example

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Photo credit: reddit user Liammm

# Today's "**fun**" Example: **Eye Sink Illusion**

*Pereidolia*

Photo credit: reddit user Liammm

# Salvador Dali — **Pareidolia**

# **Lecture 2**: Goal

To understand how images are formed

(and develop relevant mathematical concepts and abstractions)

# What is **Computer Vision**?

Compute vision, broadly speaking, is a research field aimed to enable computers to **process and interpret visual data**, as sighted humans can.

**Image** (or video)

**Interpretation**

blue sky,
trees,
fountains,
UBC, …

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Image Credit: https://www.flickr.com/photos/flamephoenix1991/8376271918

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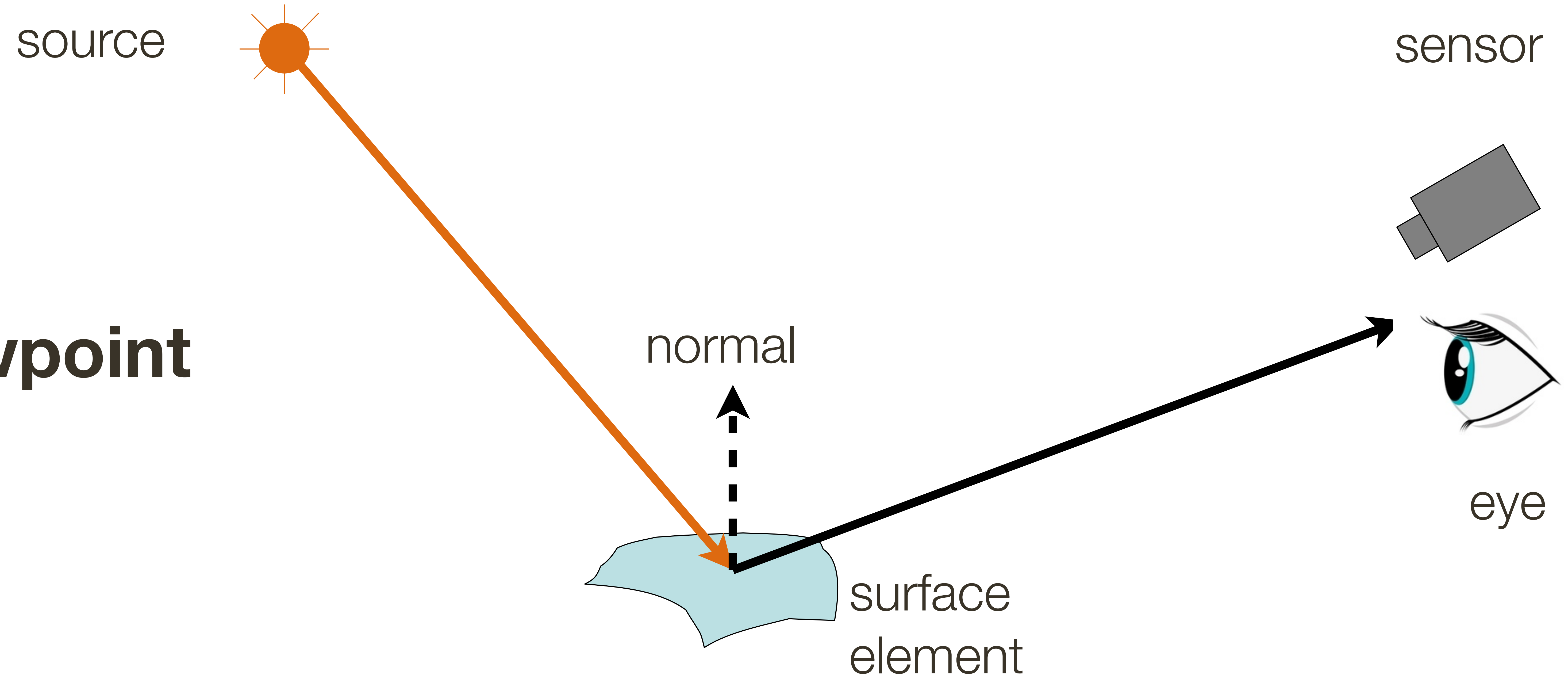
# **Overview**: Image Formation, Cameras and Lenses

The **image formation process** that produces a particular image depends on

- **Lightening** condition
- Scene **geometry**
- **Surface** properties
- Camera **optics** and **viewpoint**




Sensor (or eye) **captures amount of light** reflected from the object

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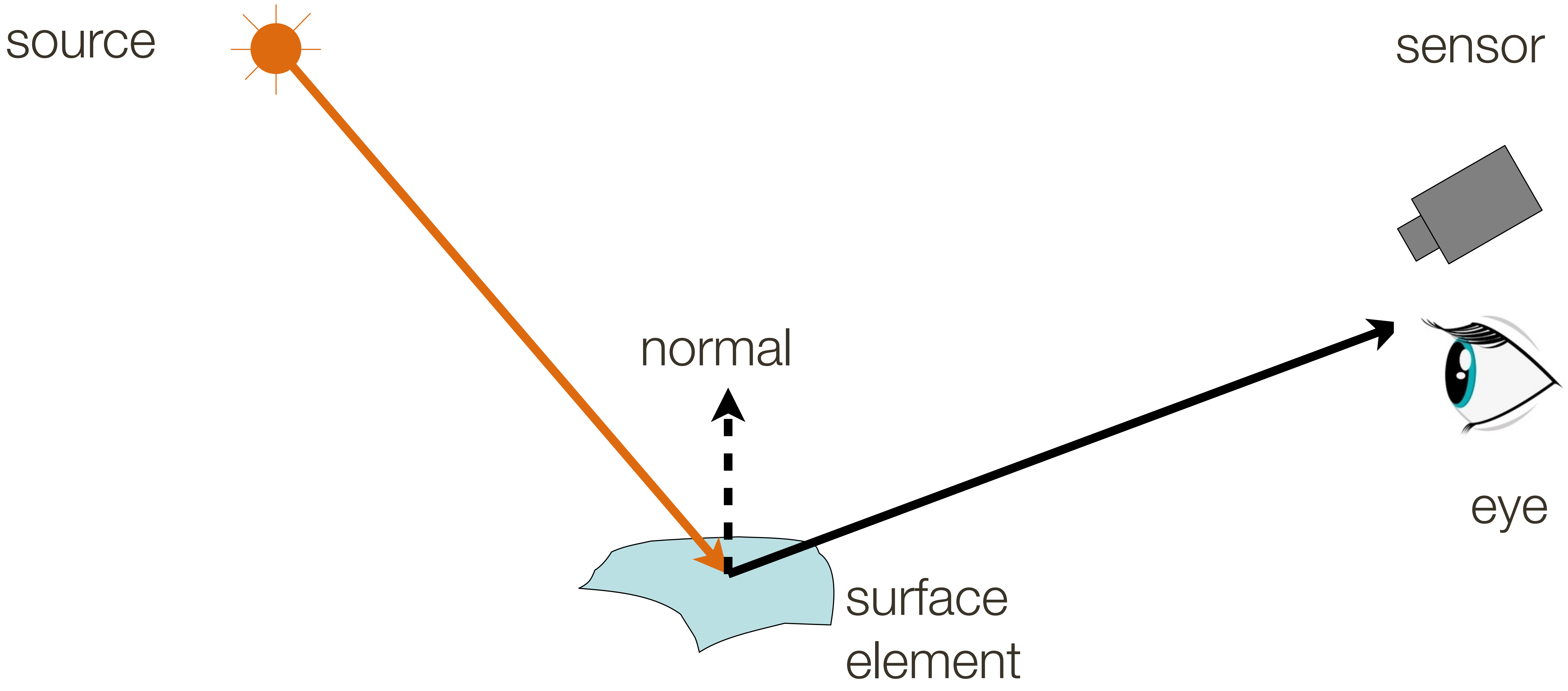


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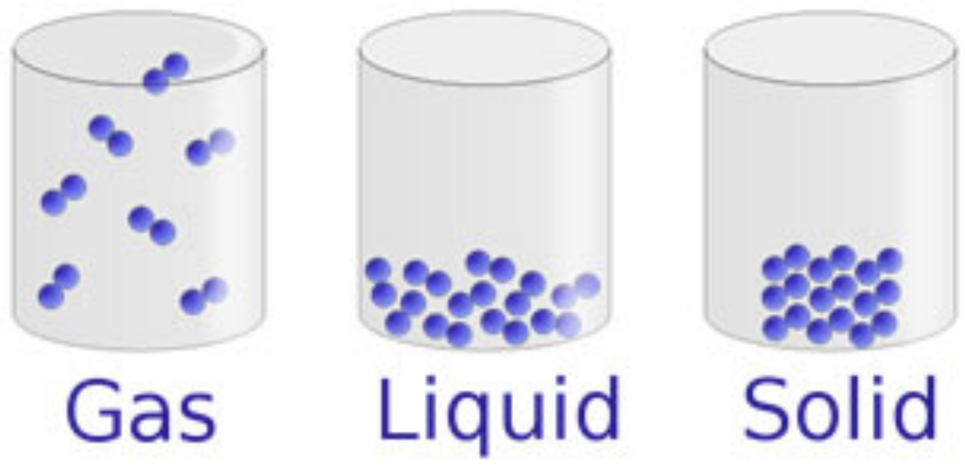
# Light

Behaves like **particles**?

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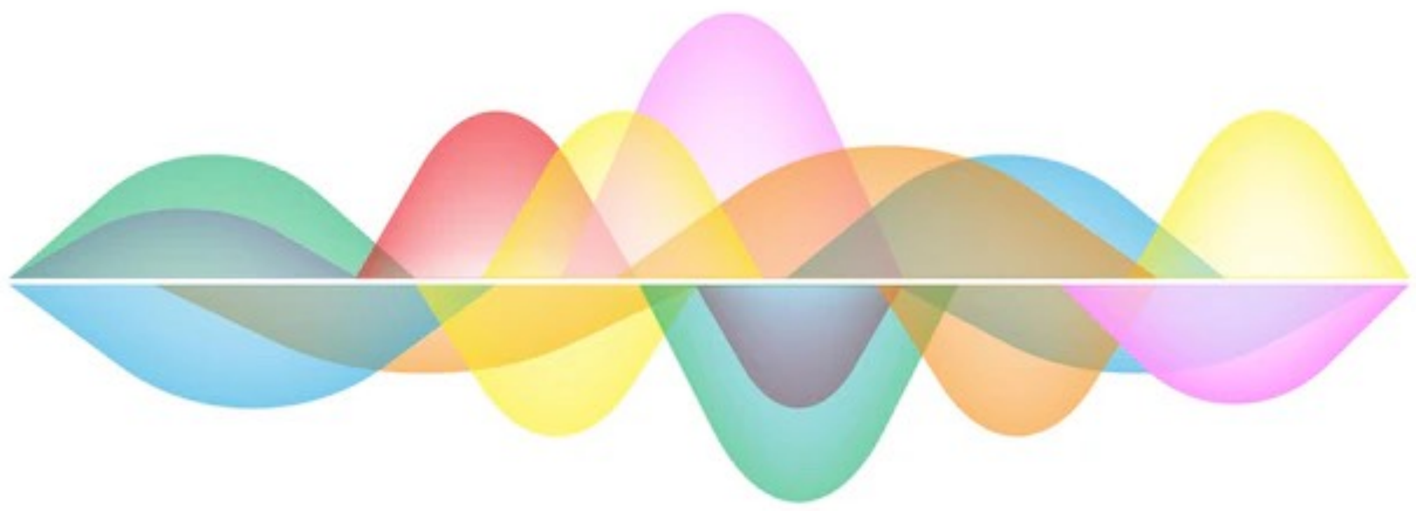
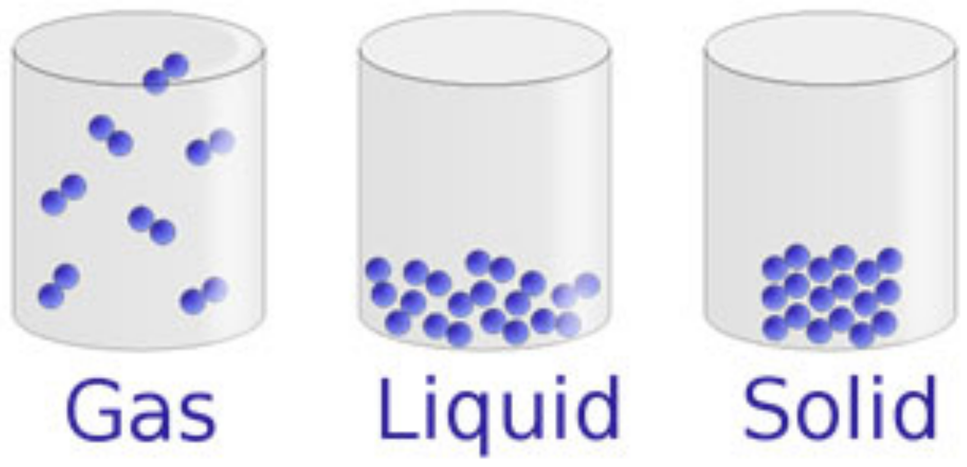
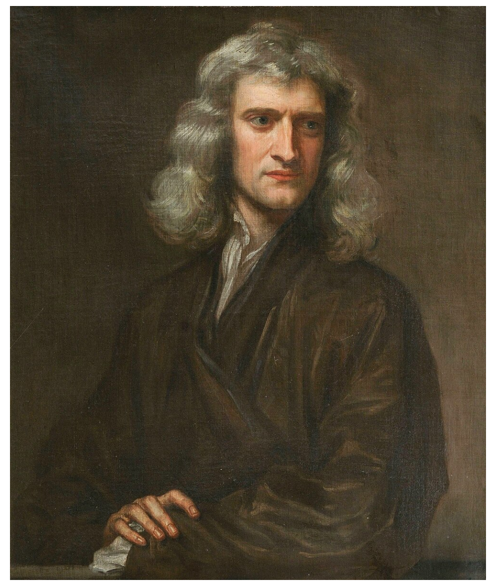


# Light

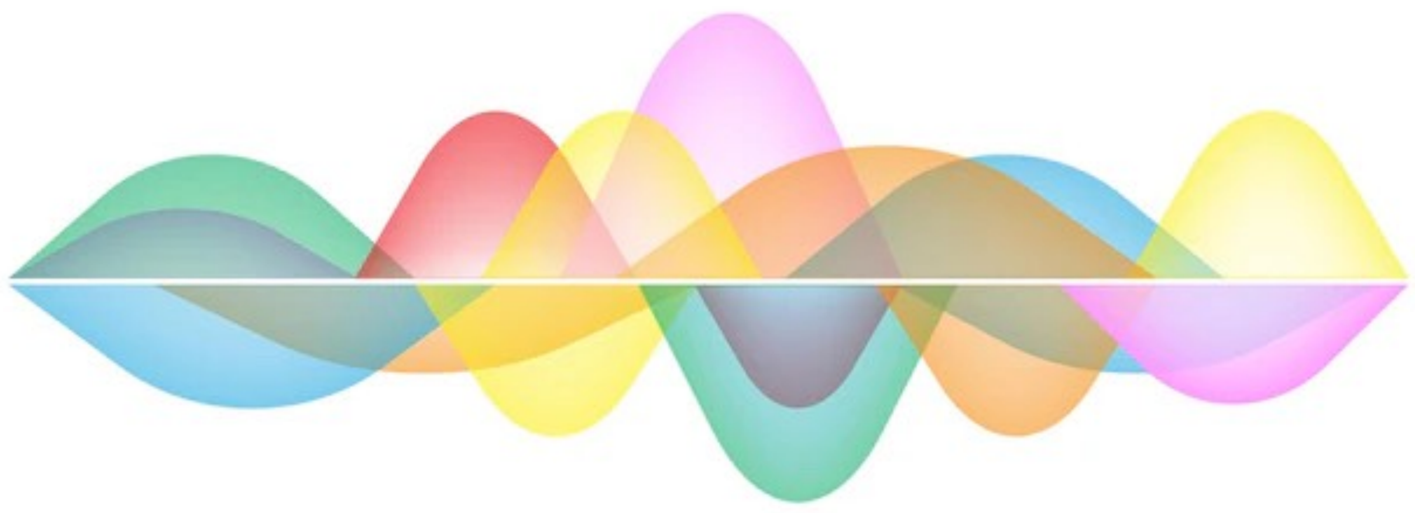
Behaves like **particles**? photons



Sir Isaac Newton

Behaves as **waves**?



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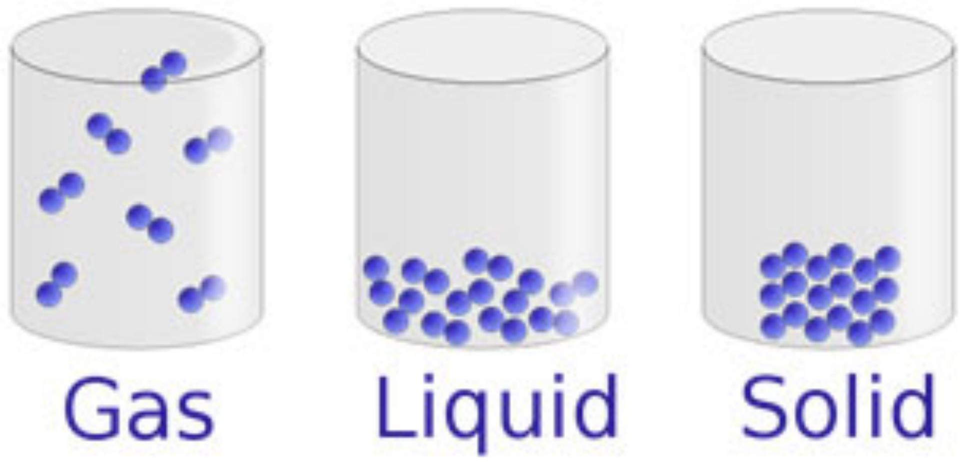


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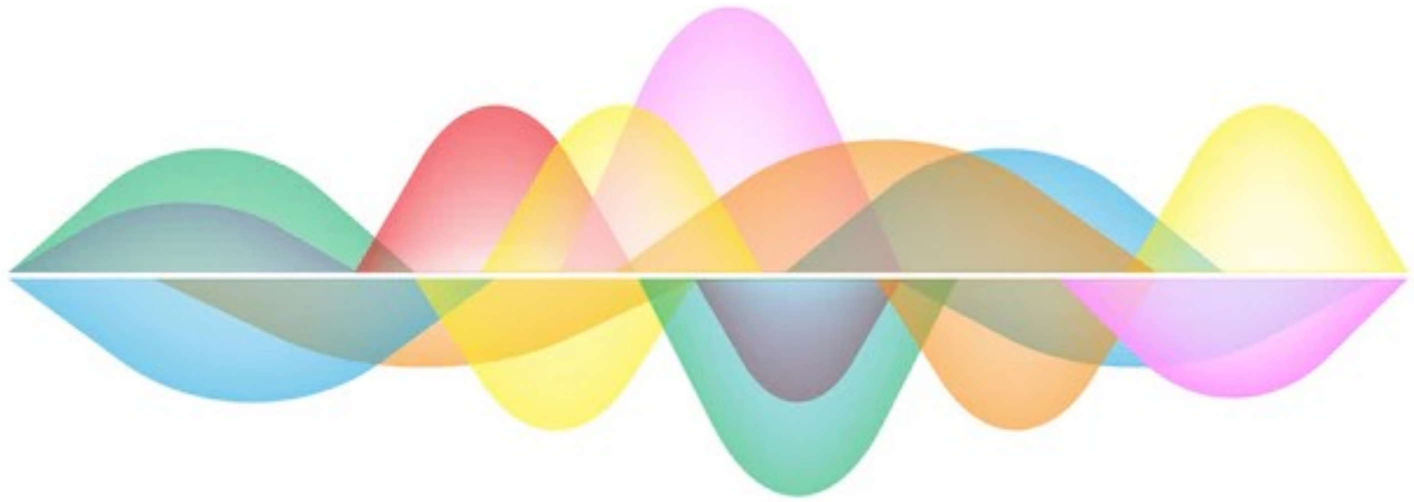

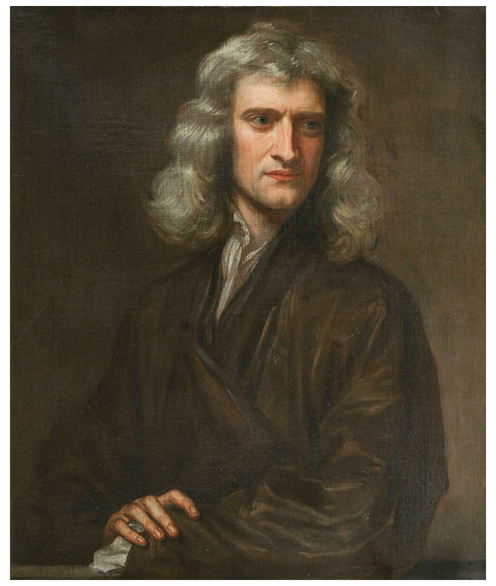

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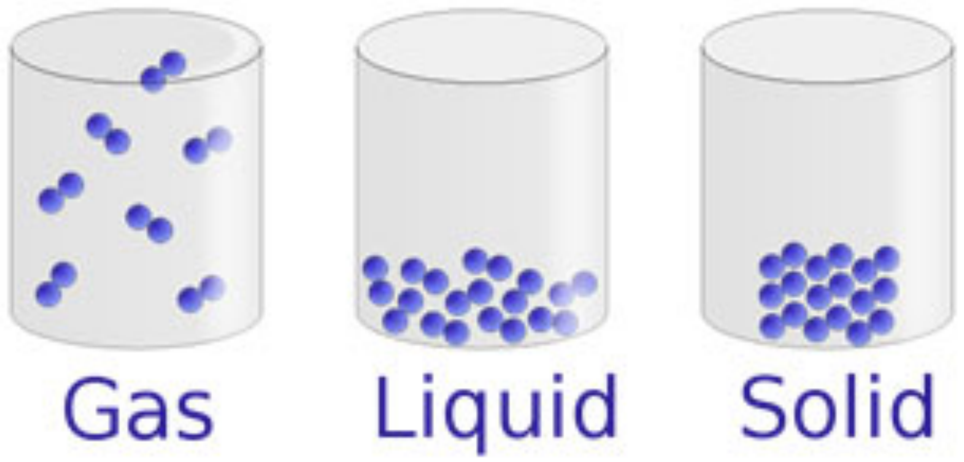


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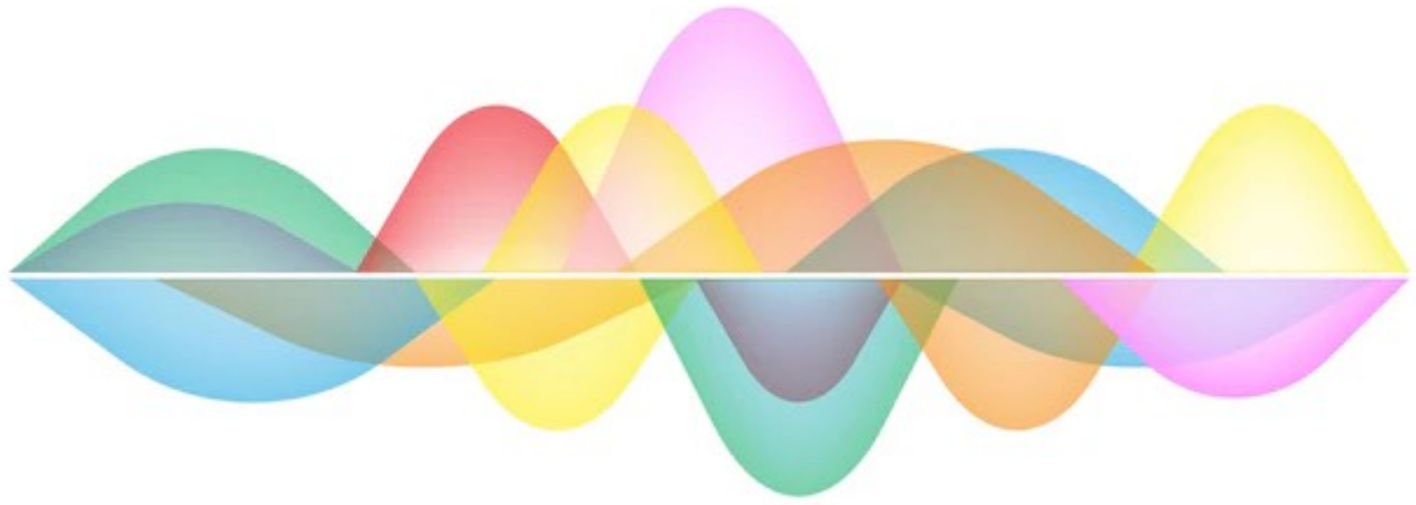
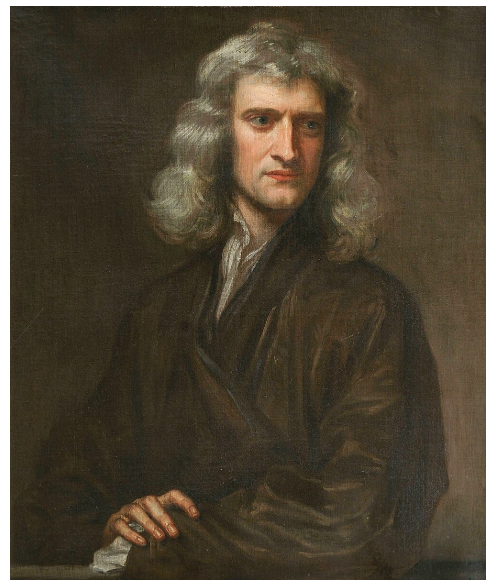


**Wave-particle Duality**: light exhibit particle or wave properties according to the experimental circumstances



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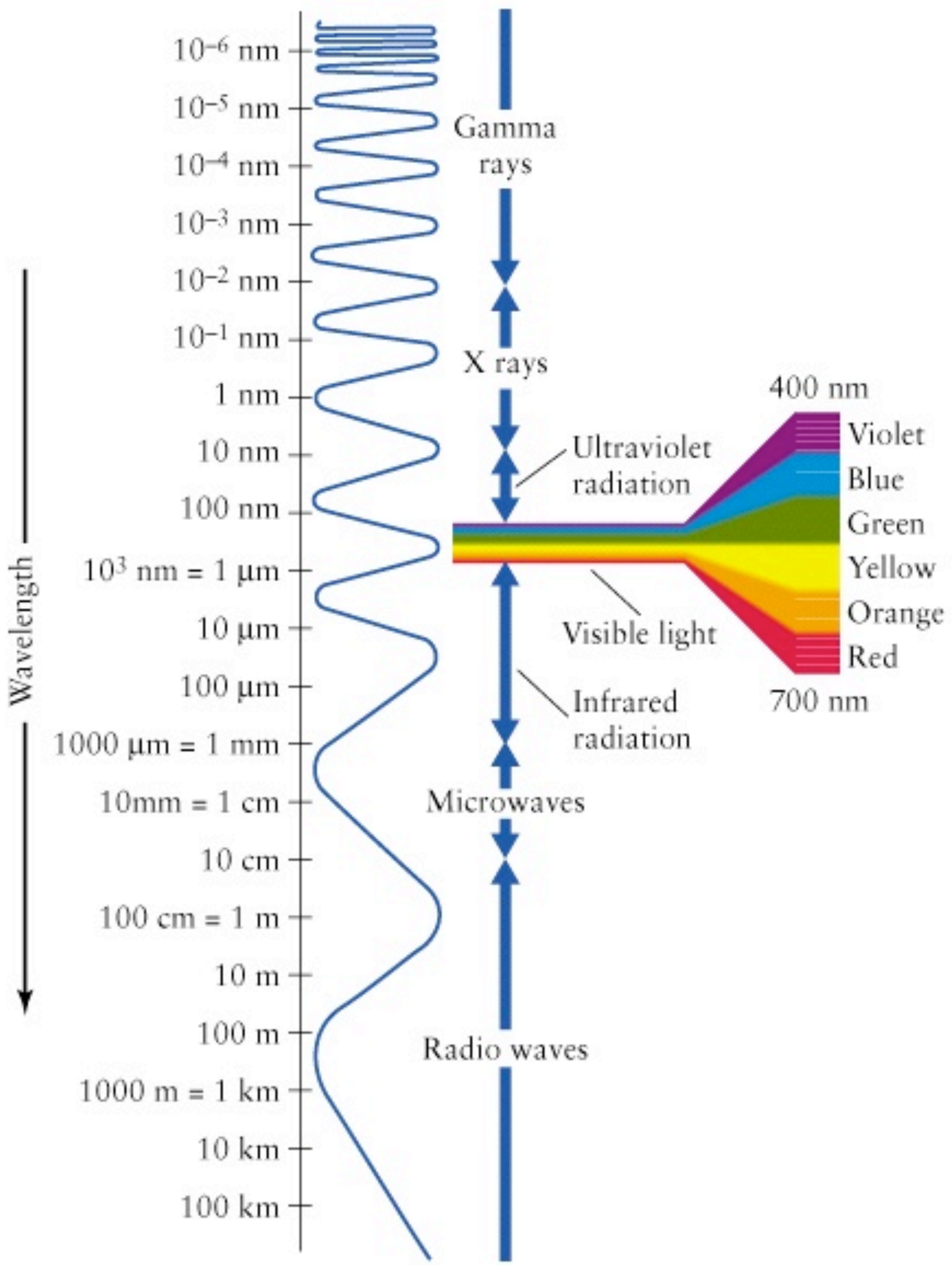
# **Light** and **Color**: A Short Preview

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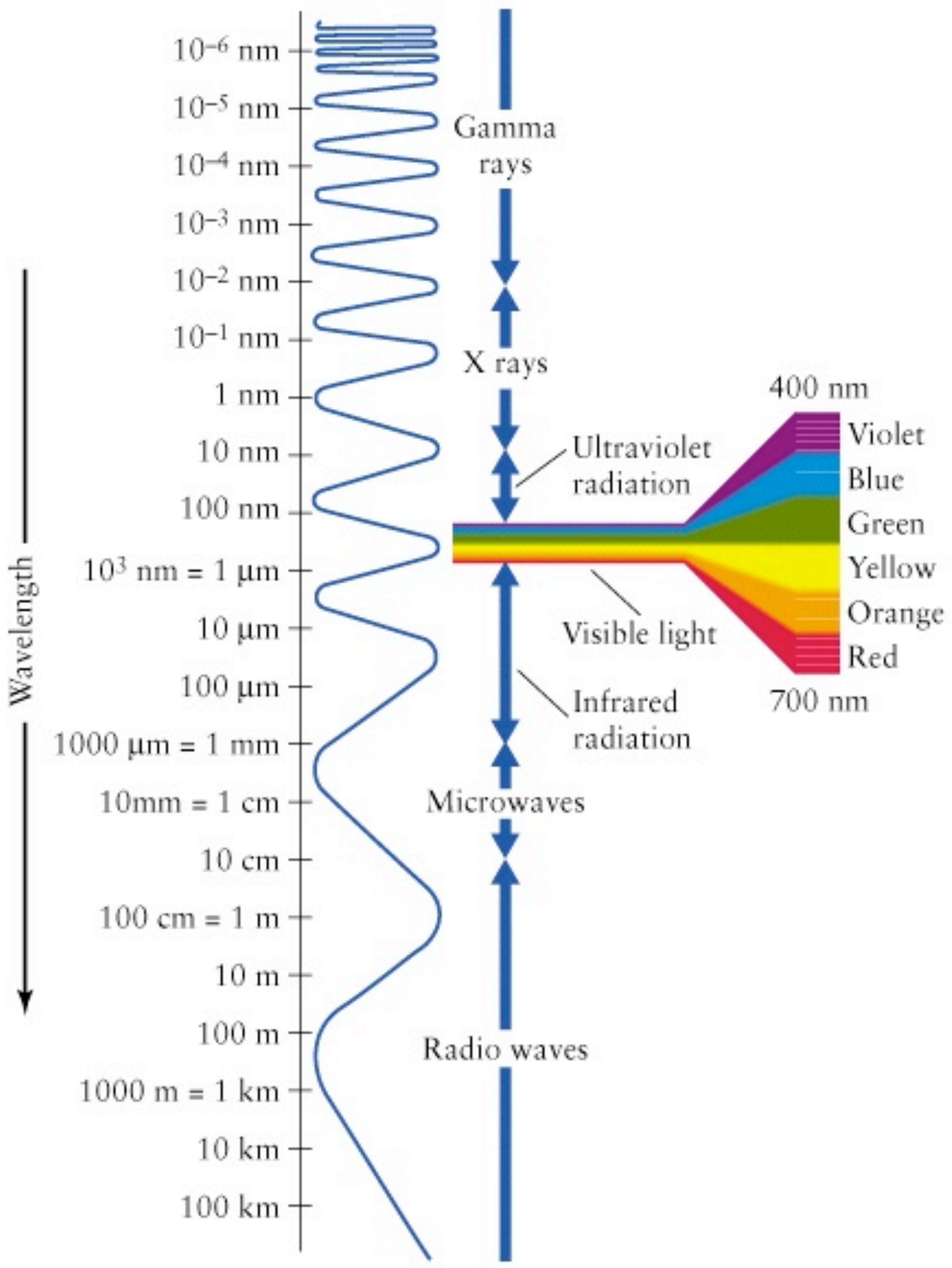


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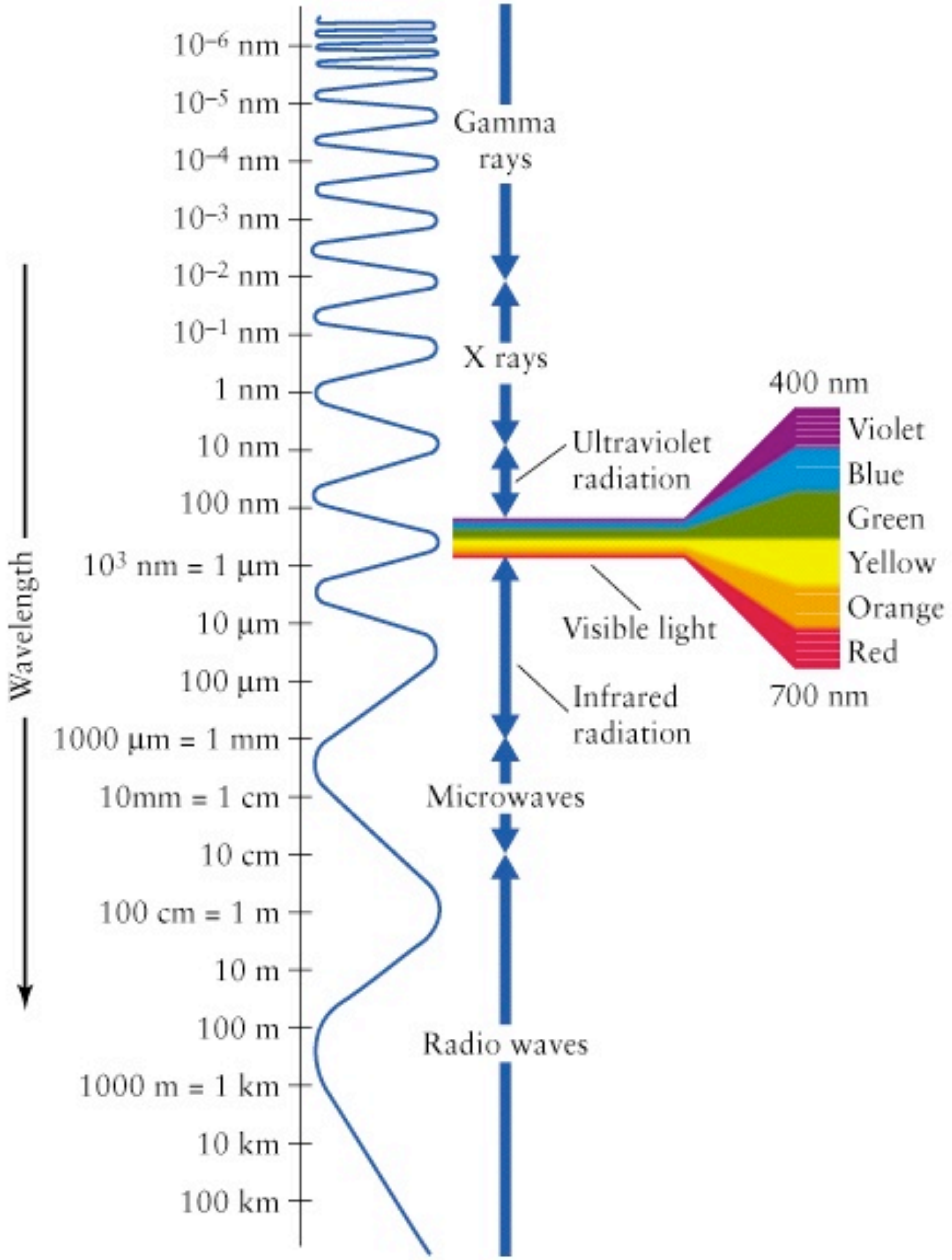


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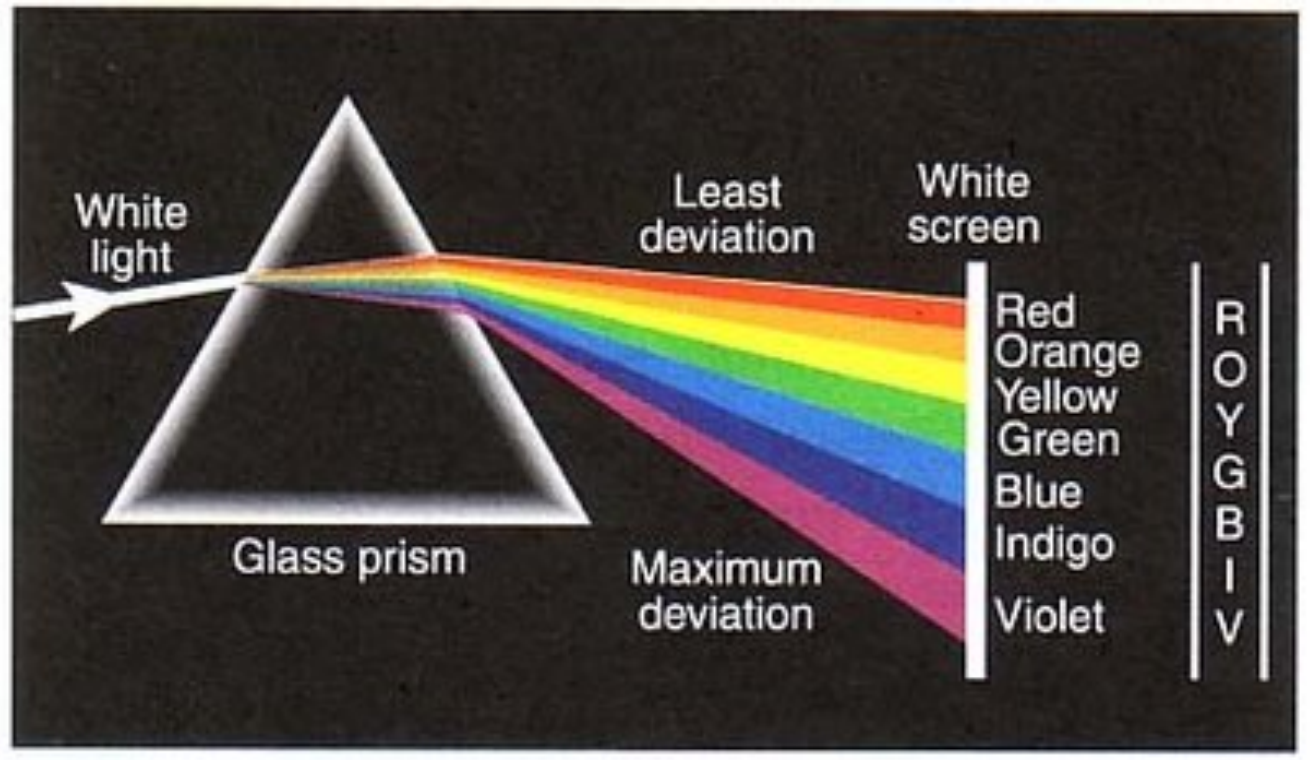
**Newton's Prism** experiment showed that white light is composed of all frequencies




**Source**: Chong Kum Ying el. all, 2006; Focus Excel Science; Form 4; pg 170

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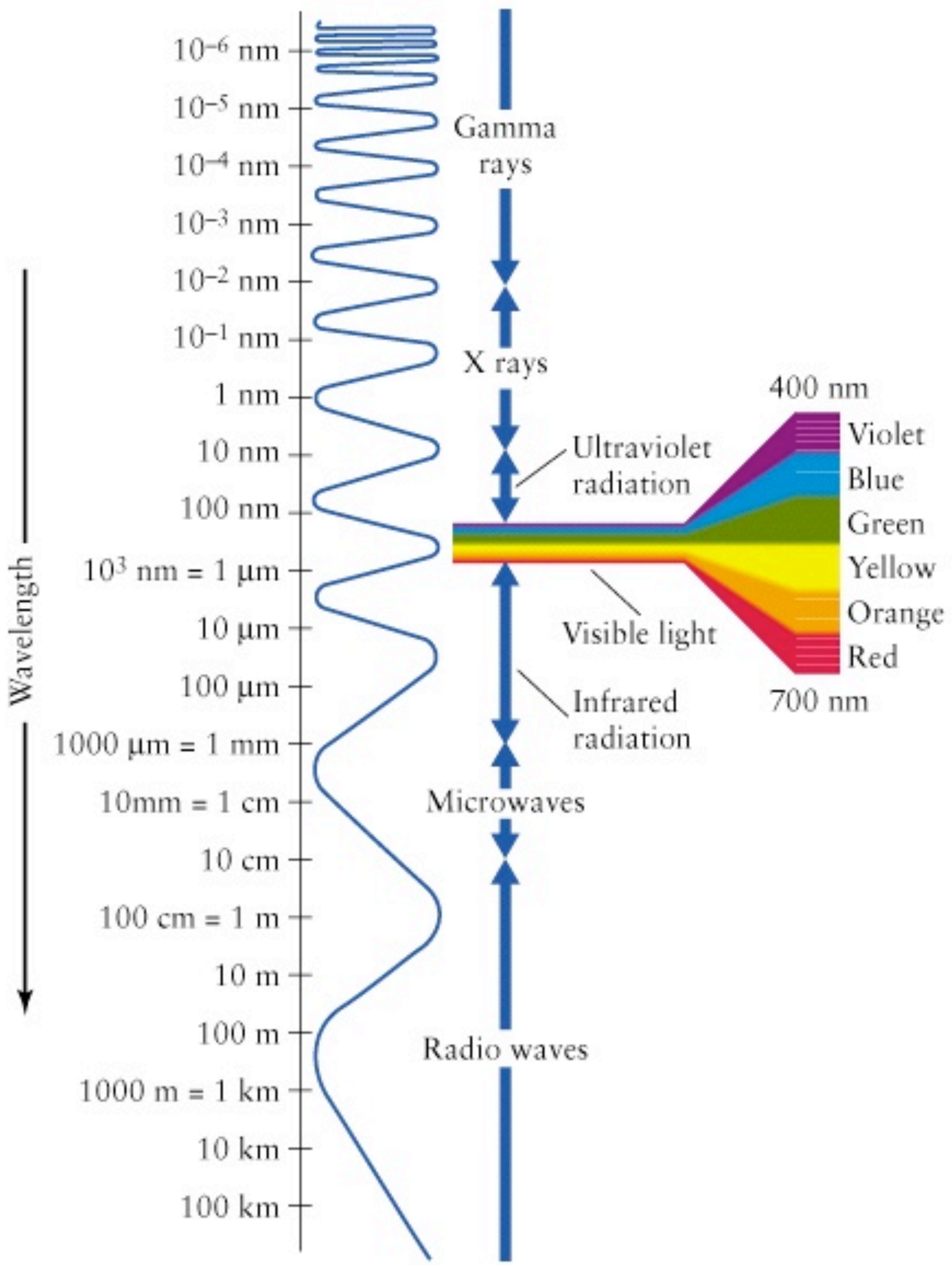


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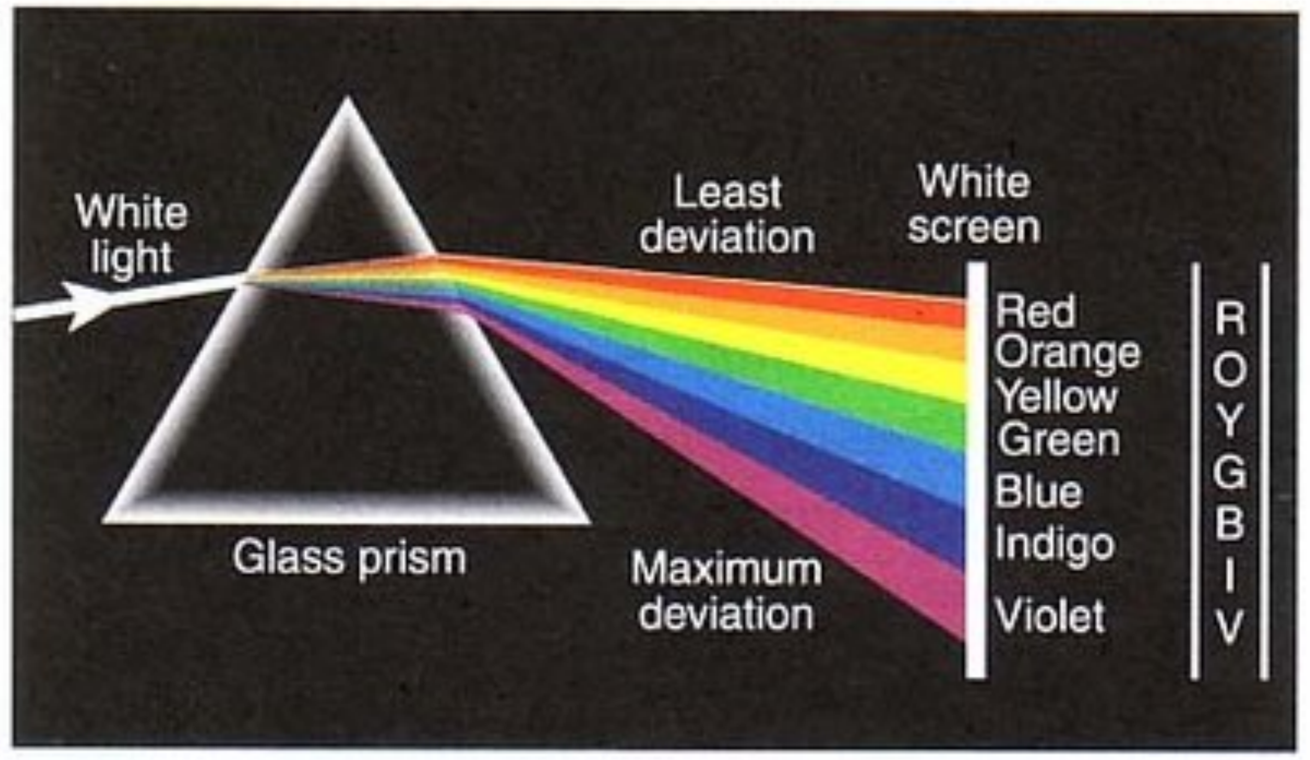


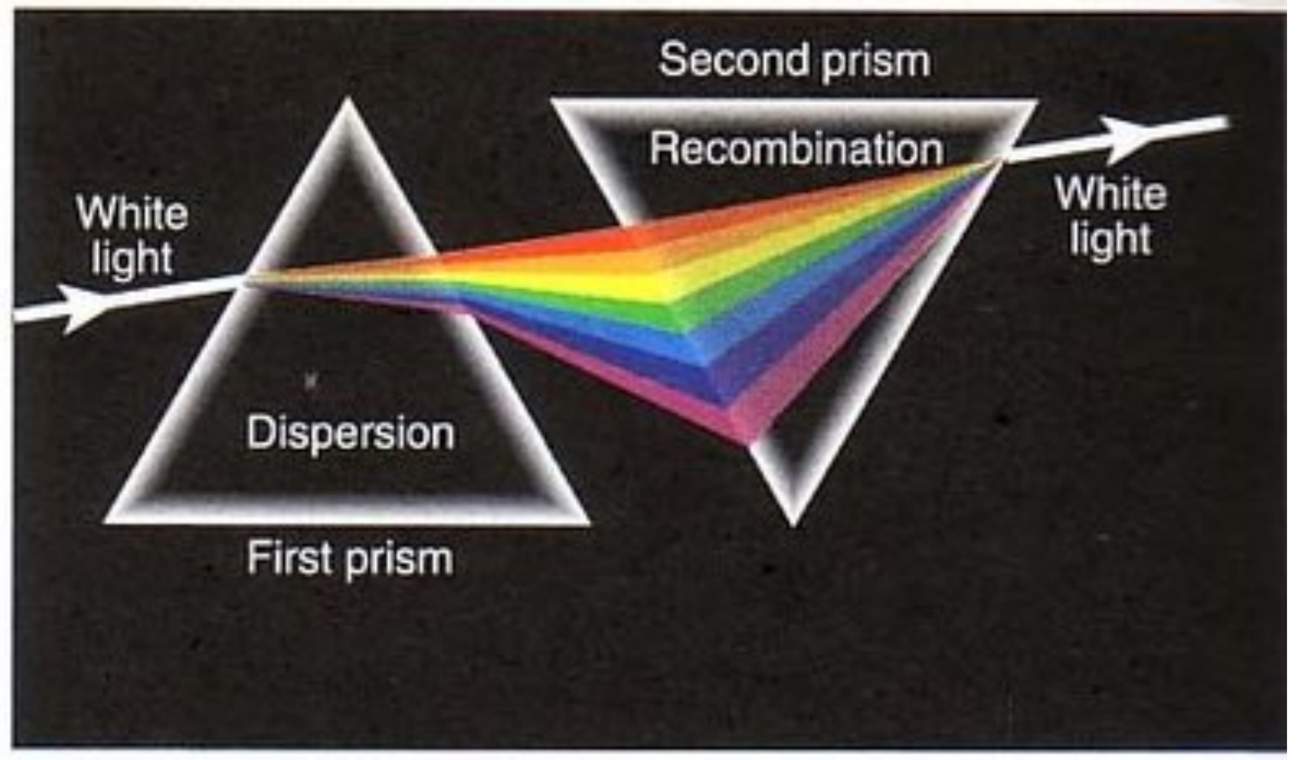


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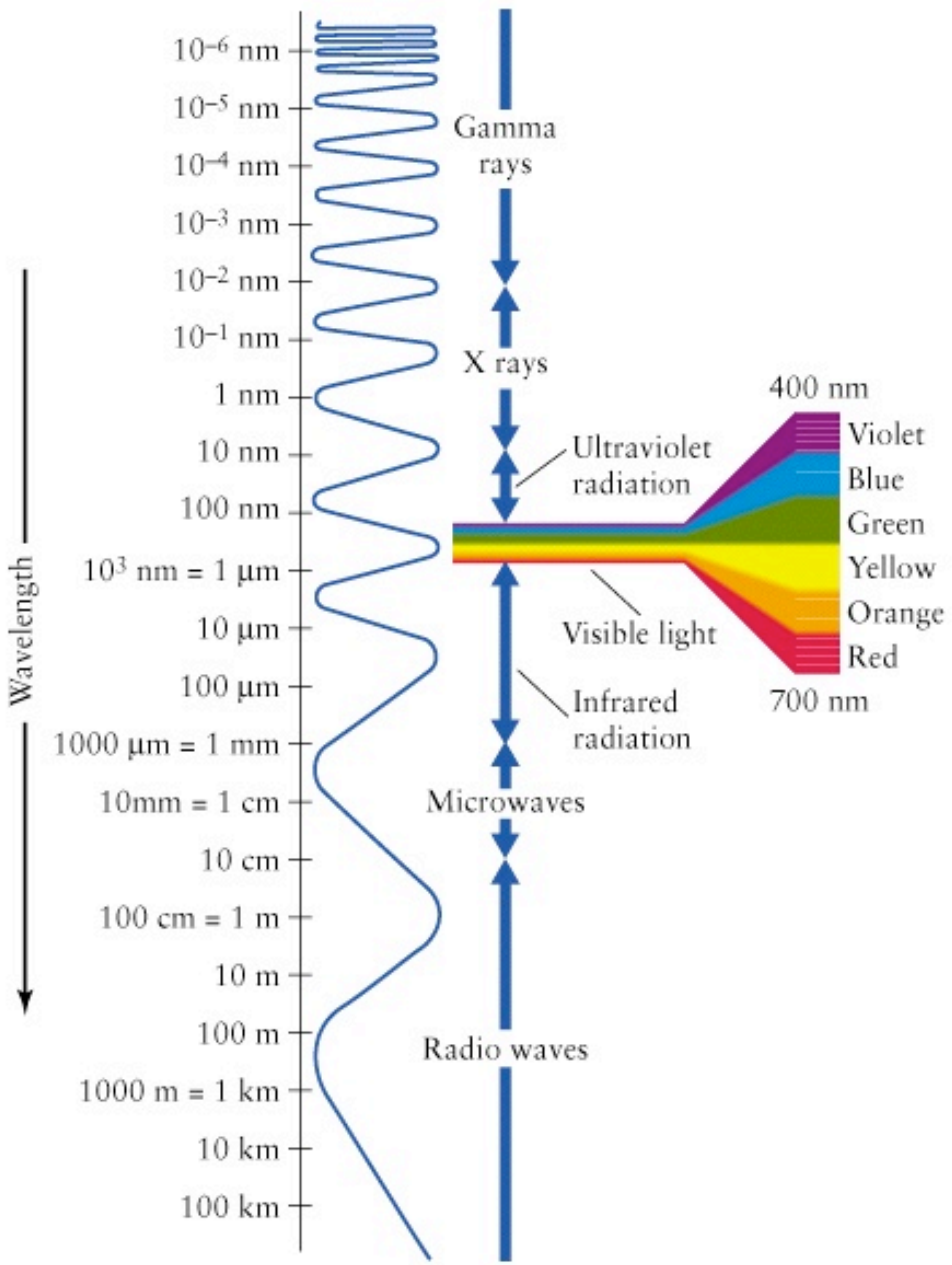


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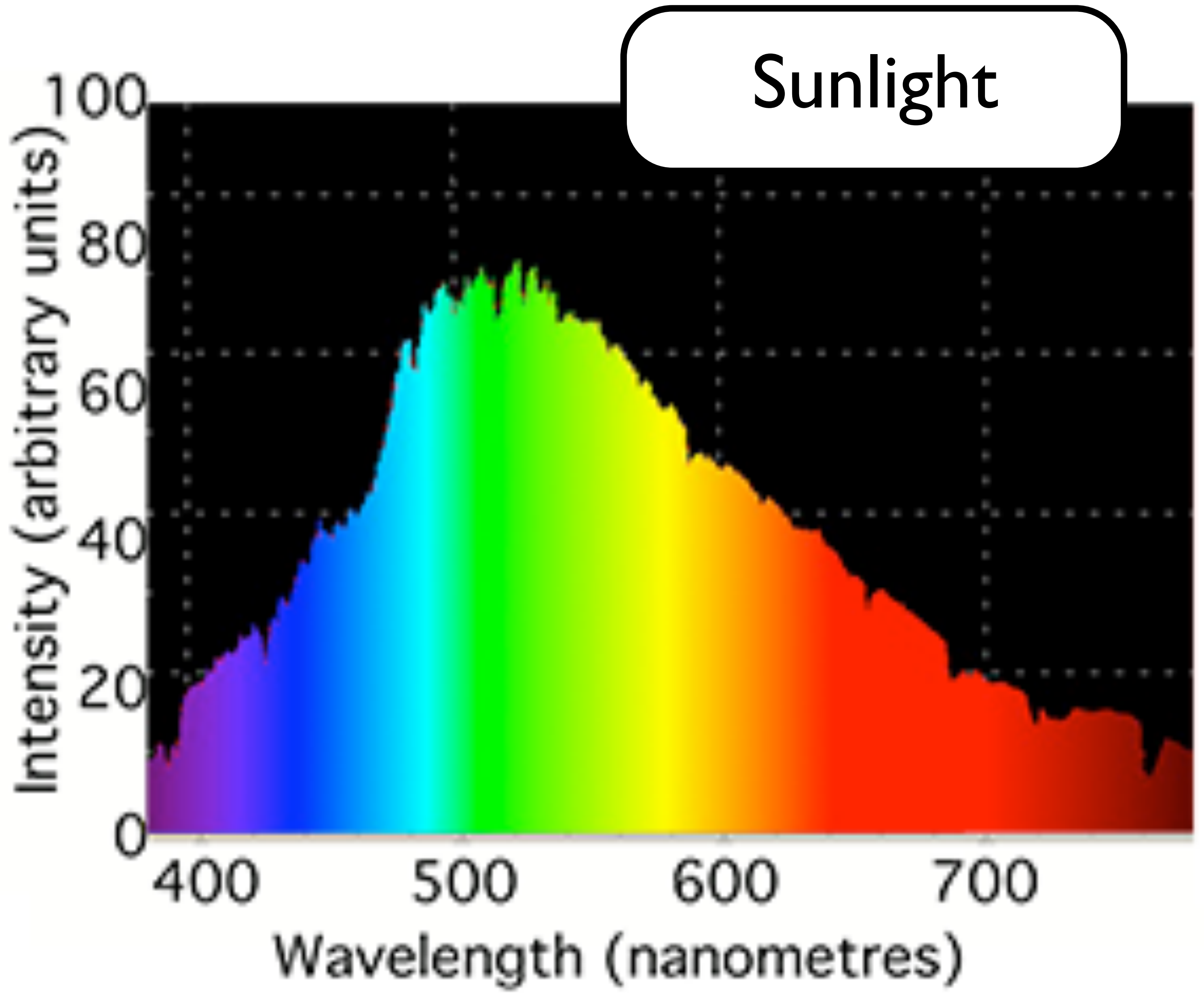

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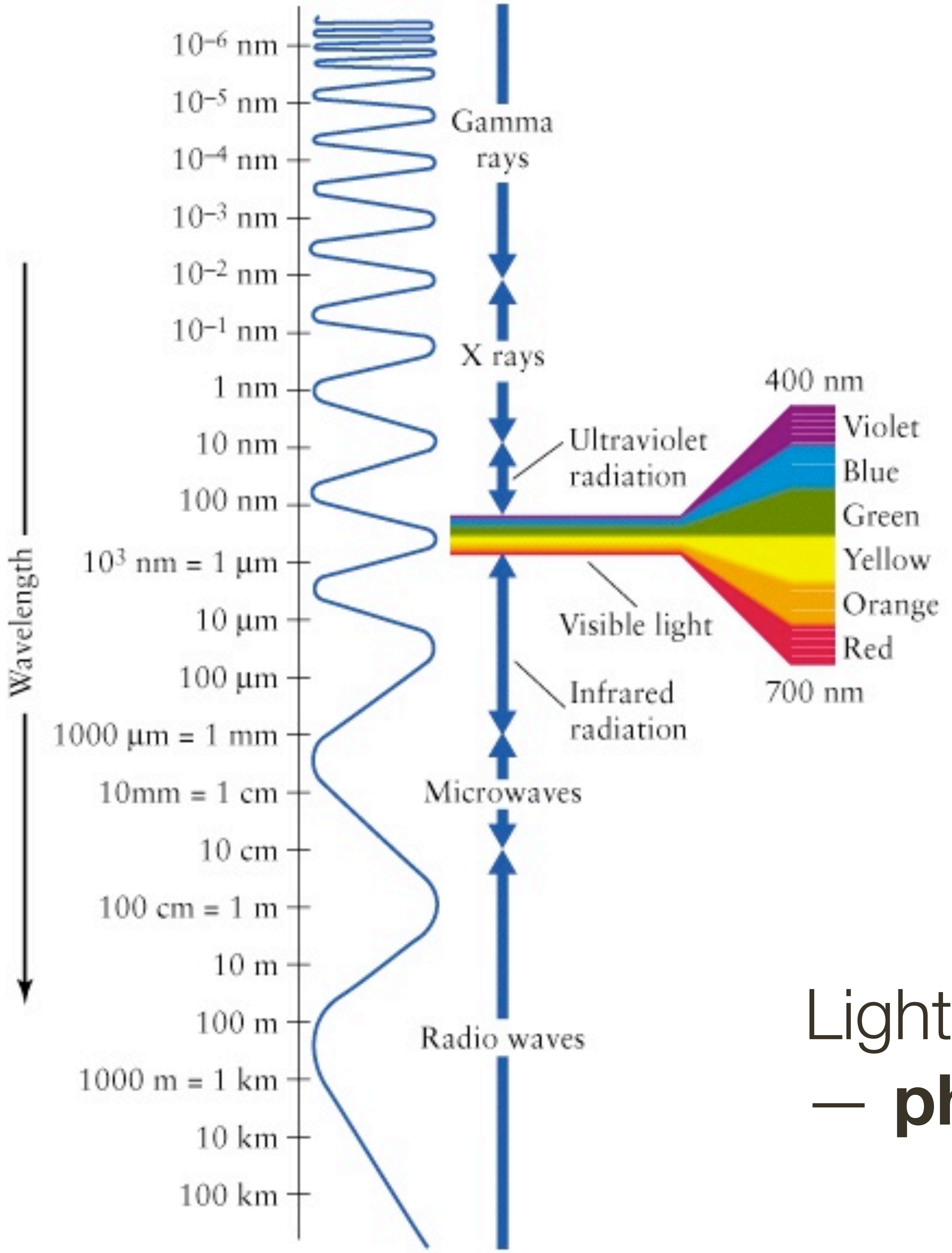


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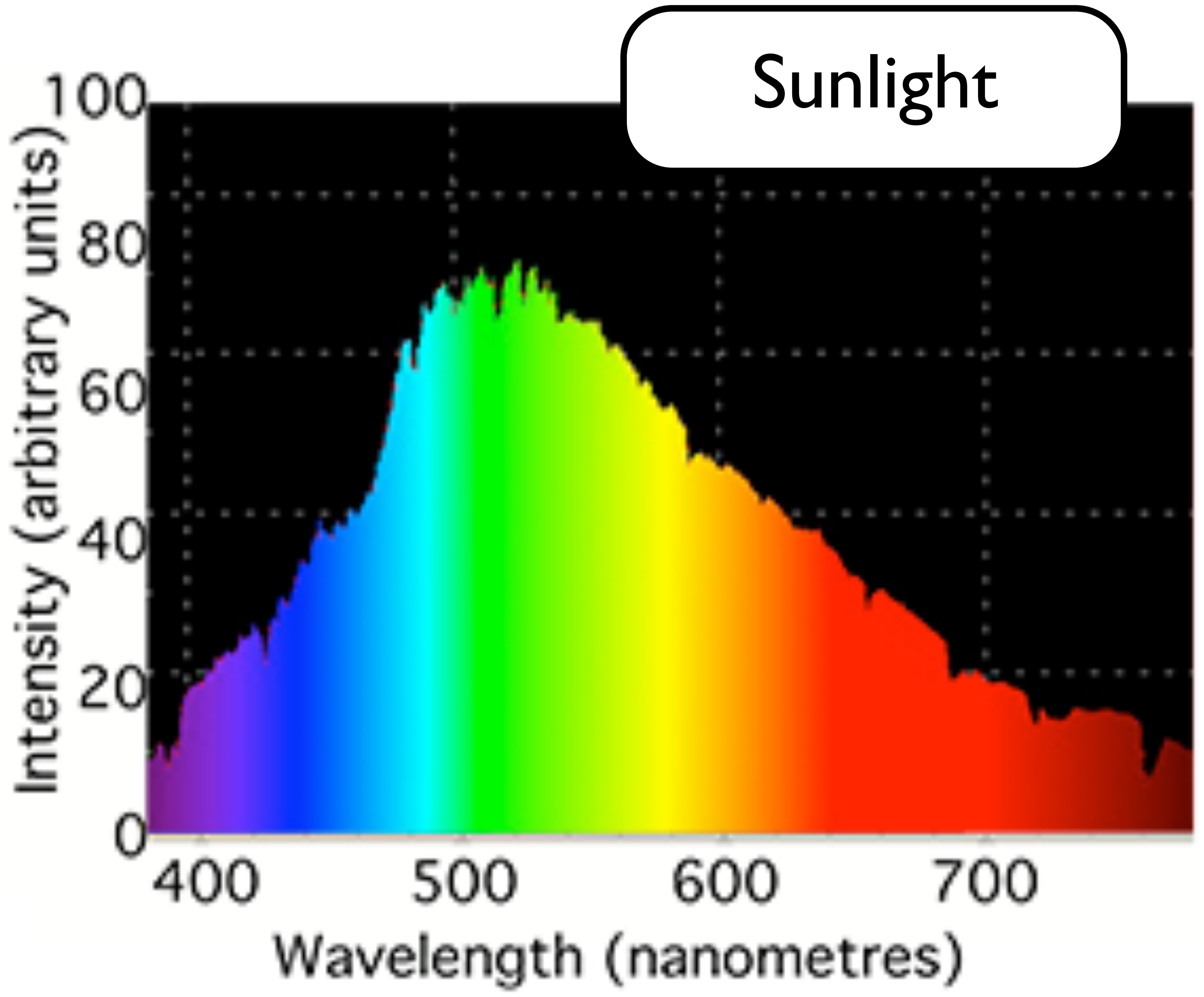
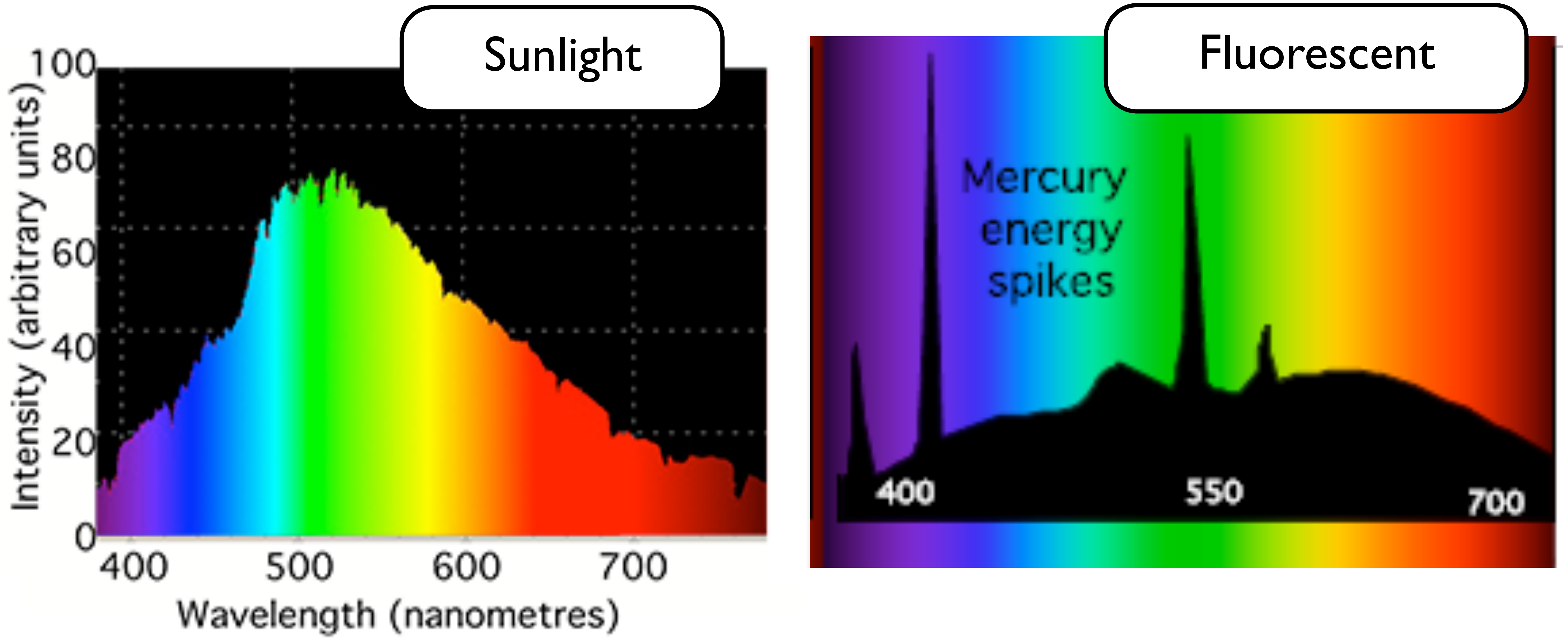



Light also behaves as particles with specific wavelengths — **photons**; that travel in straight lines within a medium

# **Spectral Power** Distribution



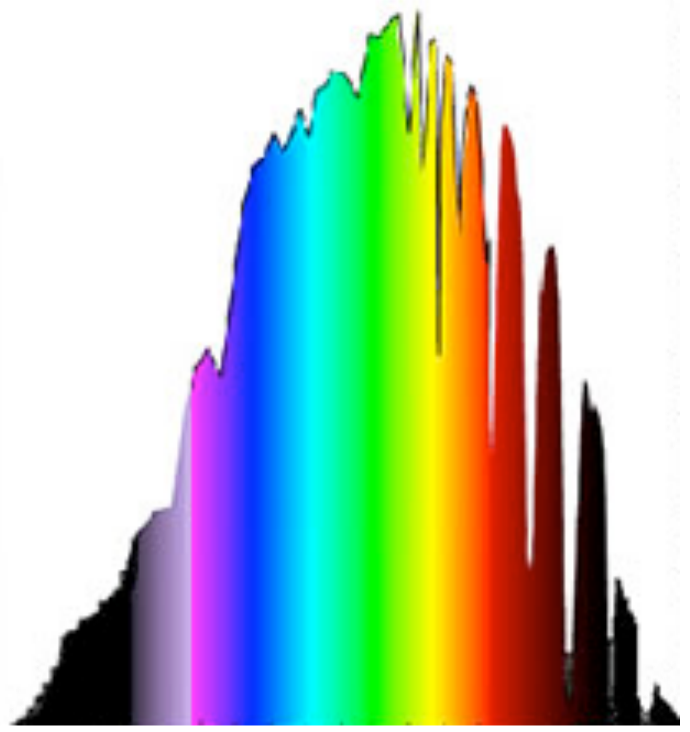

The **spectral distribution of energy** in a light ray determines its colour

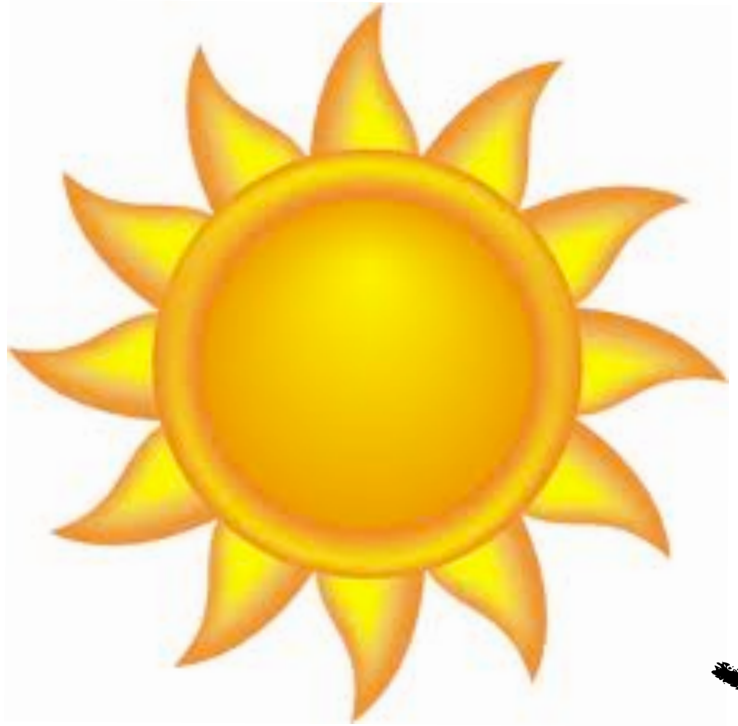
— e.g., you can have pure yellow or mixture of red and green

Surface **reflects** light energy according to a spectral distribution as well

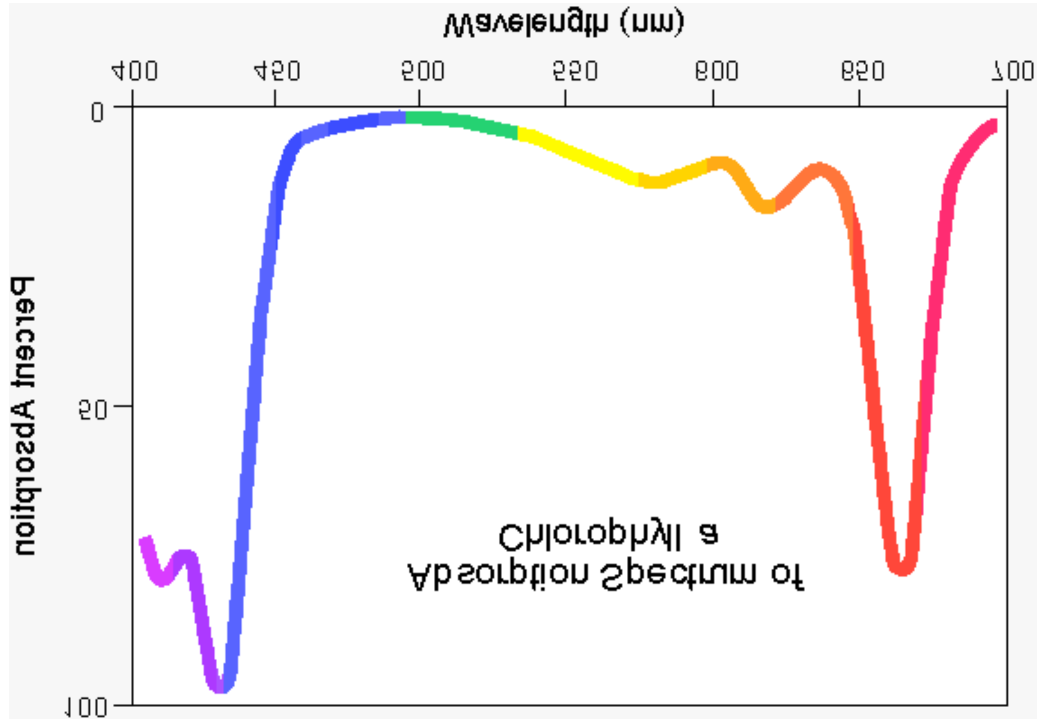
The combination of **incident** and **reflectance** spectra determines **observed colour**

# **Spectral Reflectance** Example





$E(\lambda)$




$S(\lambda)$

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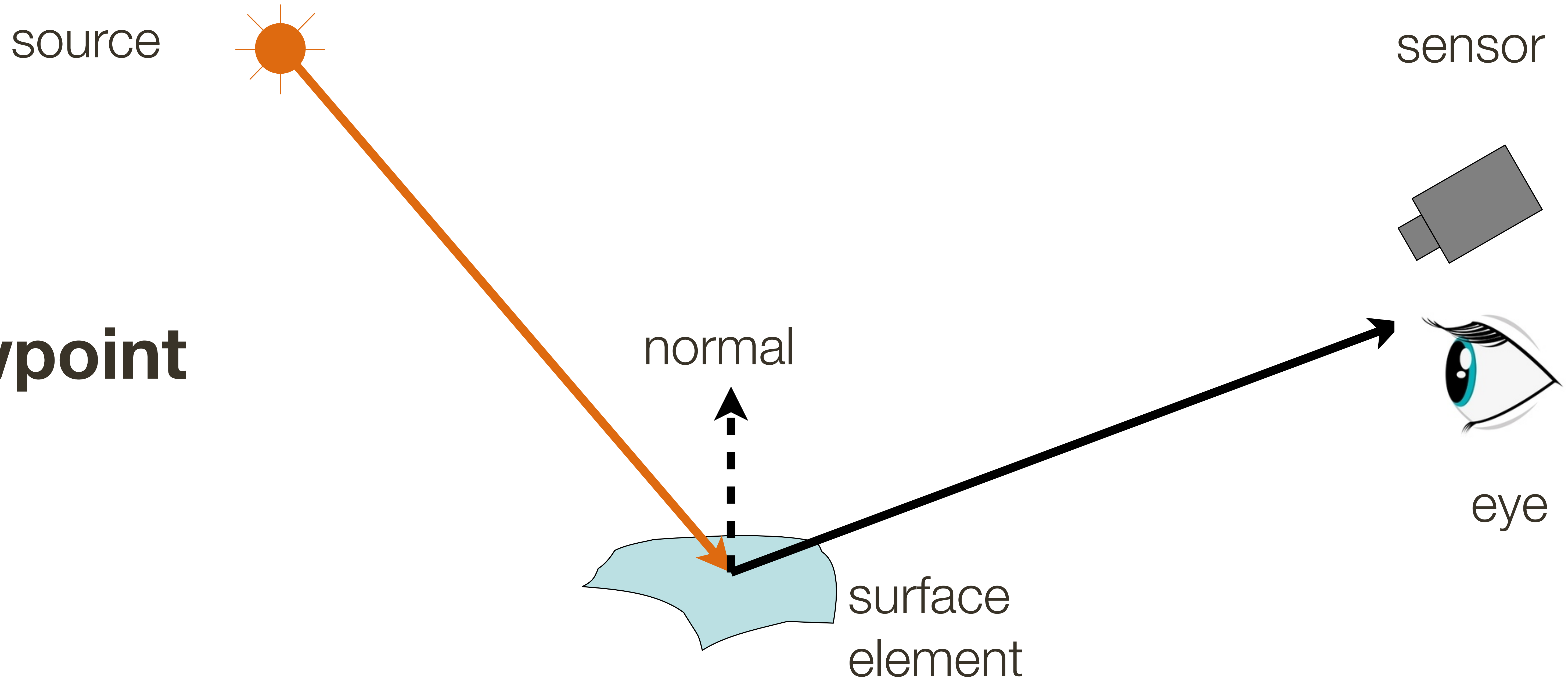


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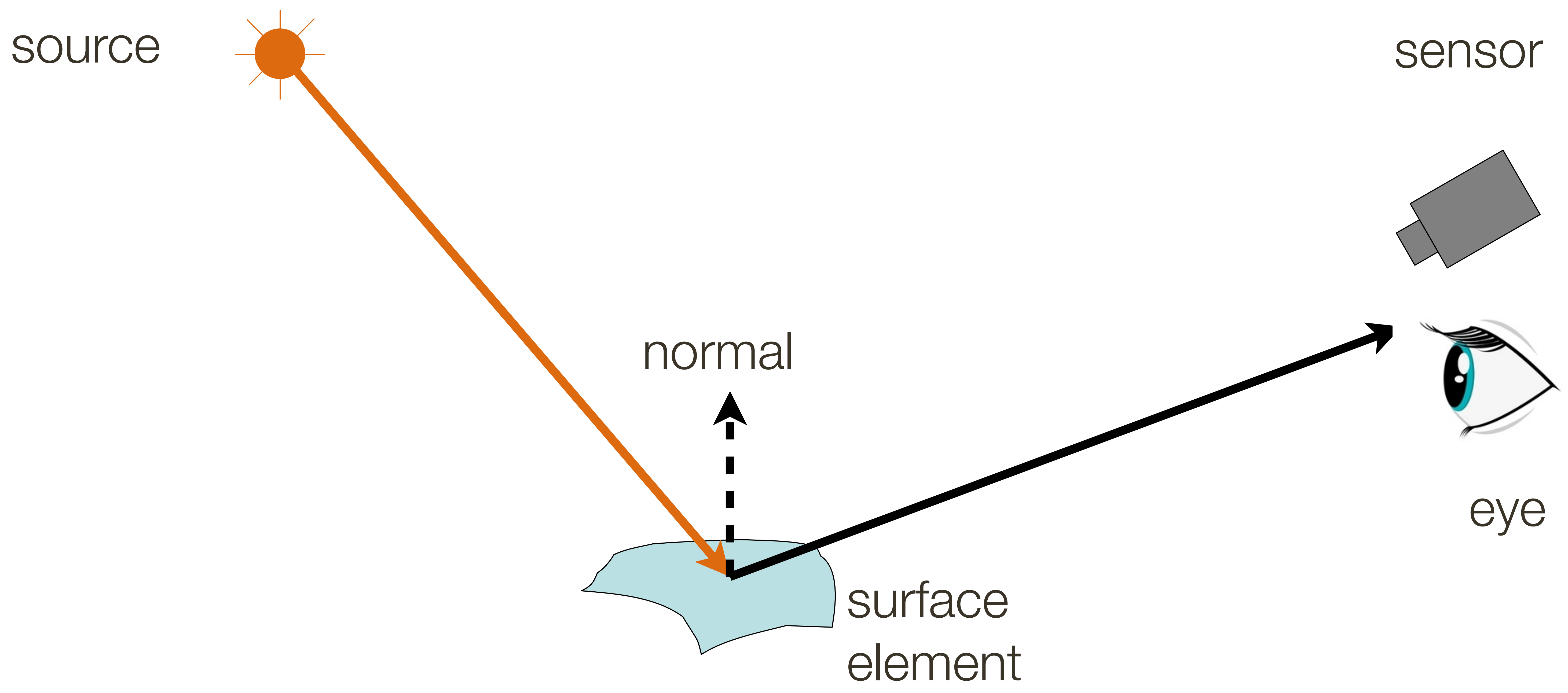


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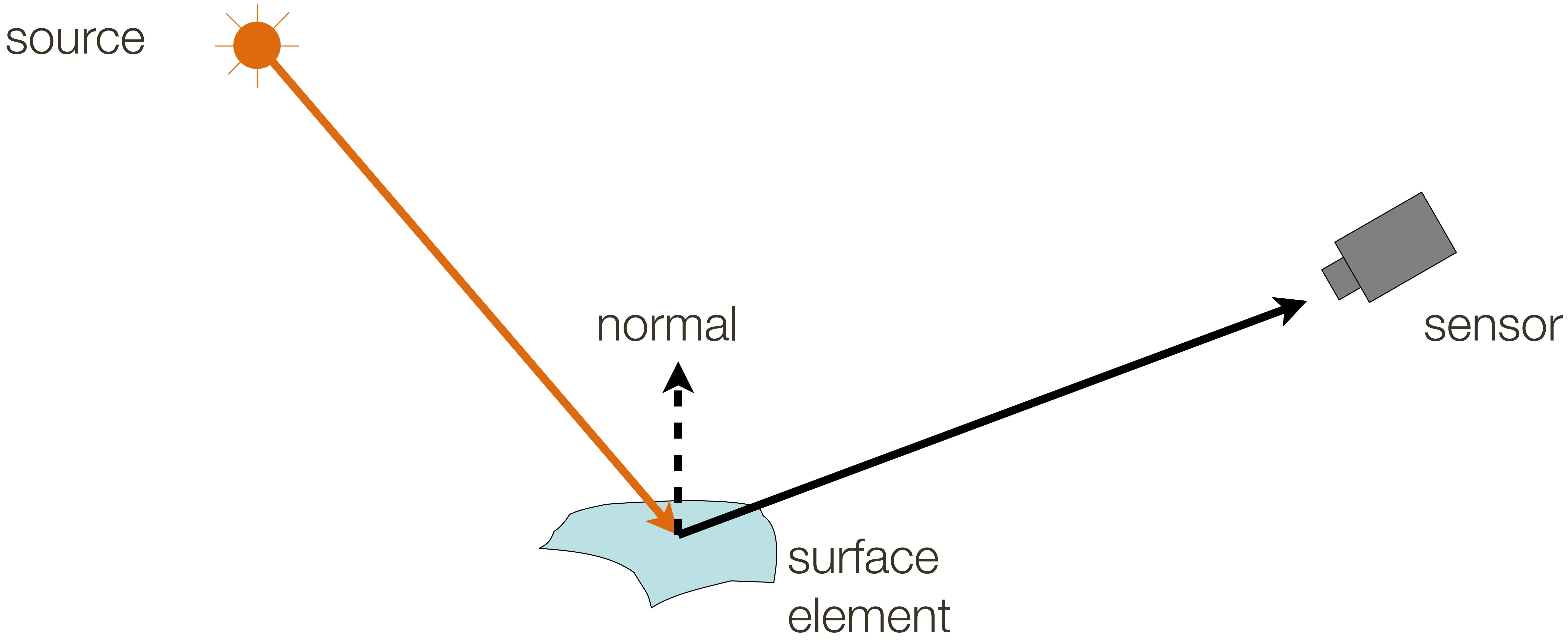

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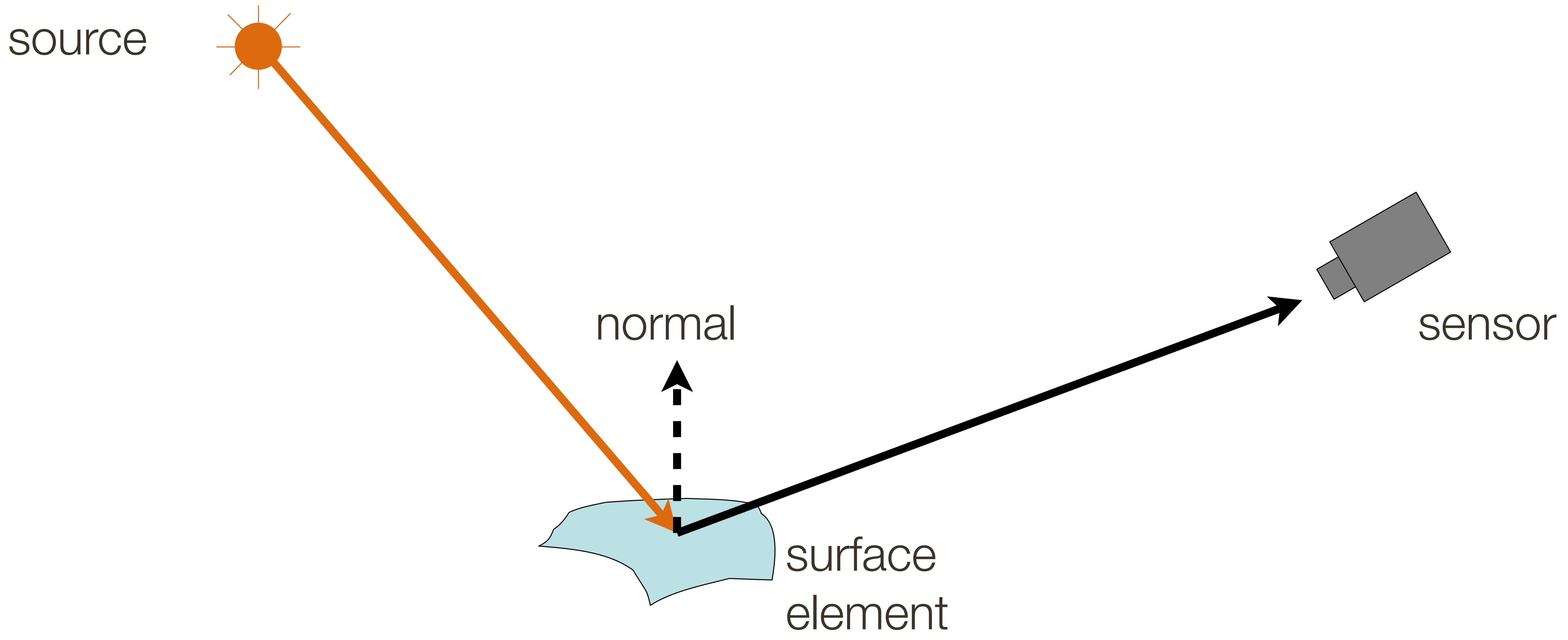

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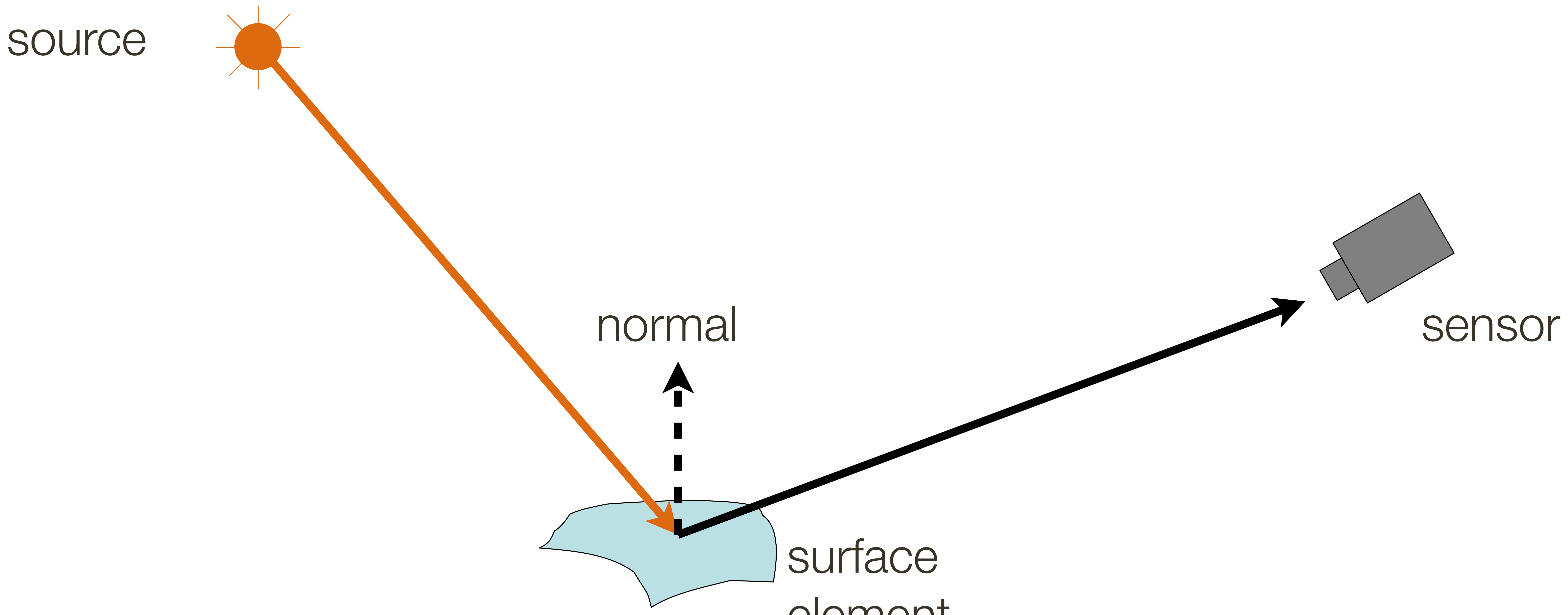

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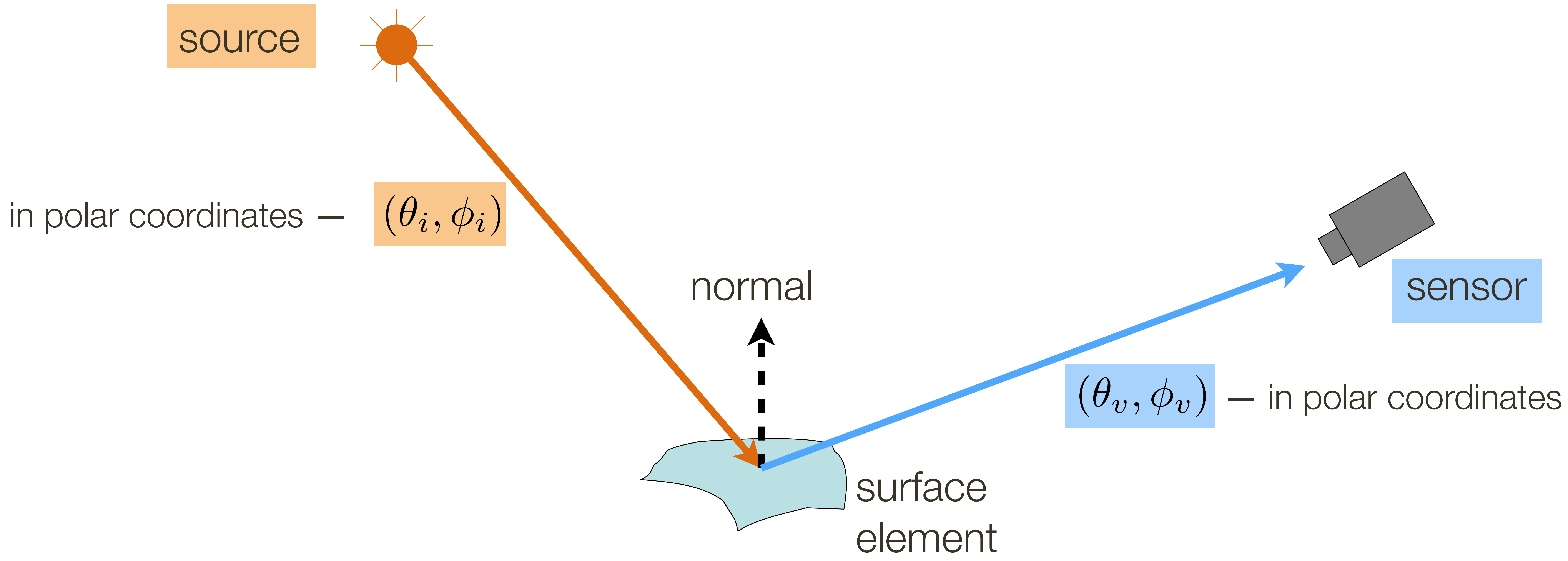
Surface reflection depends on both the **viewing** $(\theta_v, \phi_v)$ and **illumination** $(\theta_i, \phi_i)$ direction, with Bidirectional Reflection Distribution Function: $\mathbf{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v)$




**Slide adopted from**: Ioannis (Yannis) Gkioulekas (CMU)

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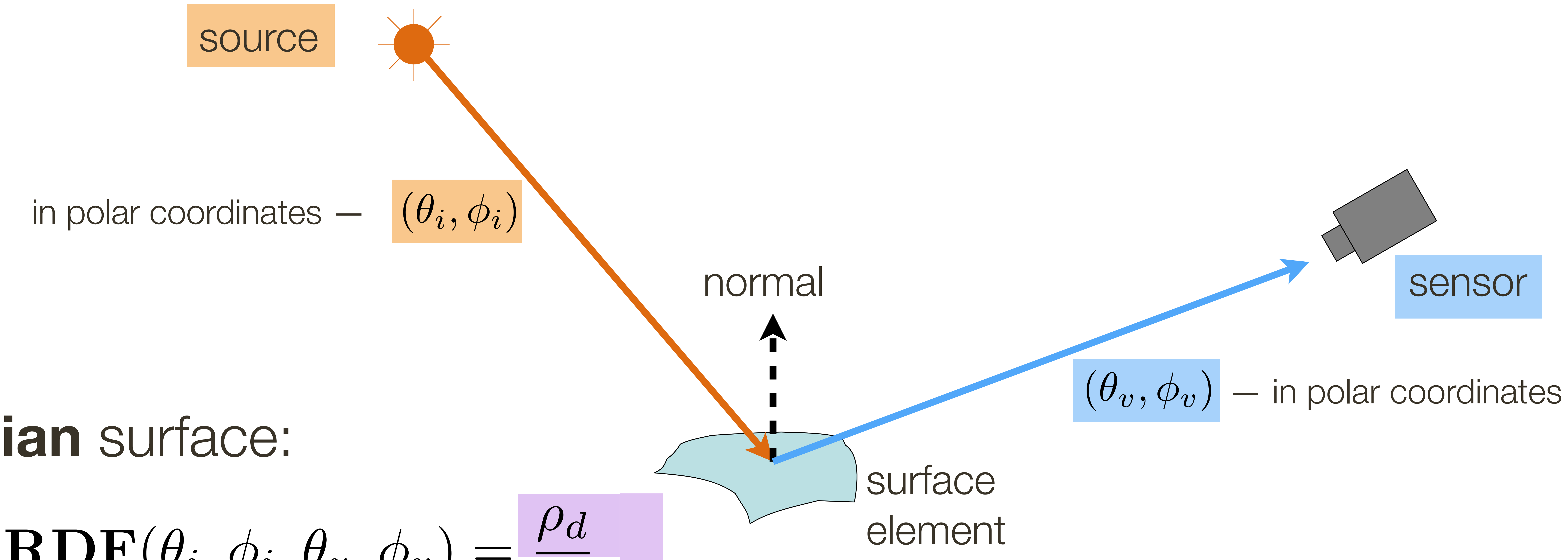


**Lambertian** surface:

$$\mathbf{BRDF}(\theta_i, \phi_i, \theta_v, \phi_v) = \frac{\rho_d}{\pi}$$

constant, called **albedo**

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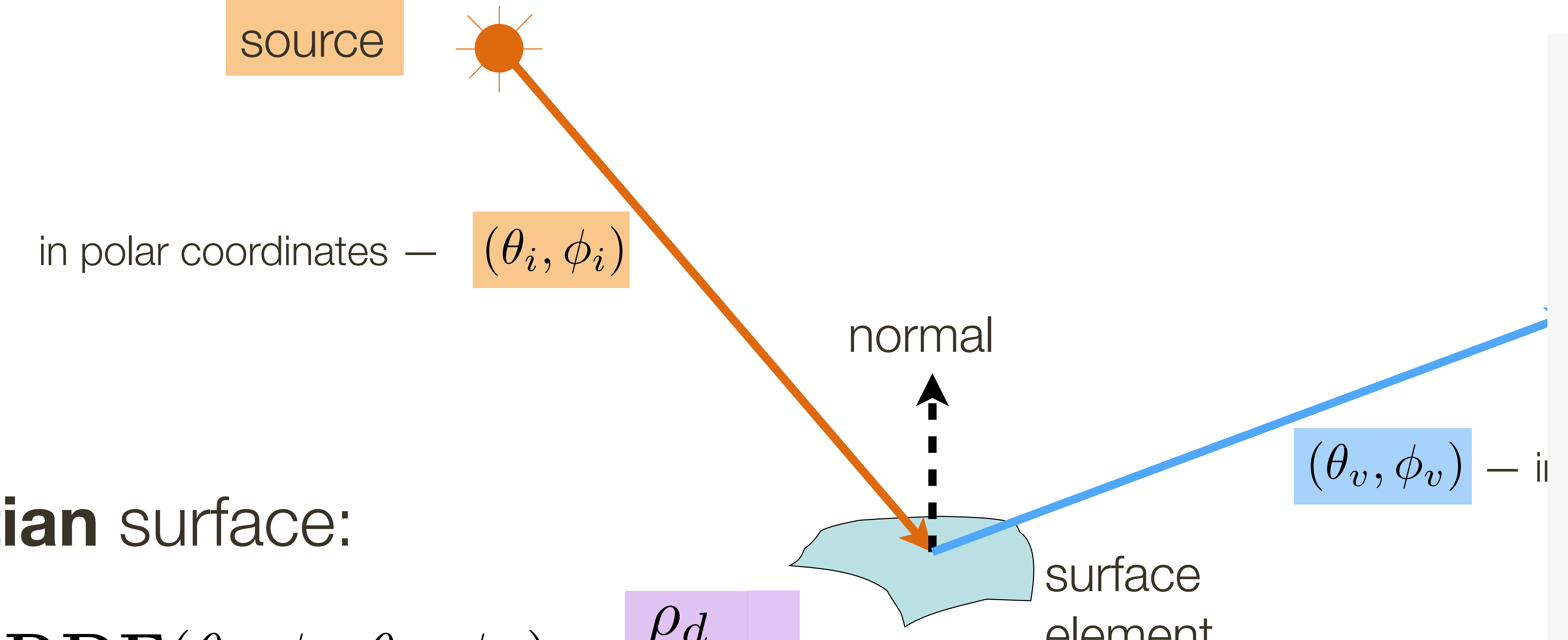


Matt Plaster Sculpture

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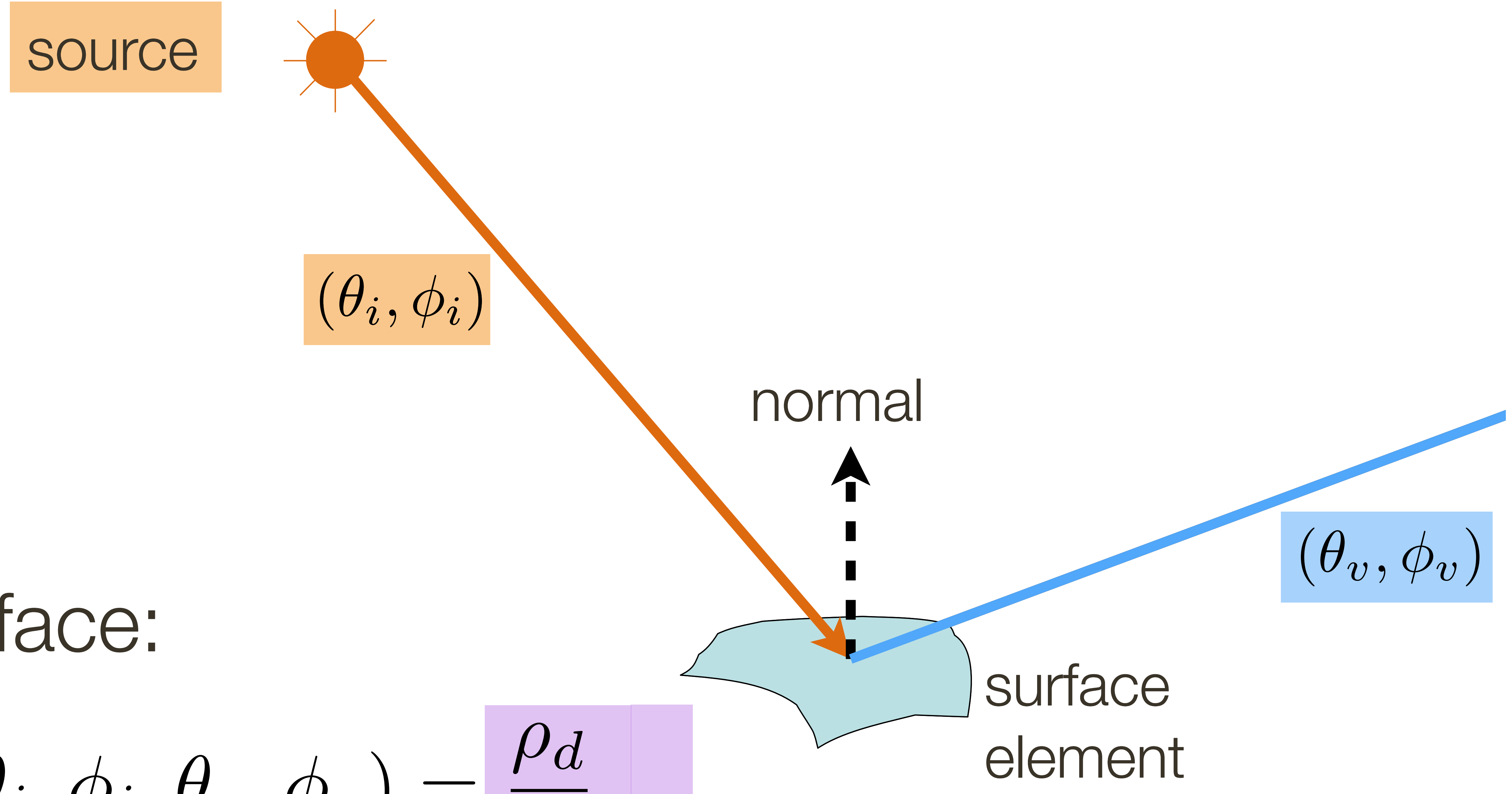


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| Surface type | Typical value |
|---|---|
| Fresh asphalt | 0.03 – 0.04 |
| Open ocean | 0.06 |
| Conifer forest (summer) | 0.08 – 0.15 |
| Worn asphalt | 0.12 |
| Deciduous trees | 0.15 – 0.18 |
| Sand | 0.15 – 0.45 |
| Tundra | 0.18 – 0.25 |
| Agricultural crops | 0.18 – 0.25 |
| Bare soil | 0.17 |
| Green grass | 0.20 – 0.25 |
| Dessert sand | 0.30 – 0.40 |
| Snow | 0.40 – 0.90 |
| Ocean ice | 0.50 – 0.70 |
| Fresh snow | 0.80 – 0.90 |

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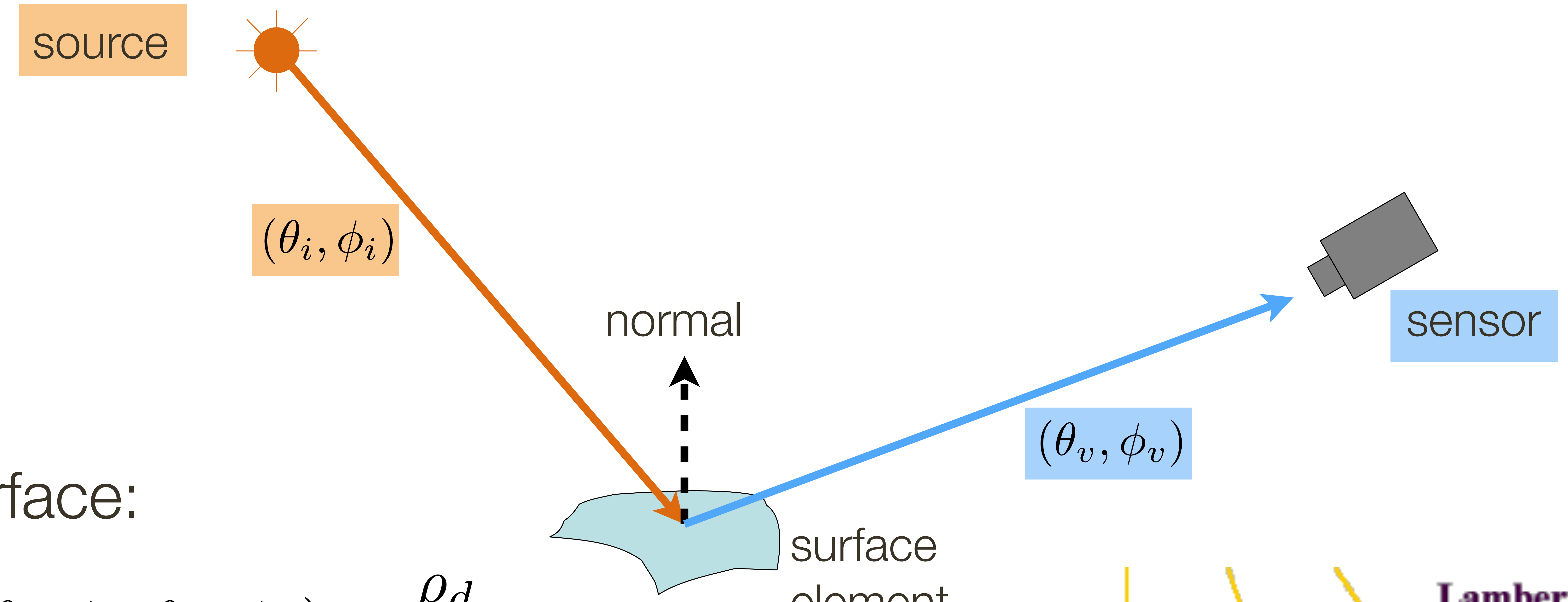


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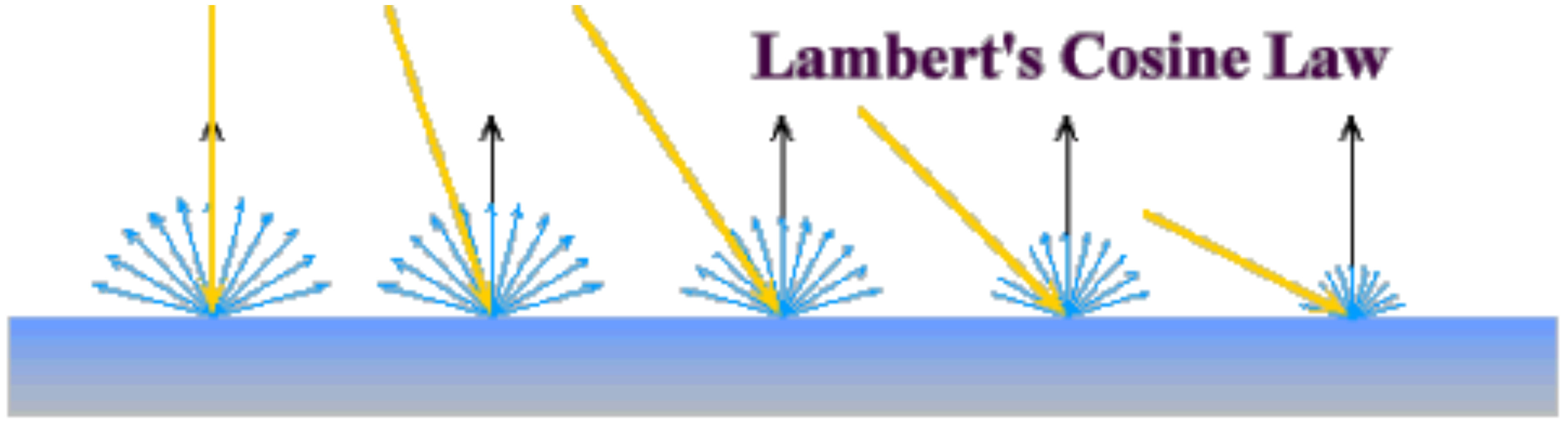


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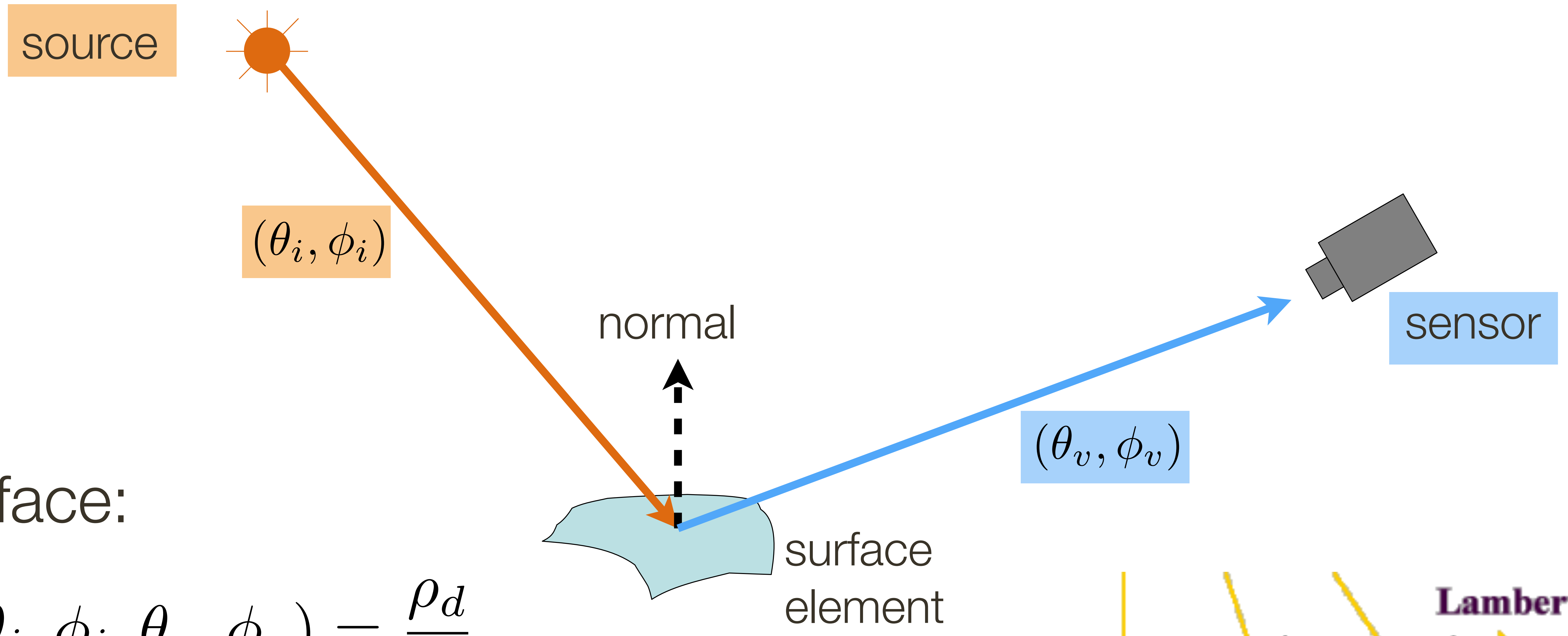
**Question**: What are the simplifying assumptions we are making here?

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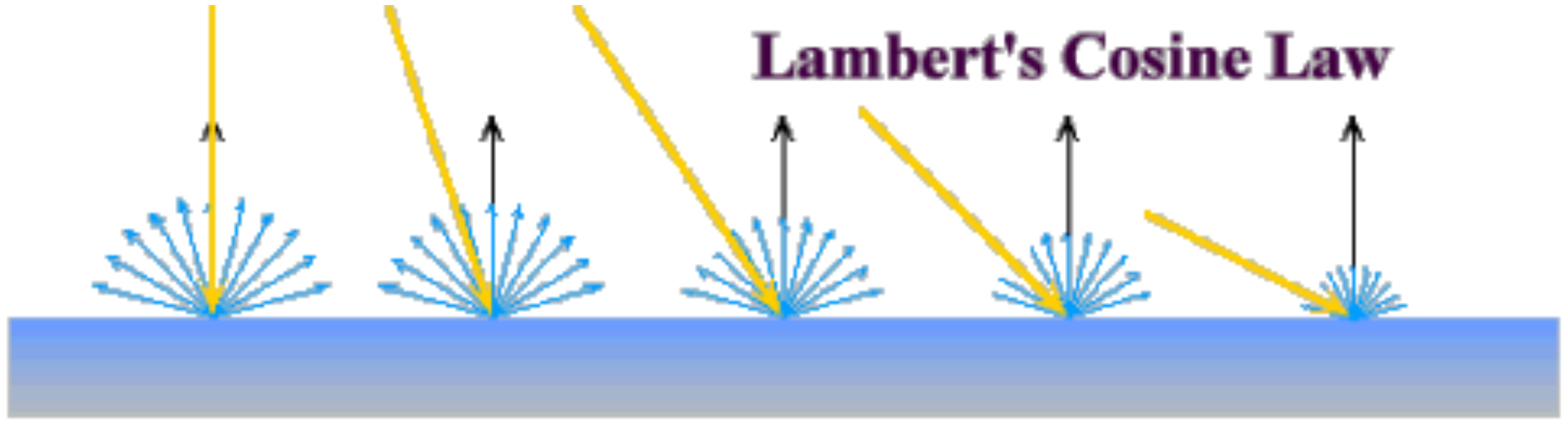


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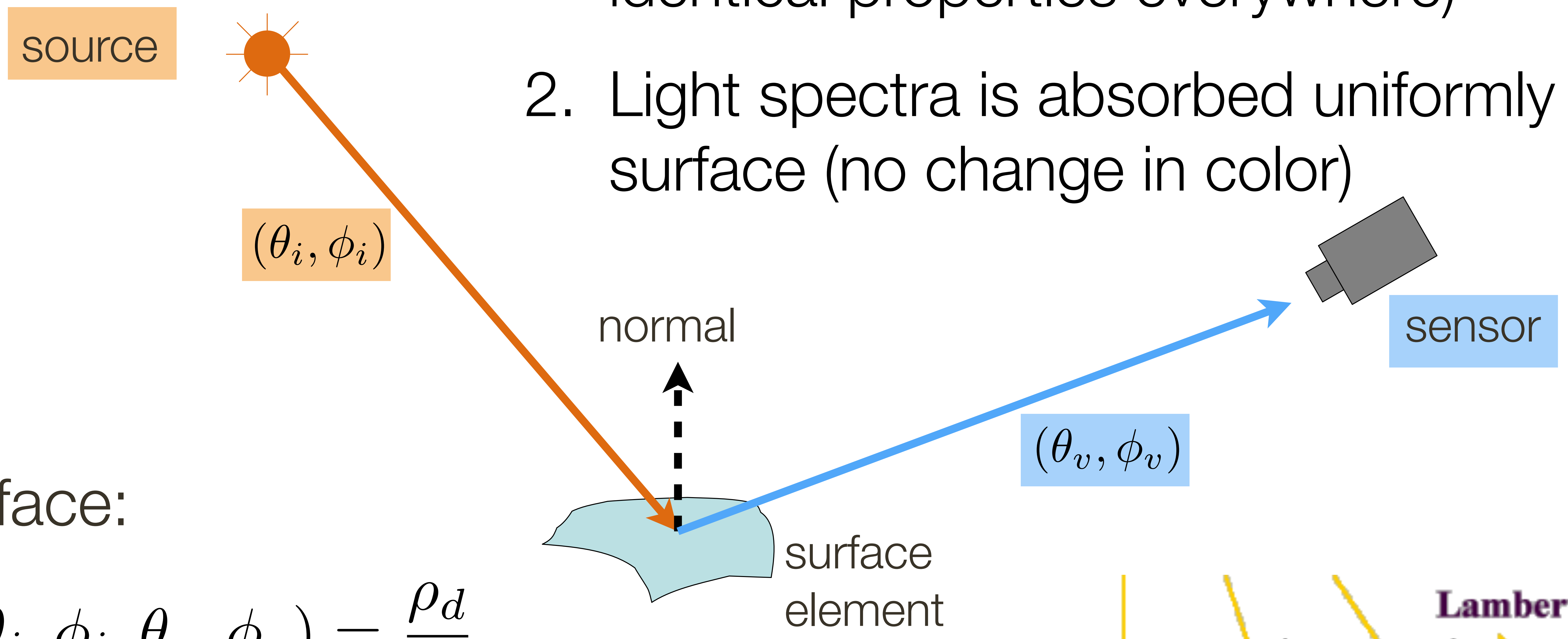
# (small) **Graphics** Review

**Question**: What are the simplifying assumptions we are making here?

1. BRDF is the same everywhere (i.e., surface has identical properties everywhere)
2. Light spectra is absorbed uniformly by the surface (no change in color)

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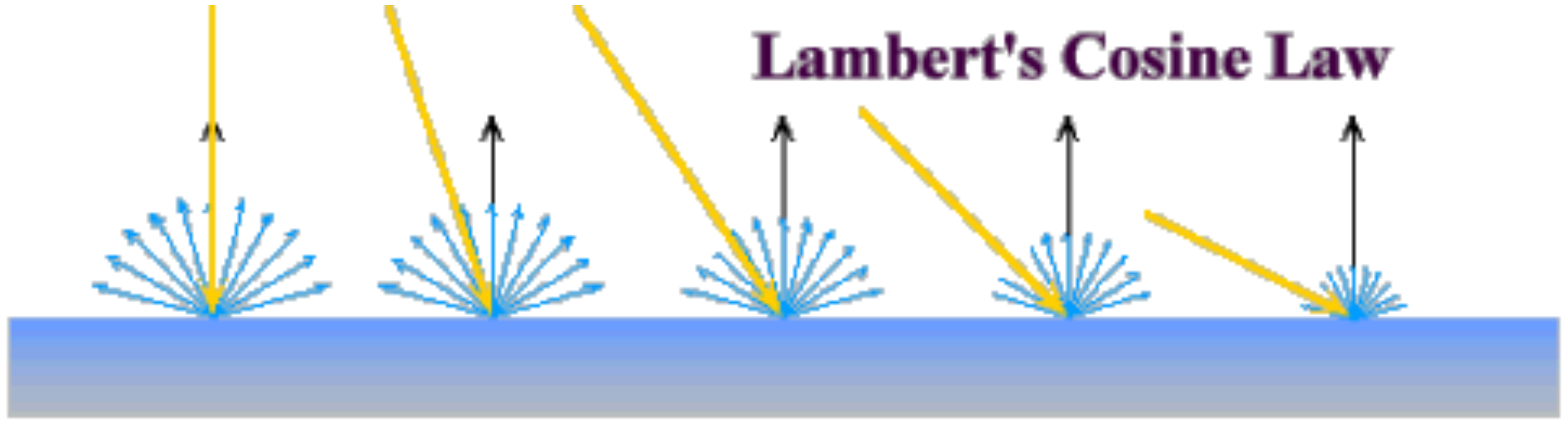


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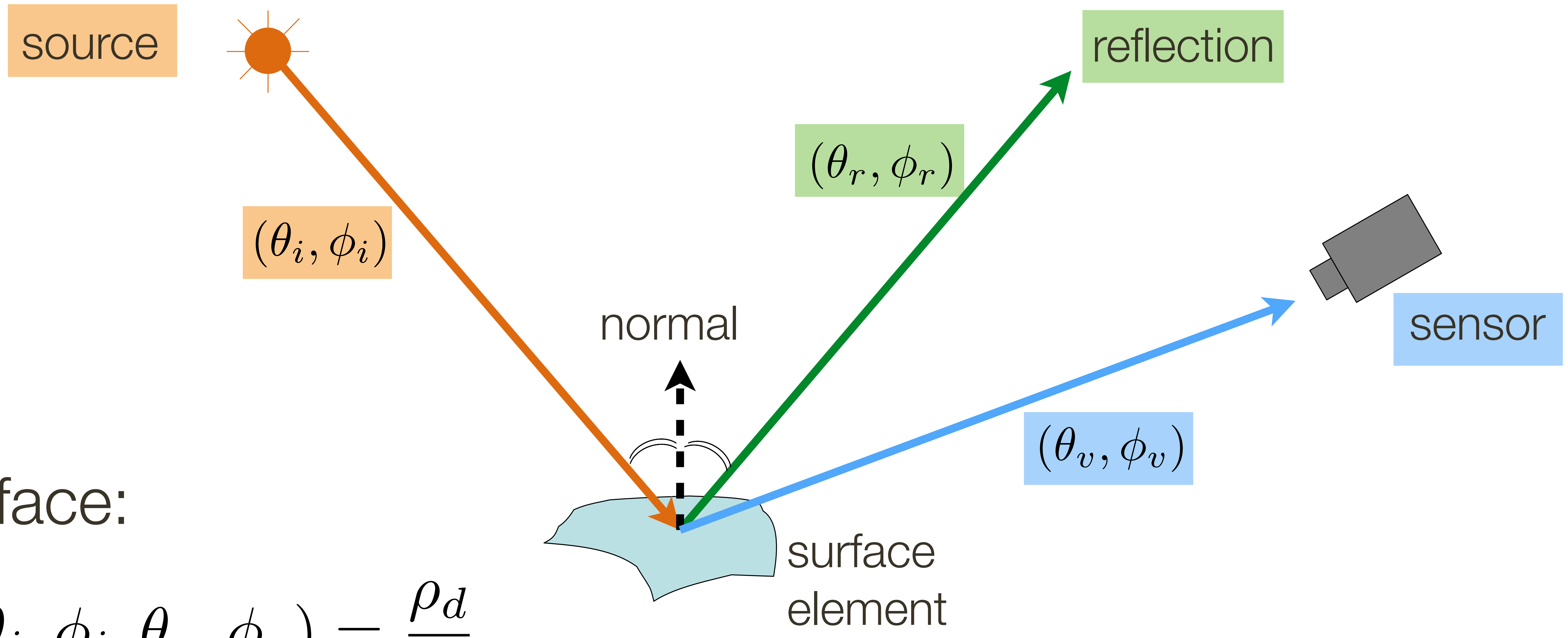


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**Mirror** surface: all incident light reflected in one directions $(\theta_v, \phi_v) = (\theta_r, \phi_r)$

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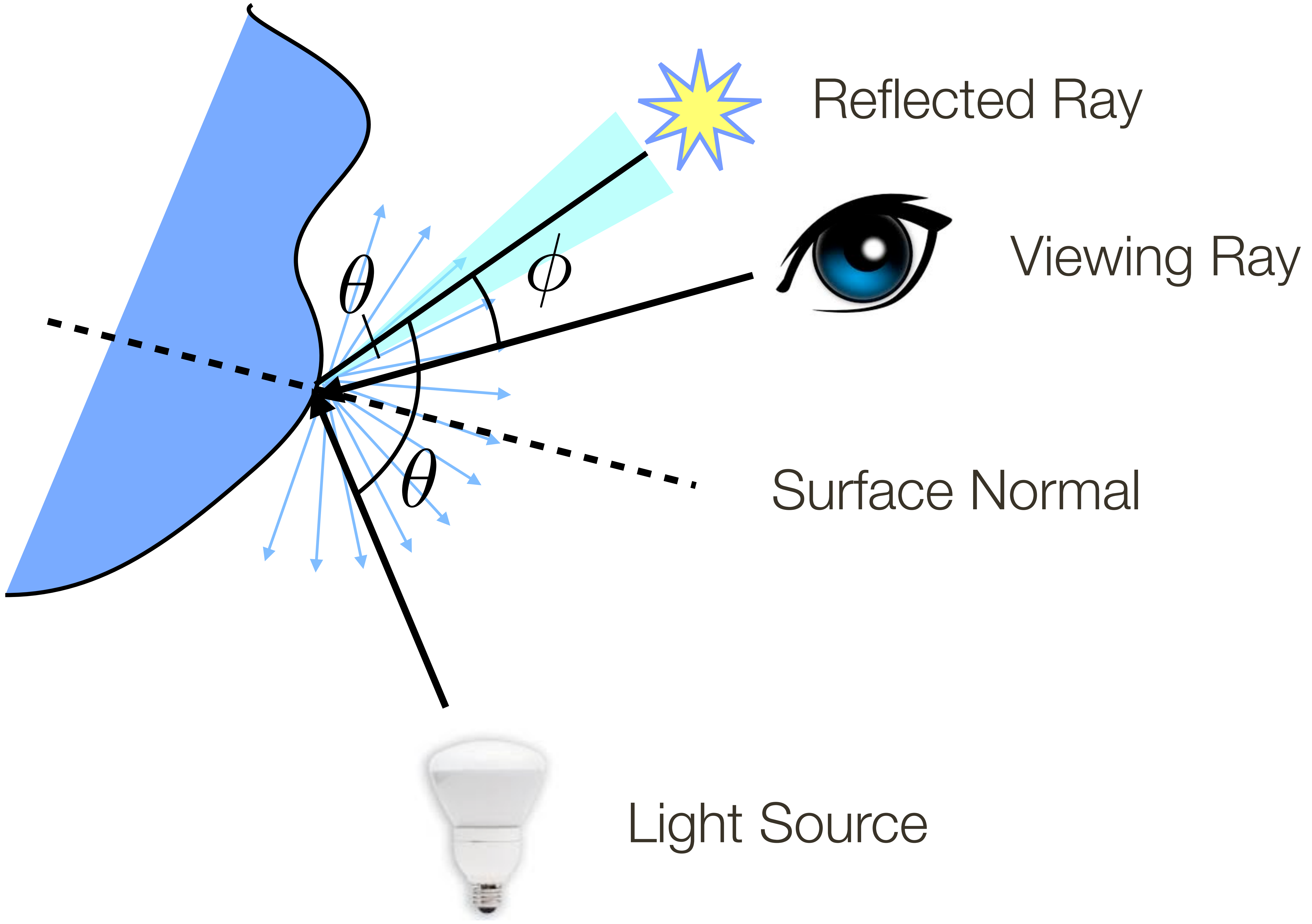
# **Phong** Illumination Model

Includes **ambient**, **diffuse** and **specular** reflection

$$I = k_a i_a + k_d i_d \cos\theta + k_s i_s \cos^\alpha \phi$$



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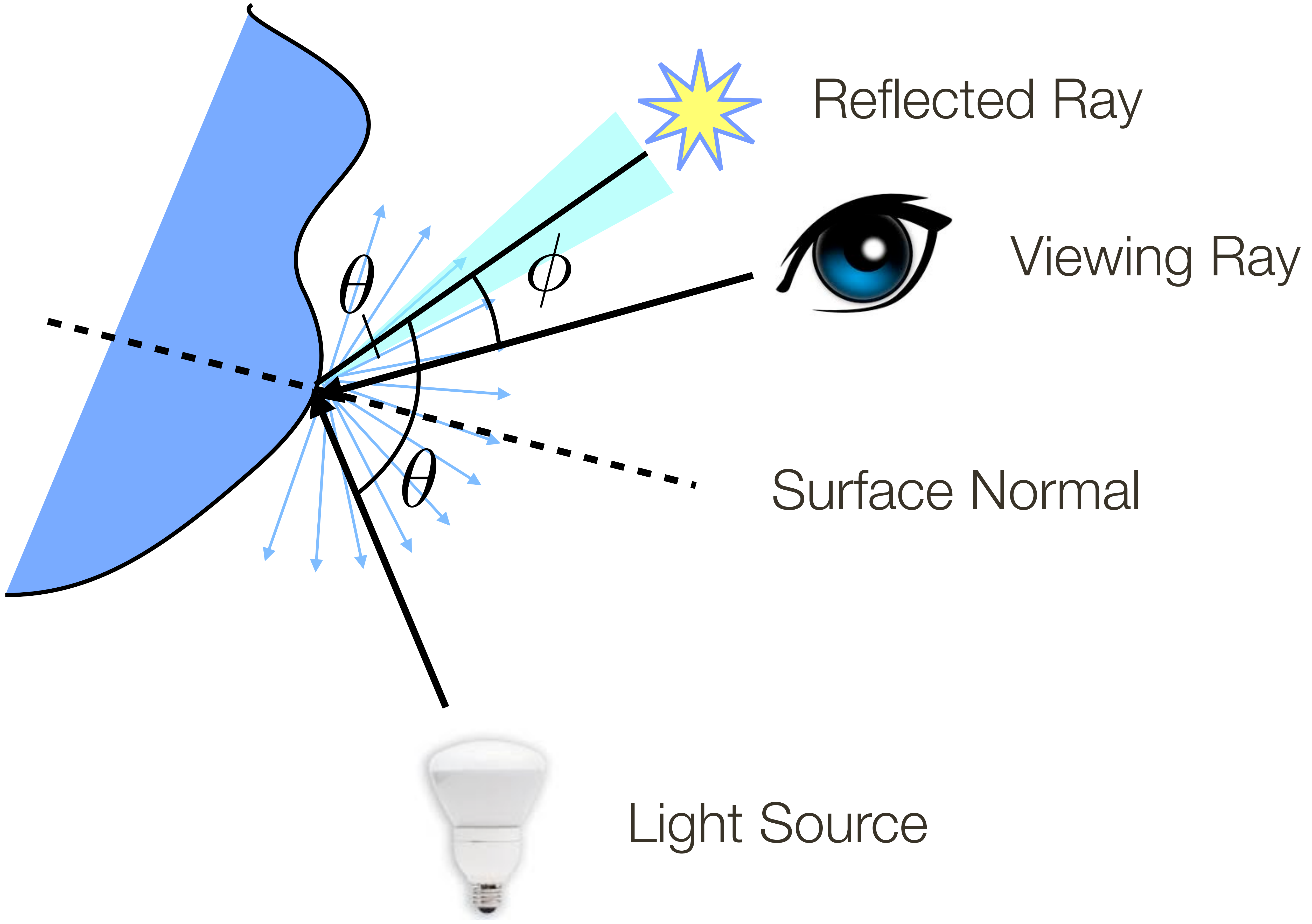
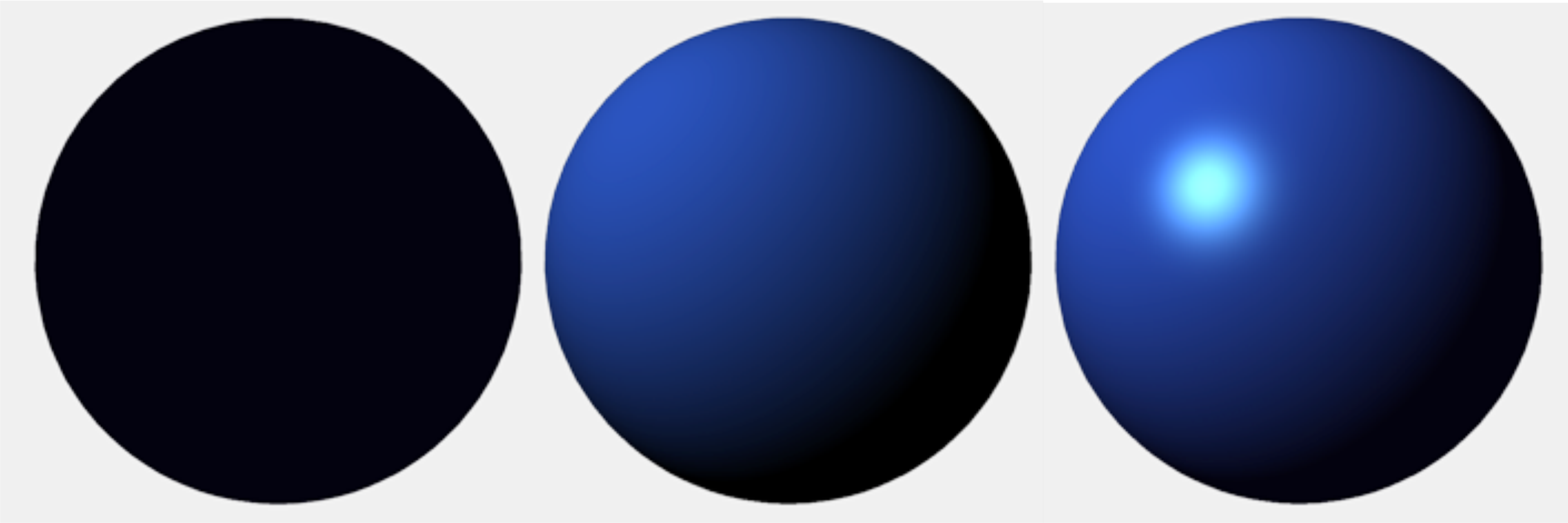


# **Phong** Illumination Model

Includes **ambient**, **diffuse** and **specular** reflection



Ambient +Diffuse +Specular

# **Phong** Illumination Model

Motivating example that uses this reflection model


[Video from https://machinelearning.apple.com/research/neural-3d-relightable reproduced for educational purposes]

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Motivating example that uses this reflection model


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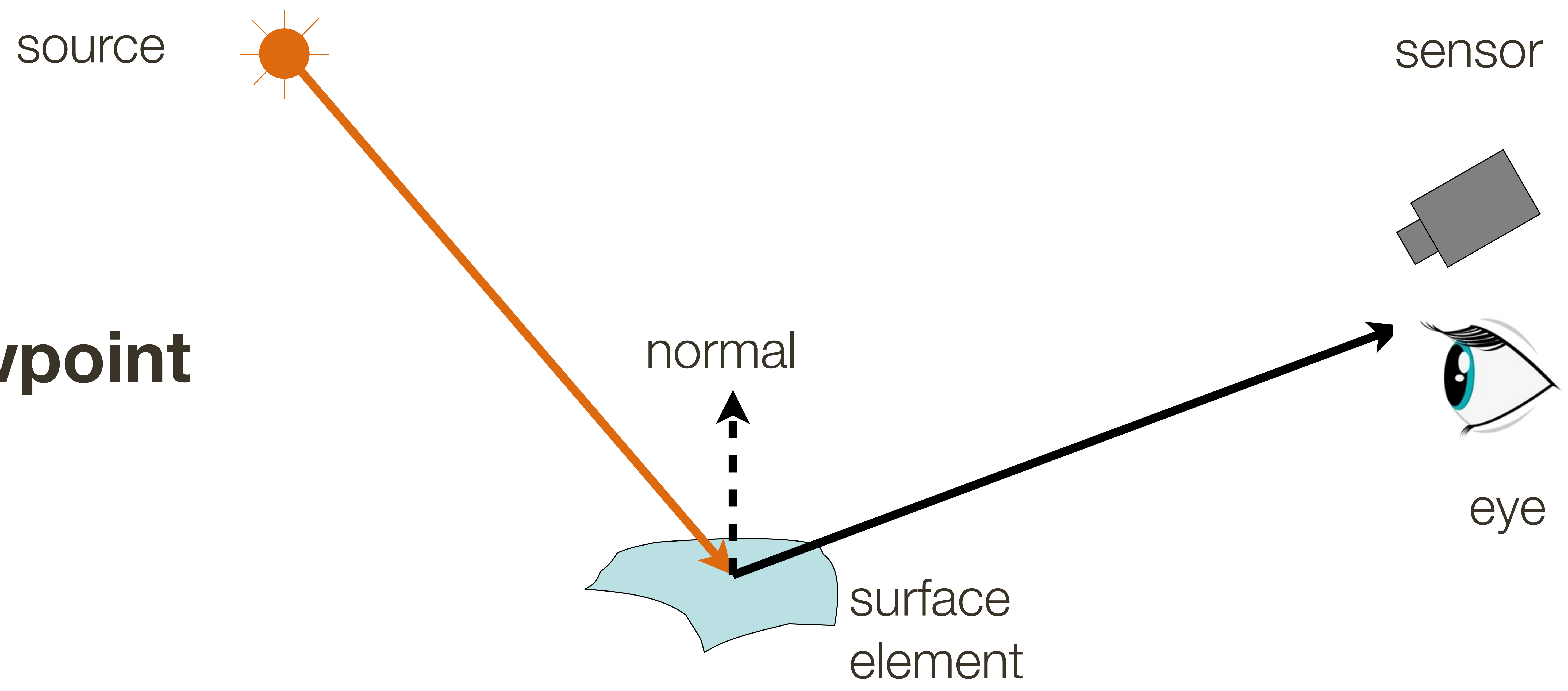
# **Overview**: Image Formation, Cameras and Lenses

The **image formation process** that produces a particular image depends on

- **Lightening** condition
- Scene **geometry**
- **Surface** properties
- Camera **optics** and **viewpoint**

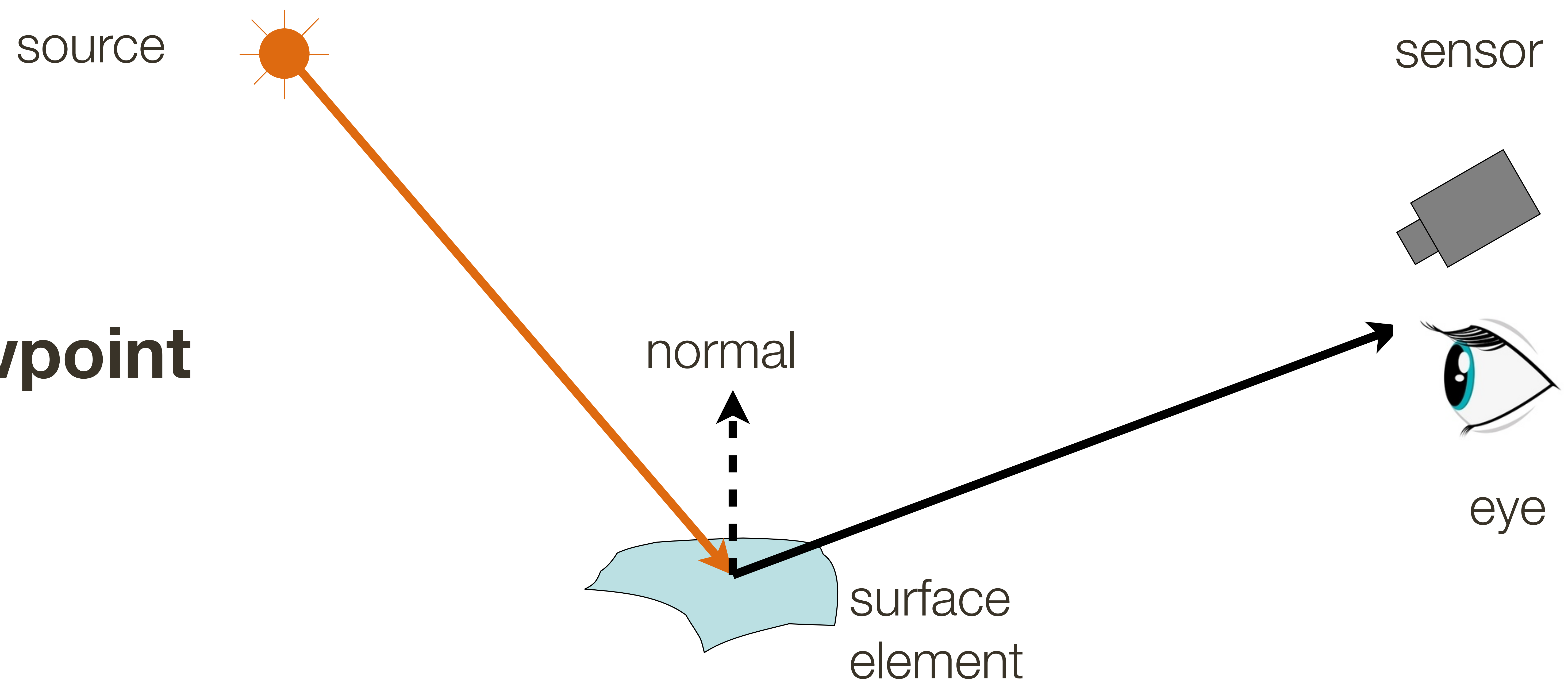



Sensor (or eye) **captures amount of light** reflected from the object

# **Overview**: Image Formation, Cameras and Lenses

The **image formation process** that produces a particular image depends on

— Camera **optics** and **viewpoint**




Sensor (or eye) **captures amount of light** reflected from the object

# Cameras

Old school **film** camera

**Digital** CCD/CMOS camera

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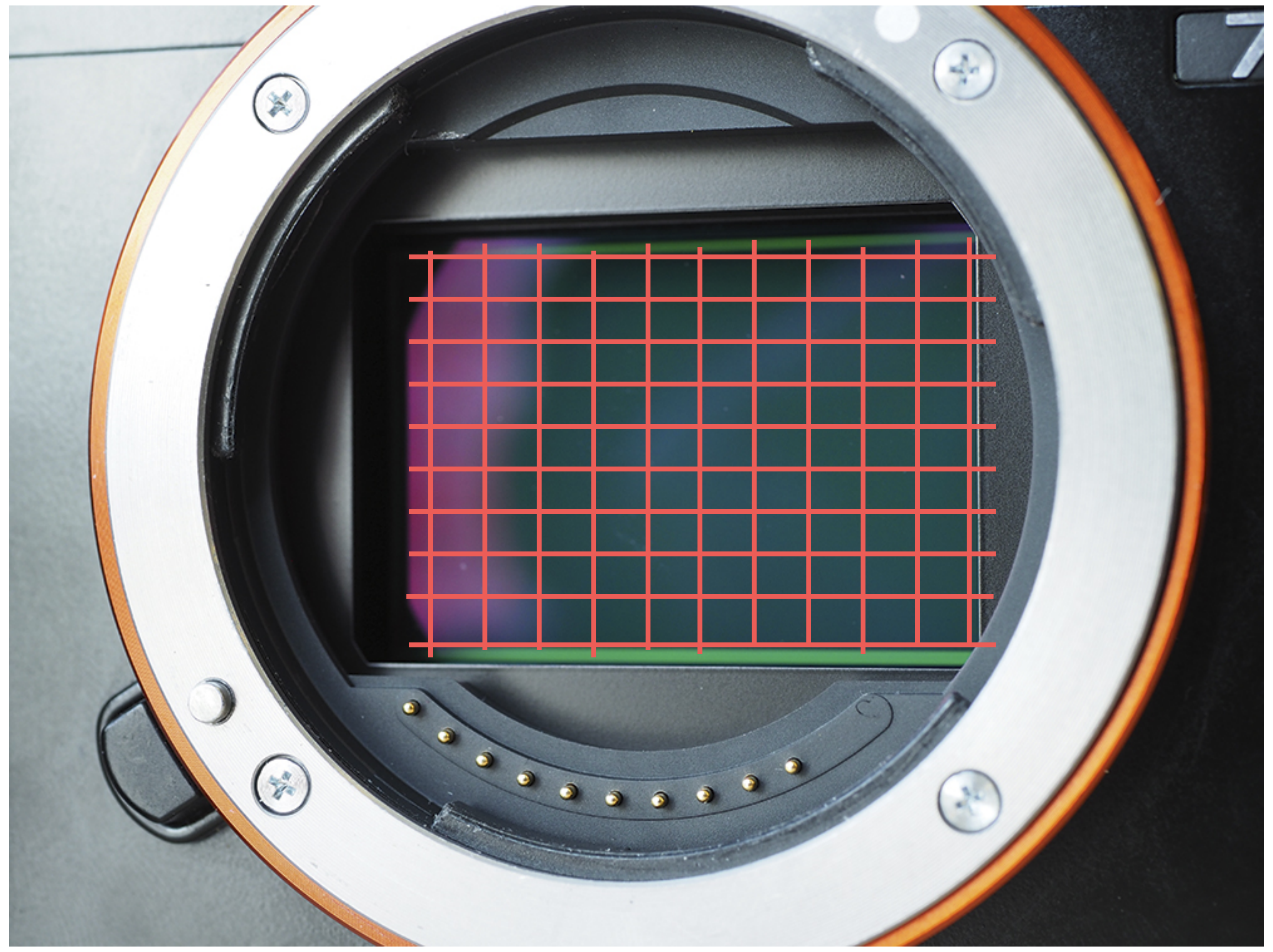
**Digital** CCD/CMOS camera



# Let’s say we have a **sensor …**

**Digital** CCD/CMOS camera

**Slide Credit**: Ioannis (Yannis) Gkioulekas (CMU)

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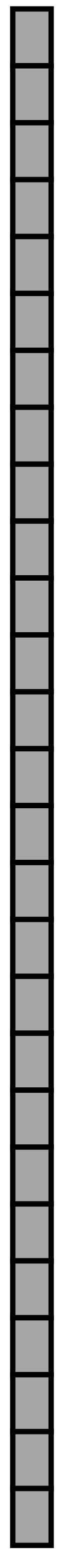
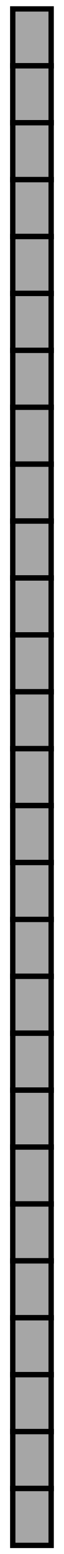



**Slide Credit:** Ioannis (Yannis) Gkioulekas (CMU)

# … and the **object** we would like to photograph

What would an image taken like this look like?

real-world object



digital sensor (CCD or CMOS)

**Slide Credit**: Ioannis (Yannis) Gkioulekas (CMU)

# **Bare-sensor** imaging


**Slide Credit**: Ioannis (Yannis) Gkioulekas (CMU)

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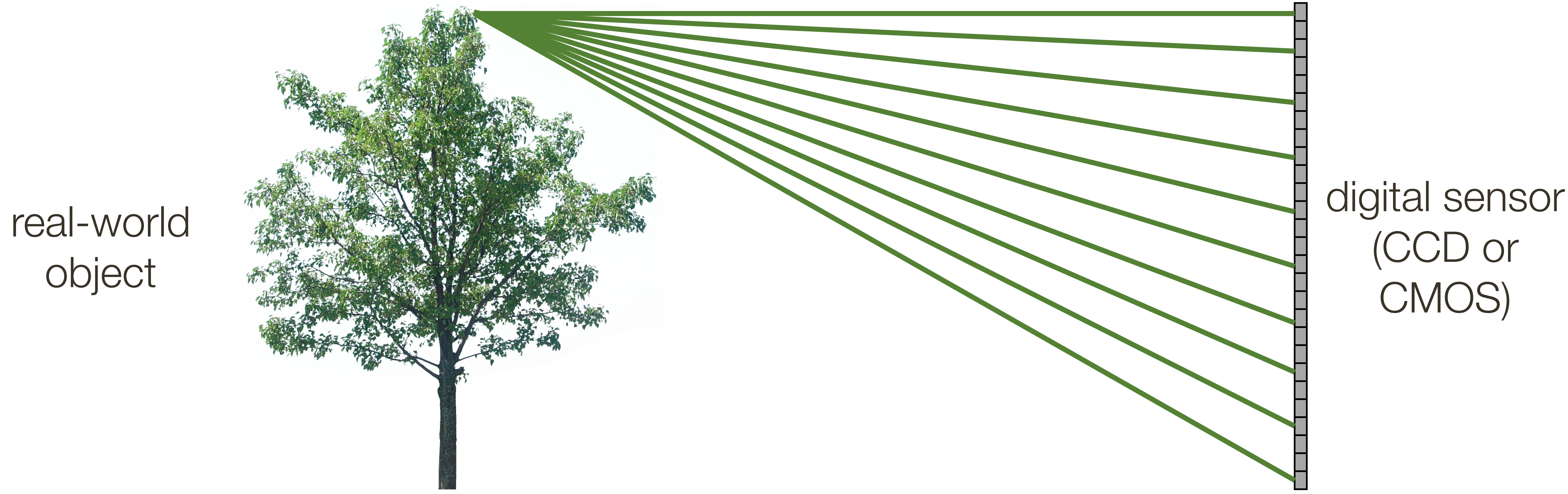


**Slide Credit**: Ioannis (Yannis) Gkioulekas (CMU)

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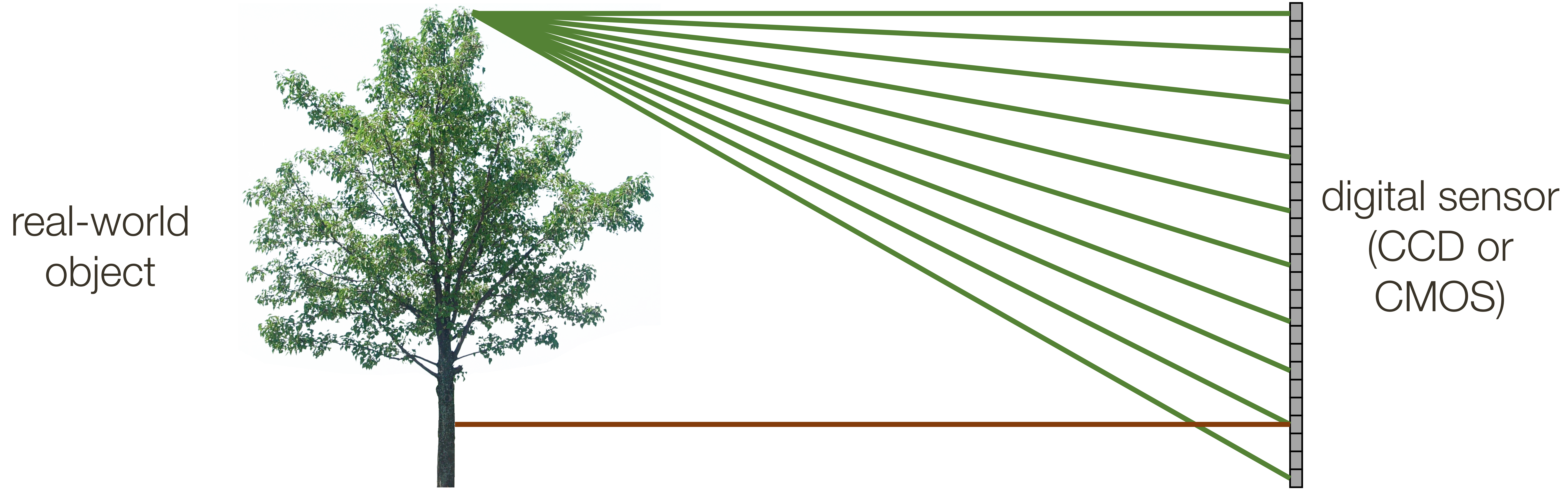


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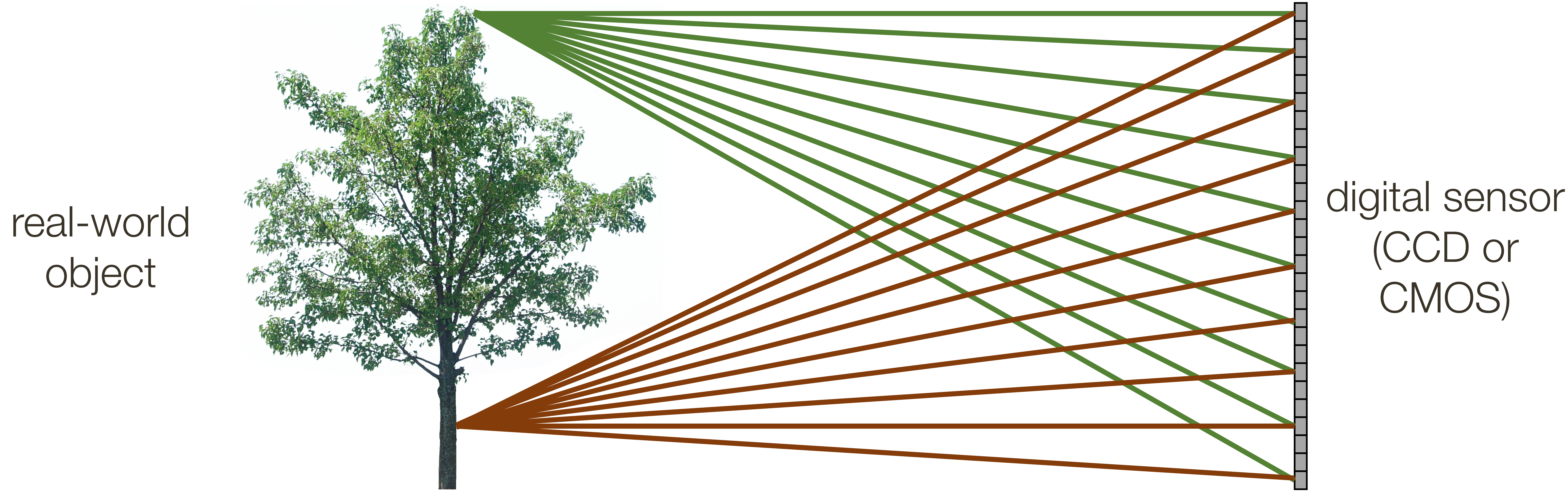


All scene points contribute to all sensor pixels

**Slide Credit**: Ioannis (Yannis) Gkioulekas (CMU)

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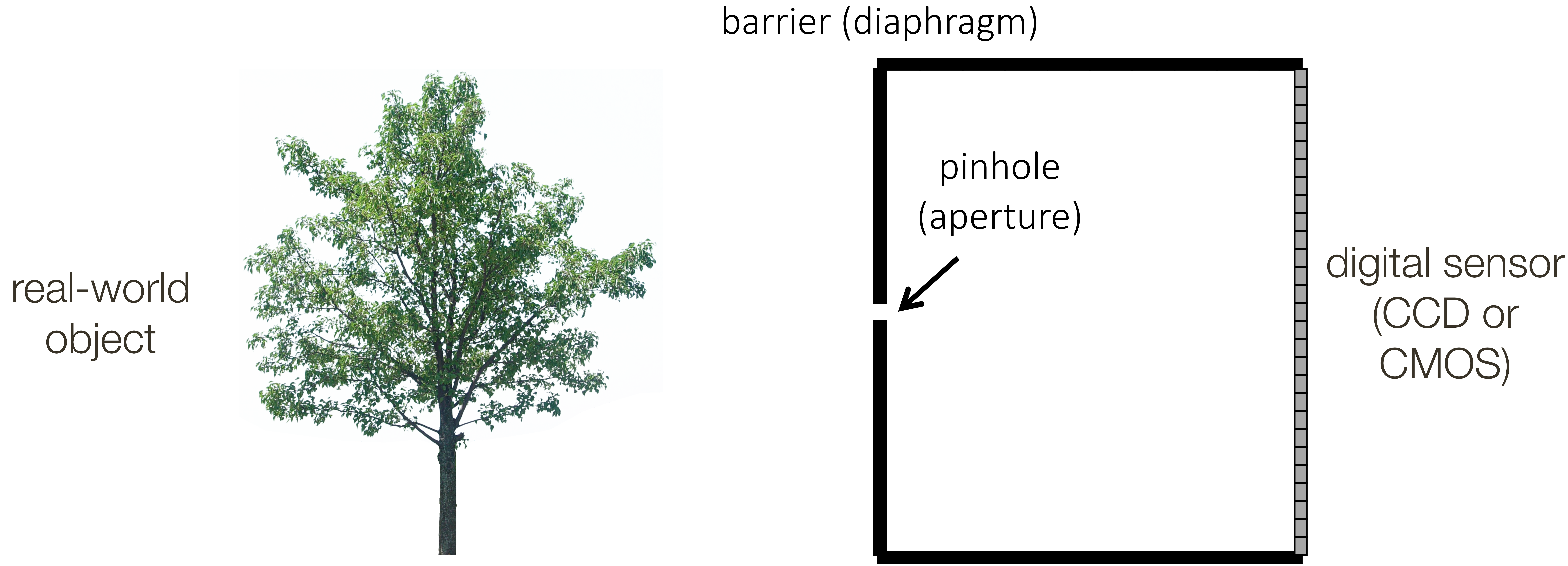
# **Pinhole** Camera

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**Slide Credit**: Ioannis (Yannis) Gkioulekas (CMU)

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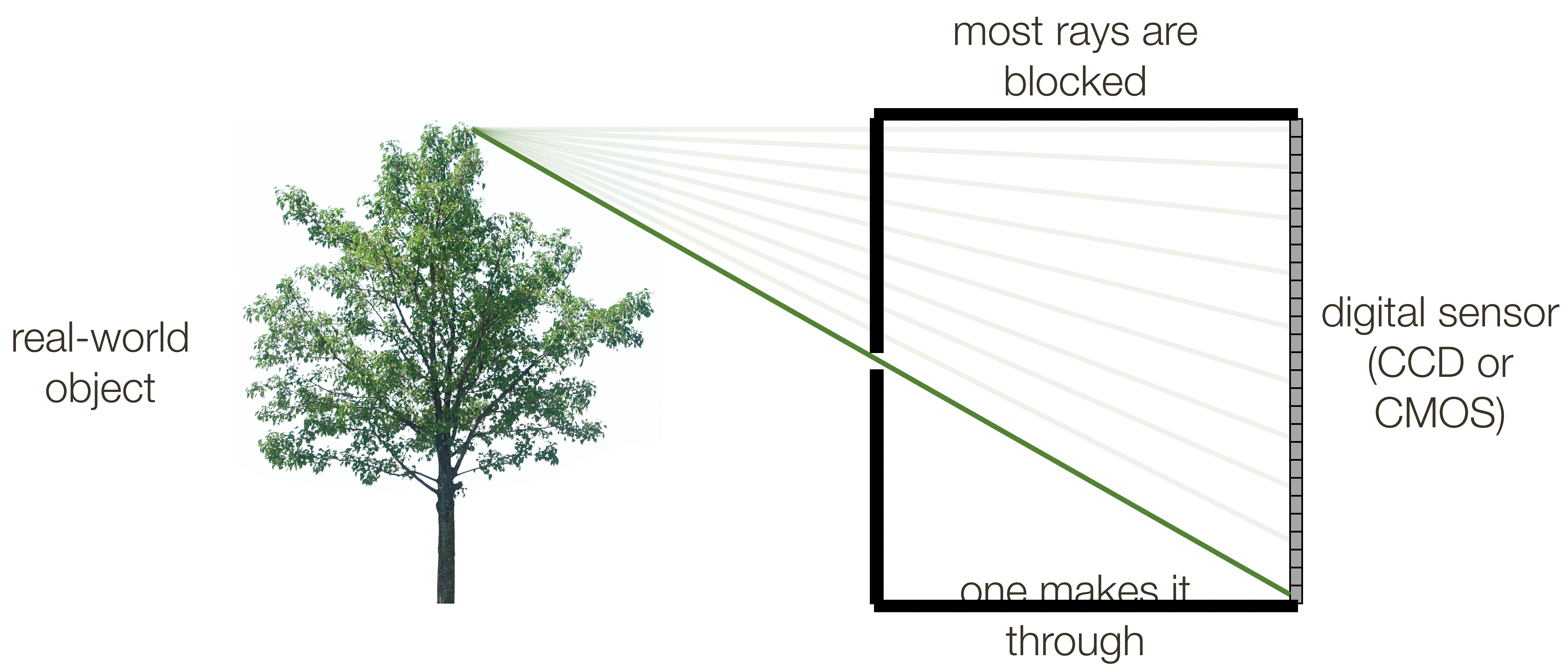



**Slide Credit**: Ioannis (Yannis) Gkioulekas (CMU)

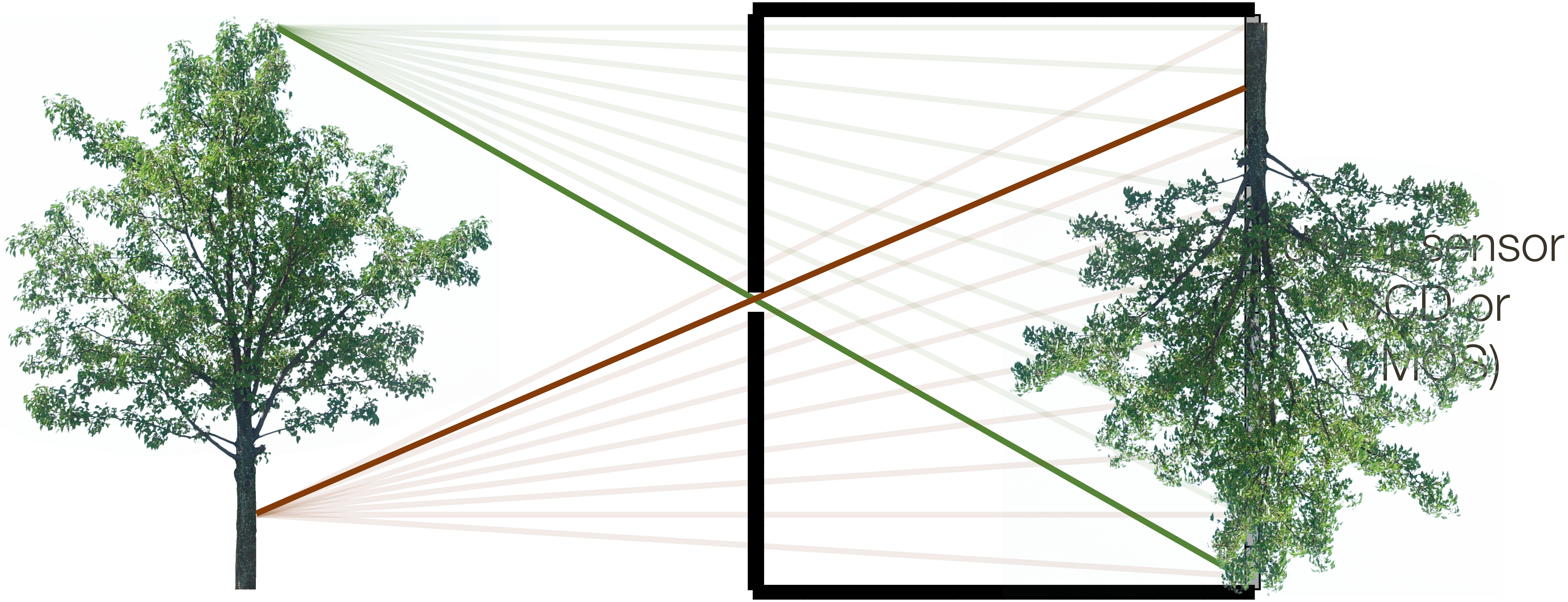
# **Pinhole** Camera


Each scene point contributes to only one sensor pixel

**Slide Credit**: Ioannis (Yannis) Gkioulekas (CMU)

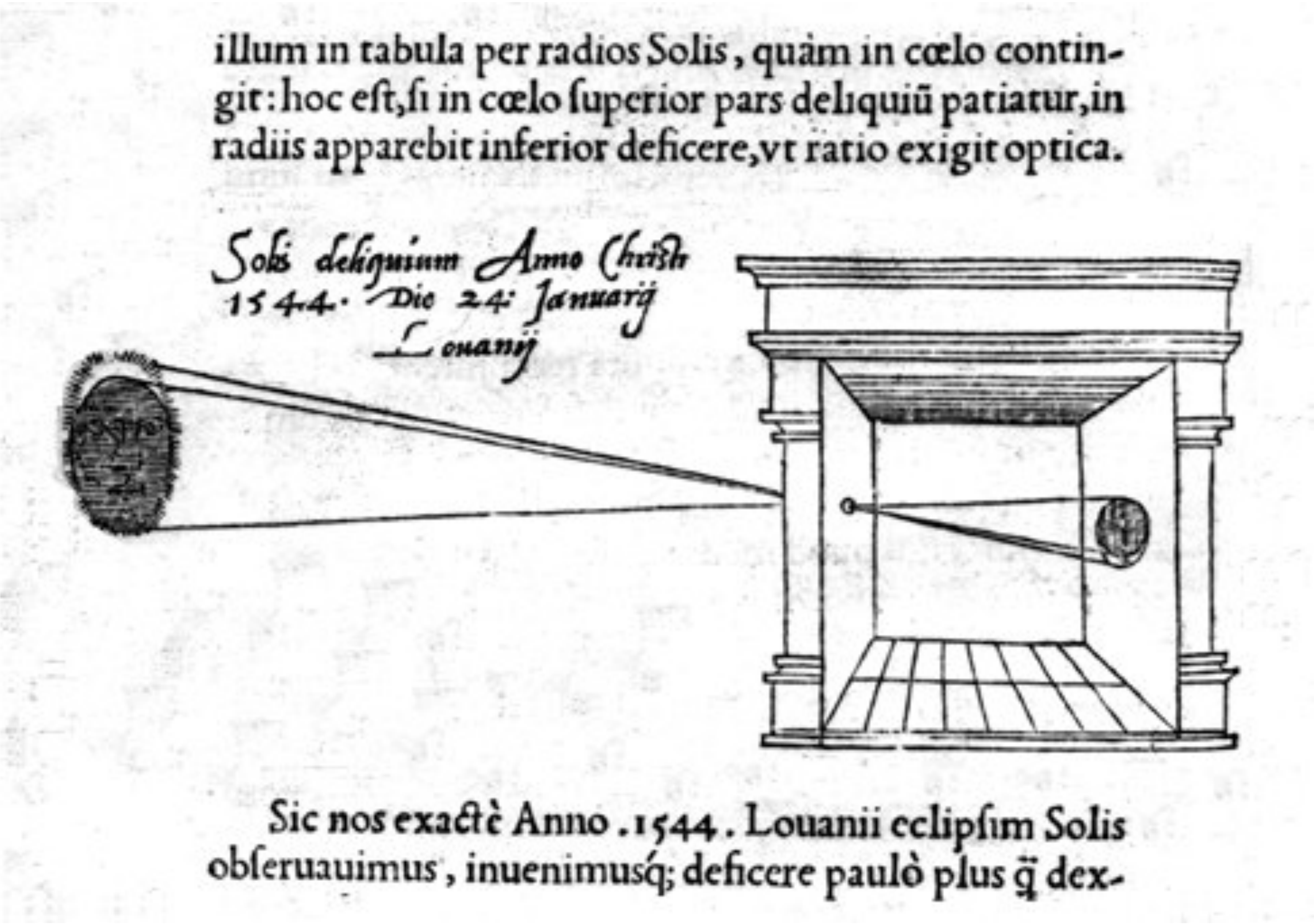
# **Pinhole** Camera

real-world
object




Each scene point contributes to only one sensor pixel

# **Camera Obscura** (latin for “dark chamber”)


illum in tabula per radios Solis, quàm in cœlo contingit: hoc eſt, ſi in cœlo ſuperior pars deliquiũ patiatur, in radiis apparebit inferior deficere, vt ratio exigit optica.



Sic nos exactè Anno .1544. Louanii eclipſim Solis obſeruauimus, inuenimusq; deficere paulò plus q̄ dex-

Reinerus Gemma-Frisius observed an eclipse of the sun at Louvain on January 24, 1544. He used this illustration in his book, “De Radio Astronomica et Geometrica,” 1545. It is thought to be the first published illustration of a camera obscura.

**Credit**: John H., Hammond, “Th Camera Obscure, A Chronicle”

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principles behind the pinhole camera or camera obscura were first mentioned by Chinese philosopher Mozi (Mo-Ti) (470 to 390 BCE)

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**Credit**: John H., Hammond, "Th Camera Obscure, A Chronicle"

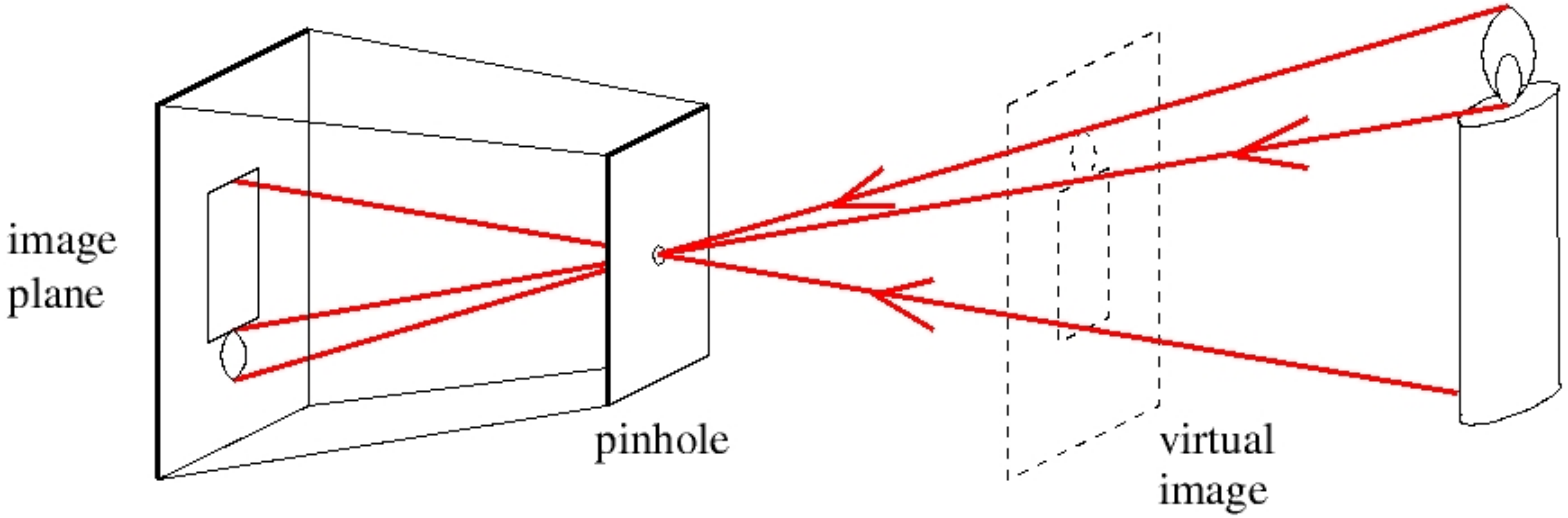
# First **Photograph** on Record

*La table servie*

**Credit**: Nicéphore Niepce, 1822

# **Pinhole** Camera

A pinhole camera is a box with a small hall (**aperture**) in it




Forsyth & Ponce (2nd ed.) Figure 1.2

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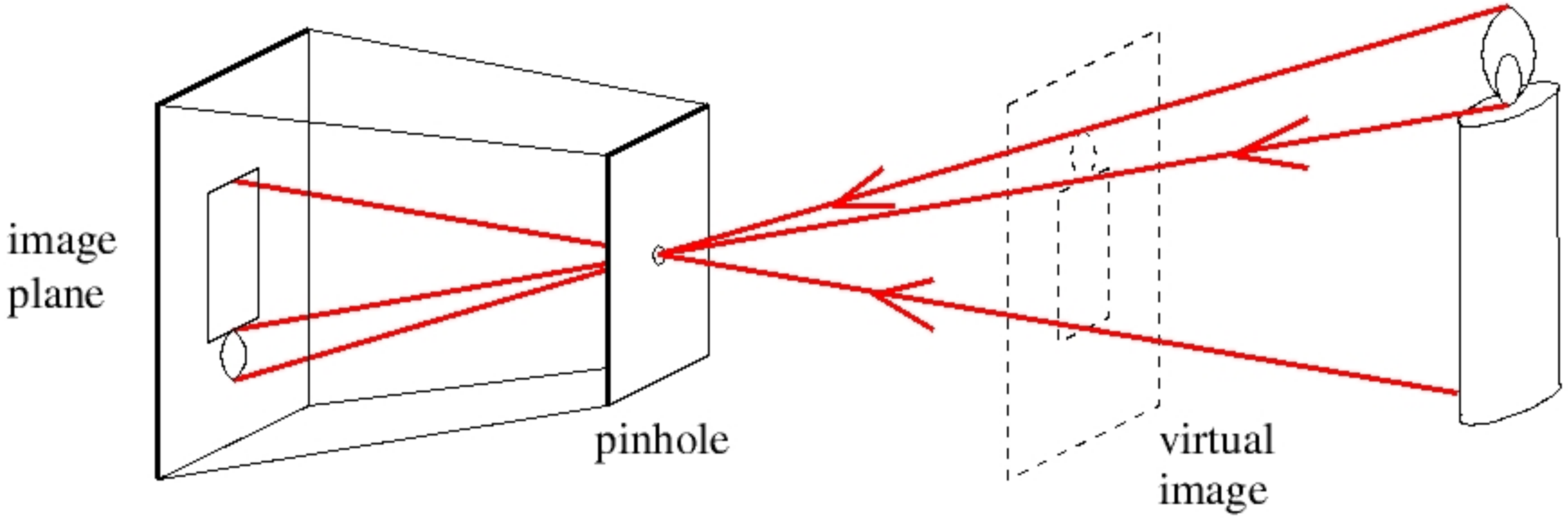
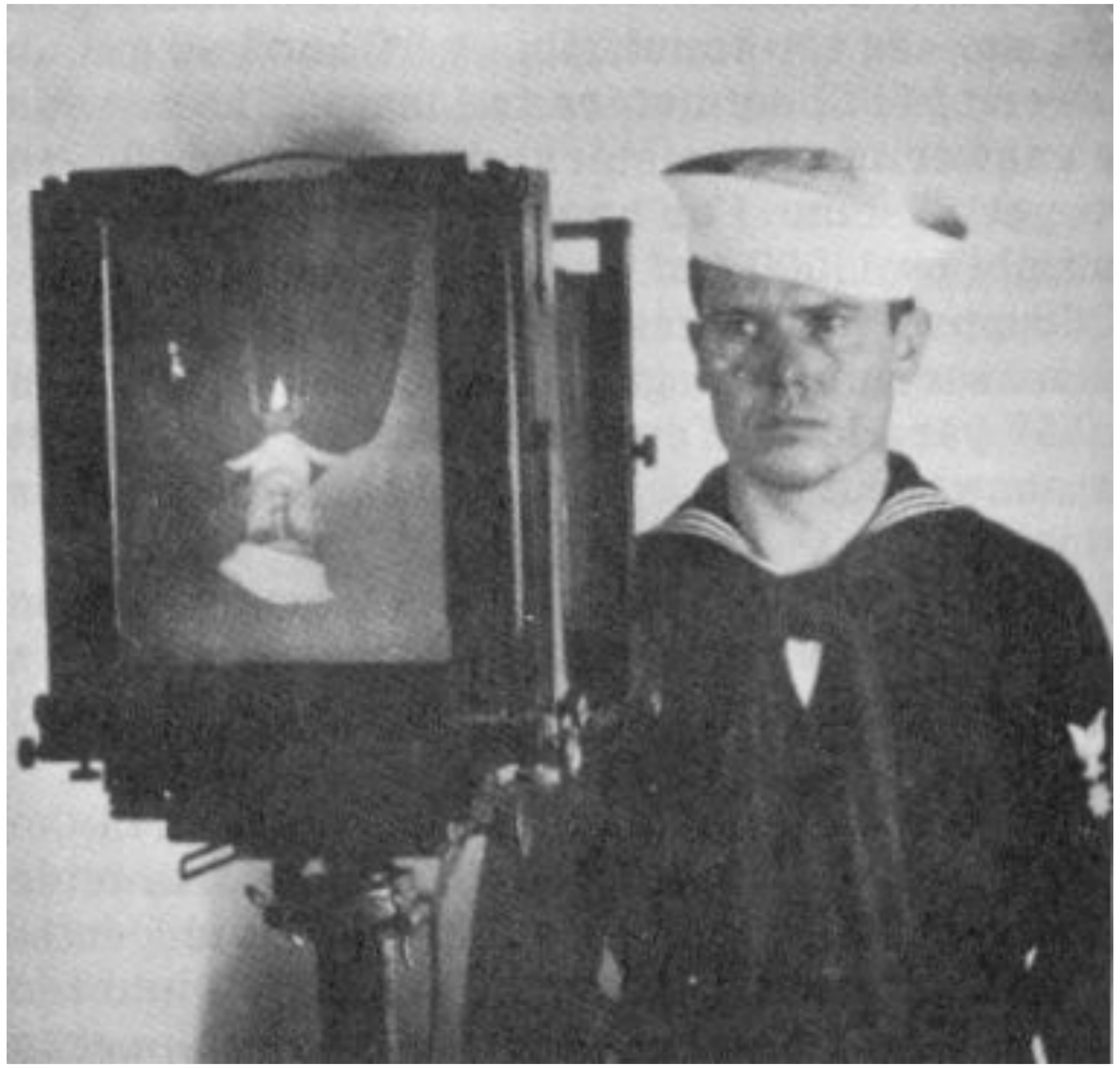



Forsyth & Ponce (2nd ed.) Figure 1.2

# Image **Formation**



Forsyth & Ponce (2nd ed.) Figure 1.1

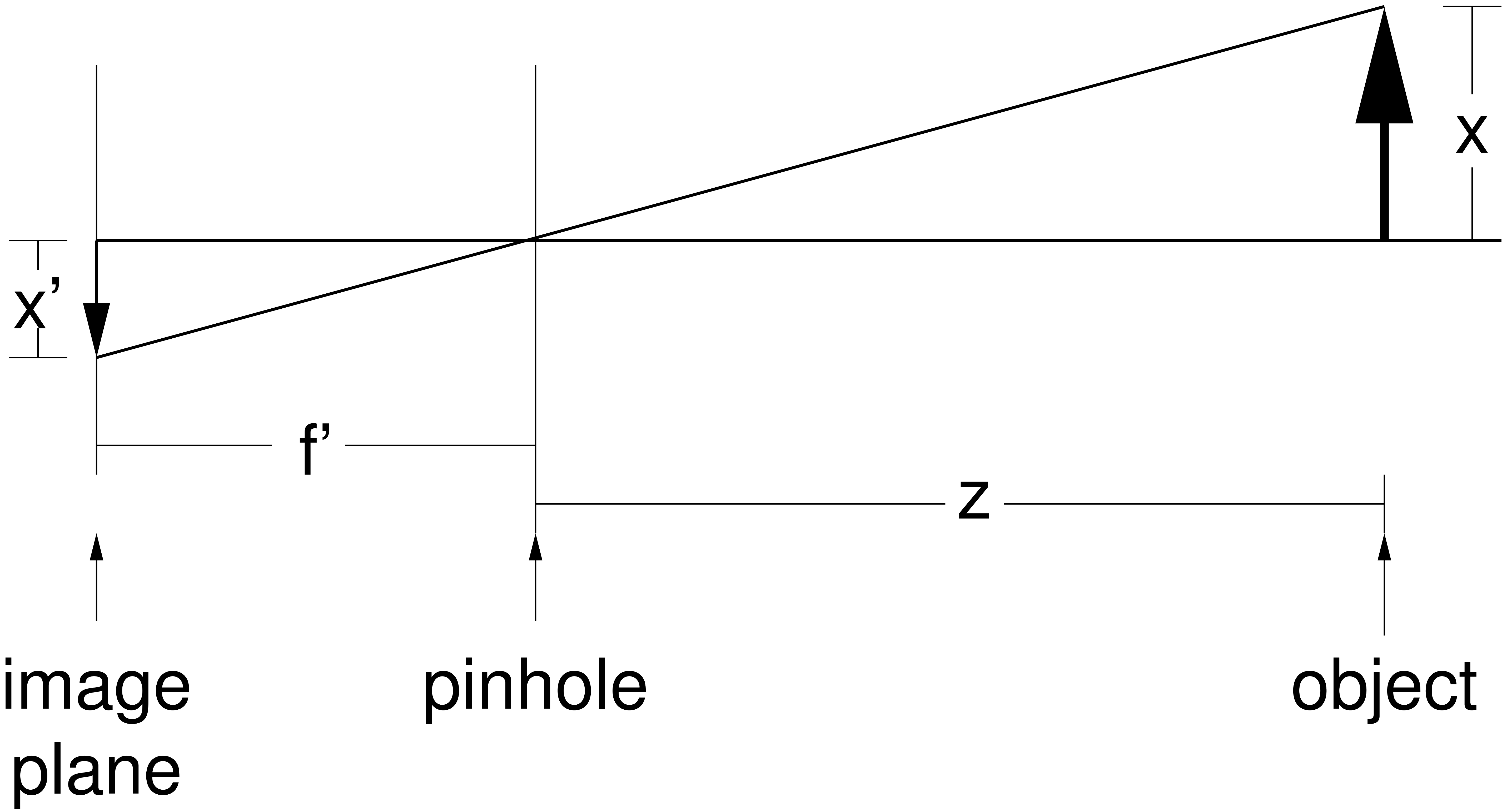
**Credit**: US Navy, Basic Optics and Optical Instruments. Dover, 1969

# **Accidental** Pinhole Camera

**Image Credit**: Ioannis (Yannis) Gkioulekas (CMU)

# **Pinhole** Camera (Simplified 1D)

f’ is the **focal length** of the camera



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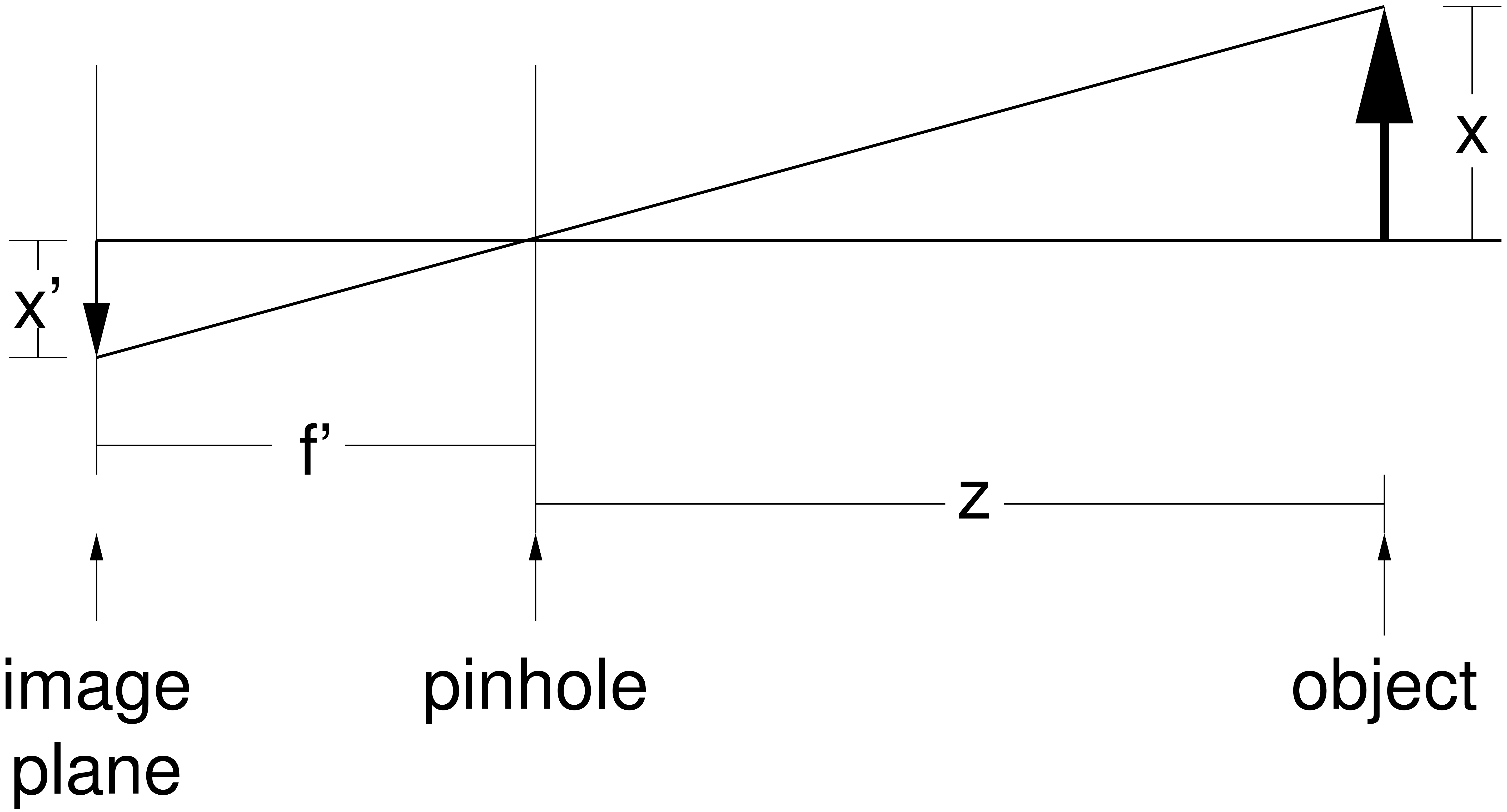
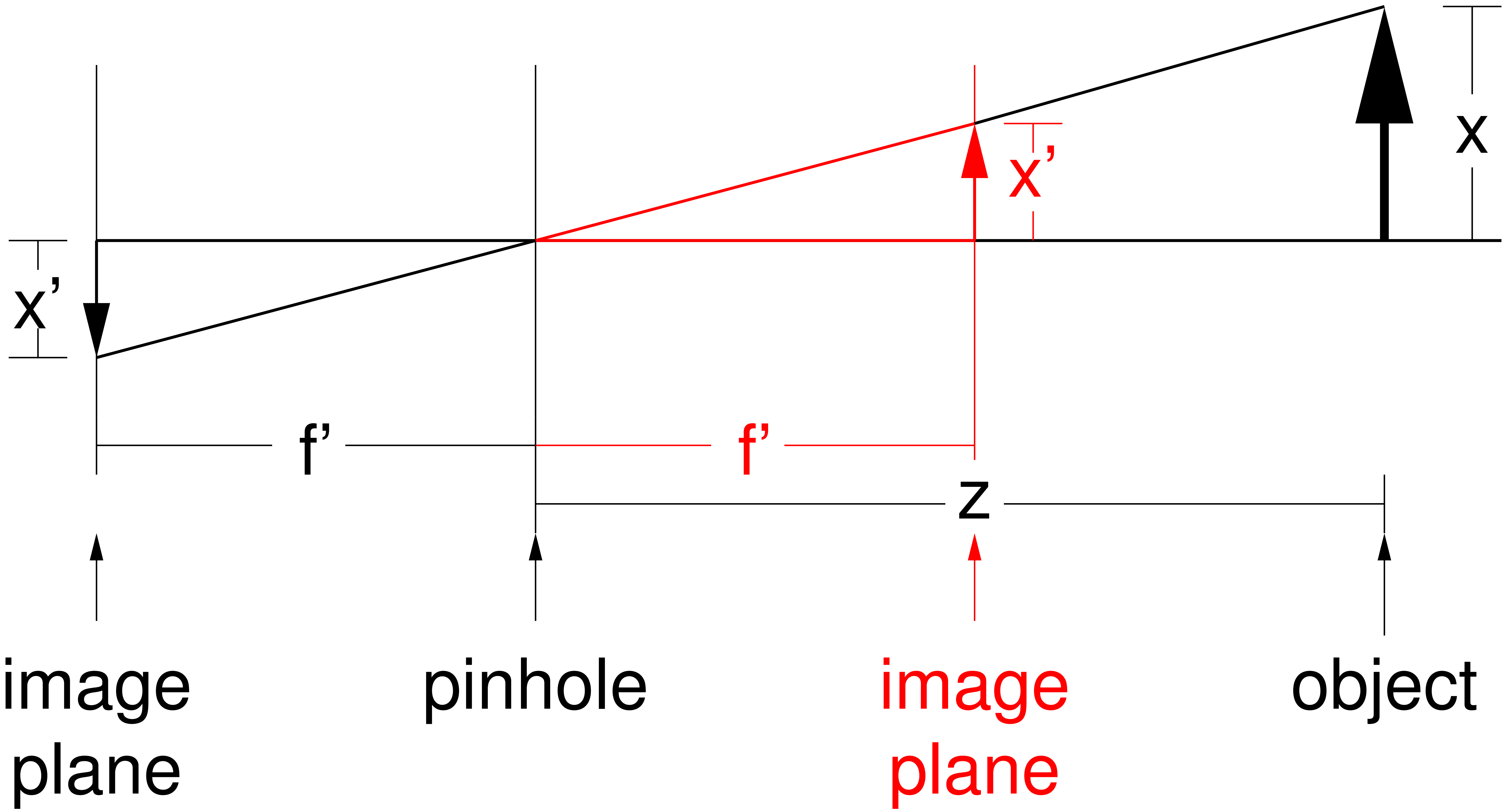



**Note:** In a pinhole camera we can adjust the focal length, all this will do is change the **size** of the resulting image

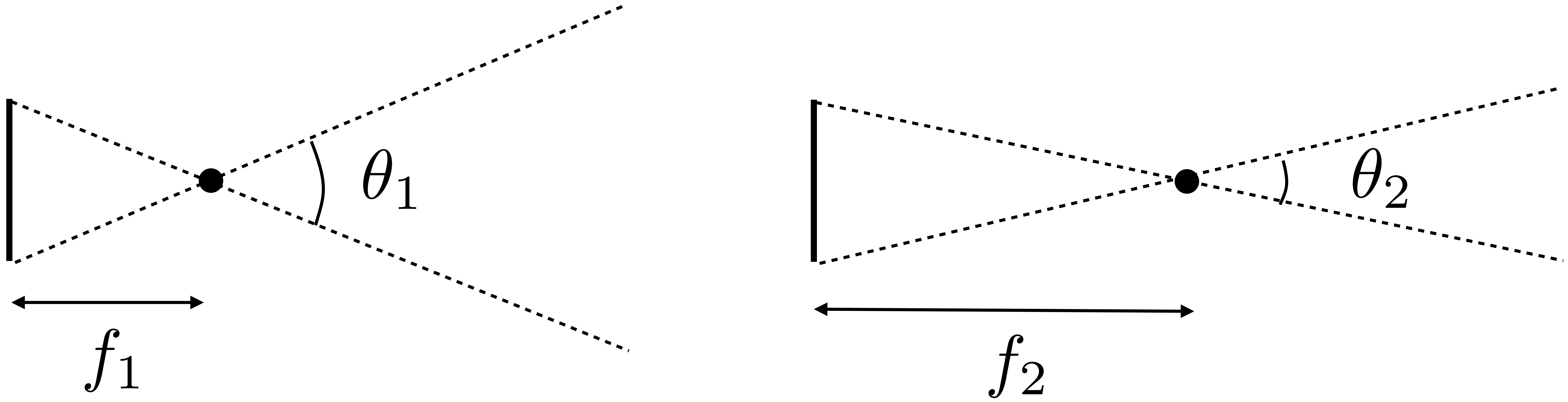
# **Pinhole** Camera (Simplified 1D)

It is convenient to think of the **image plane** which is in from of the pinhole



# **Focal** Length

For a fixed sensor size, focal length determines the **field of view** (FoV)



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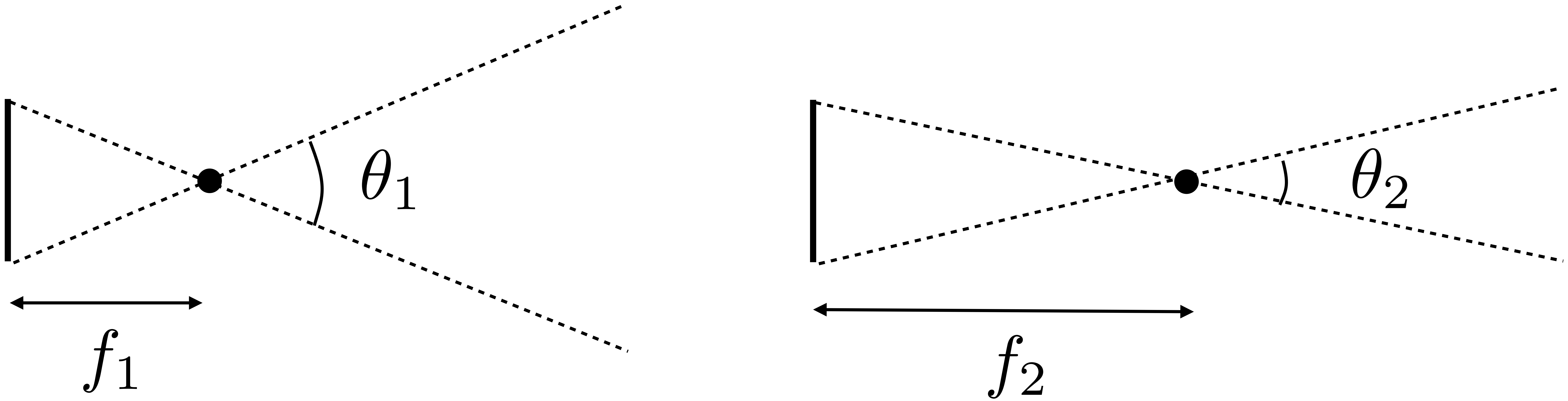



Sensor size

**Exercise:** What is the field of view of a full frame (35mm) camera with a 50mm lens? 100mm lens?

Focal length

# **Focal** Length

28 mm

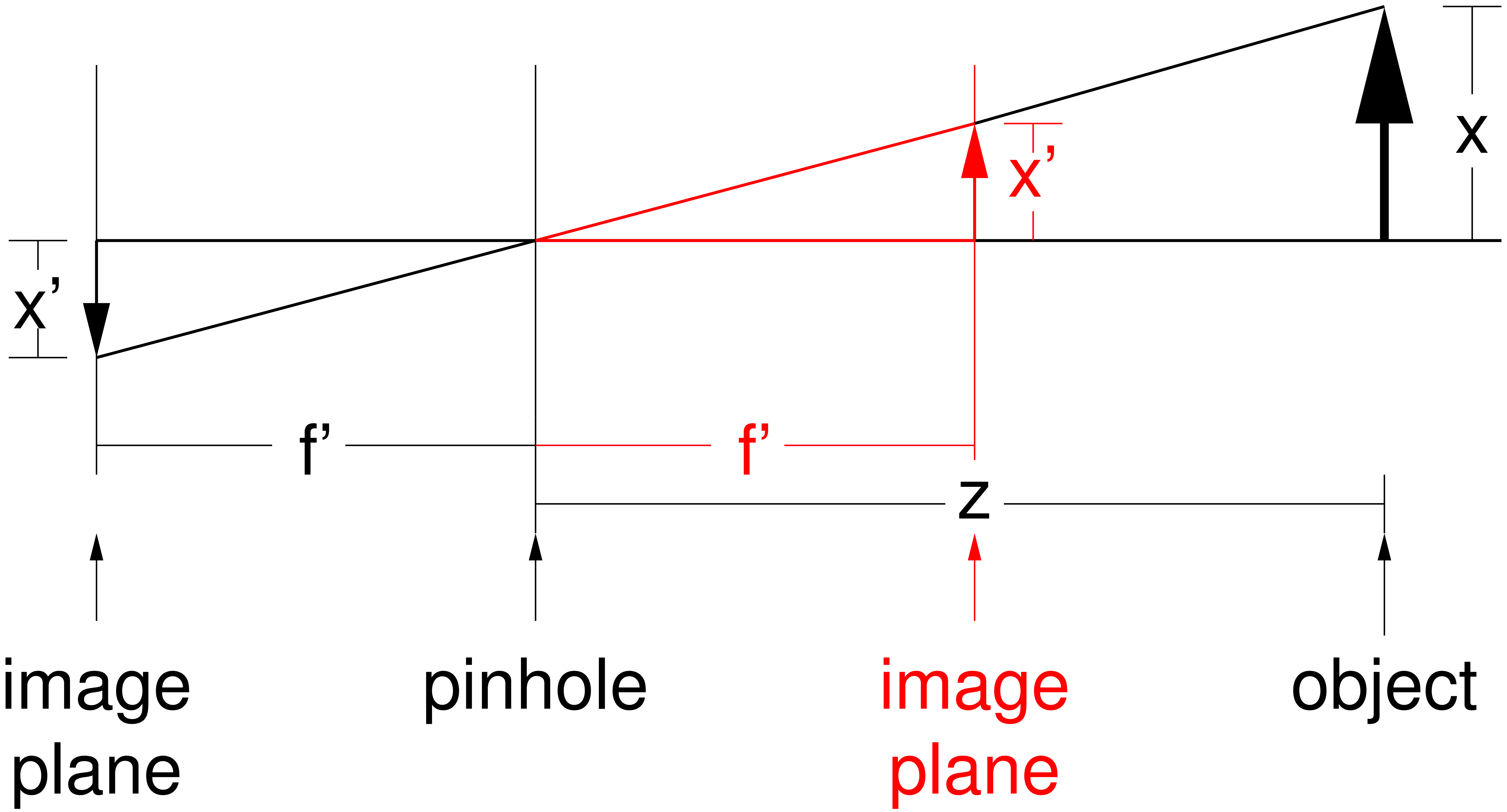
35 mm

50 mm

70 mm

# **Pinhole** Camera (Simplified 1D)

It is convenient to think of the **image plane** which is in from of the pinhole



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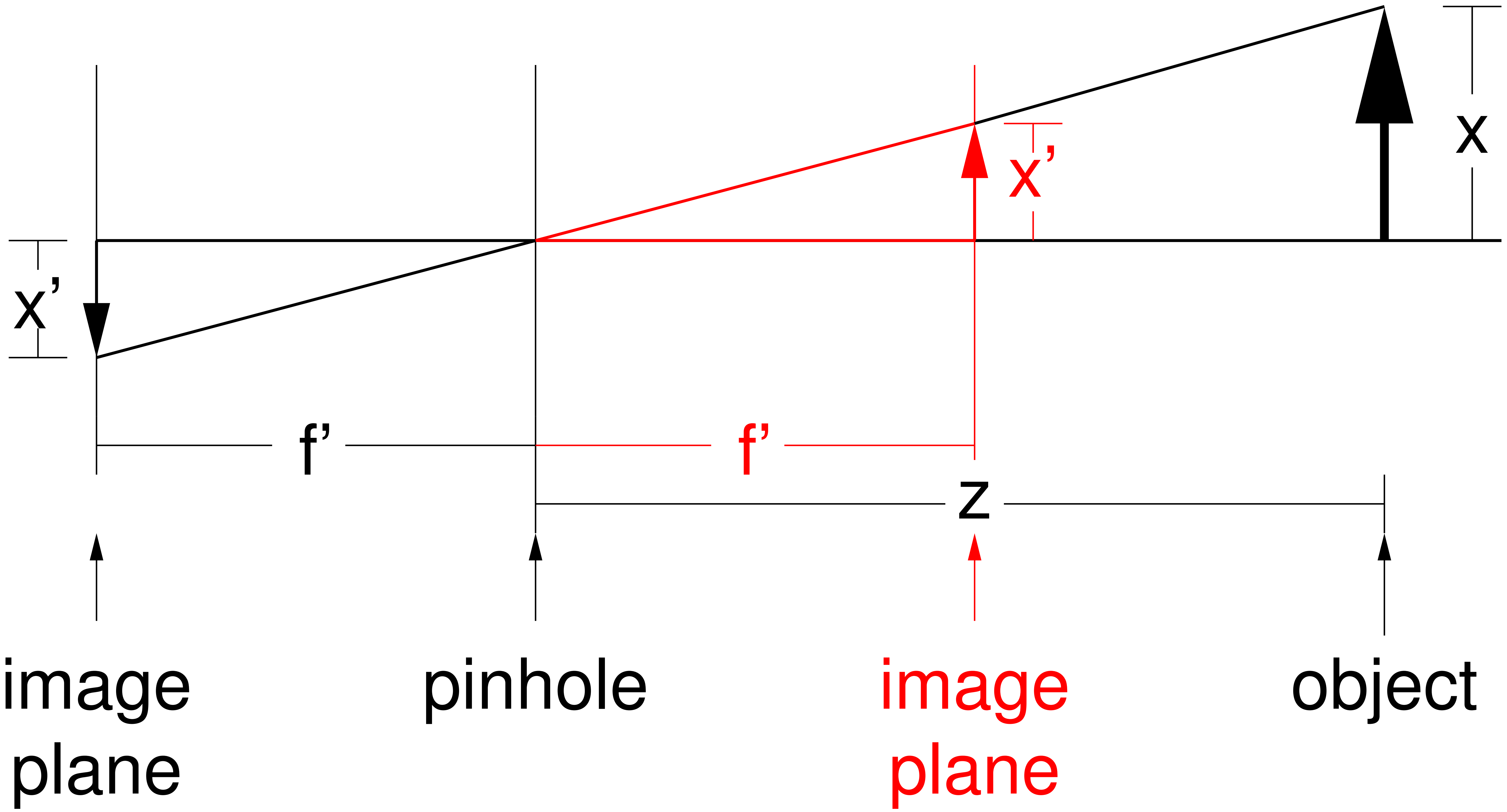
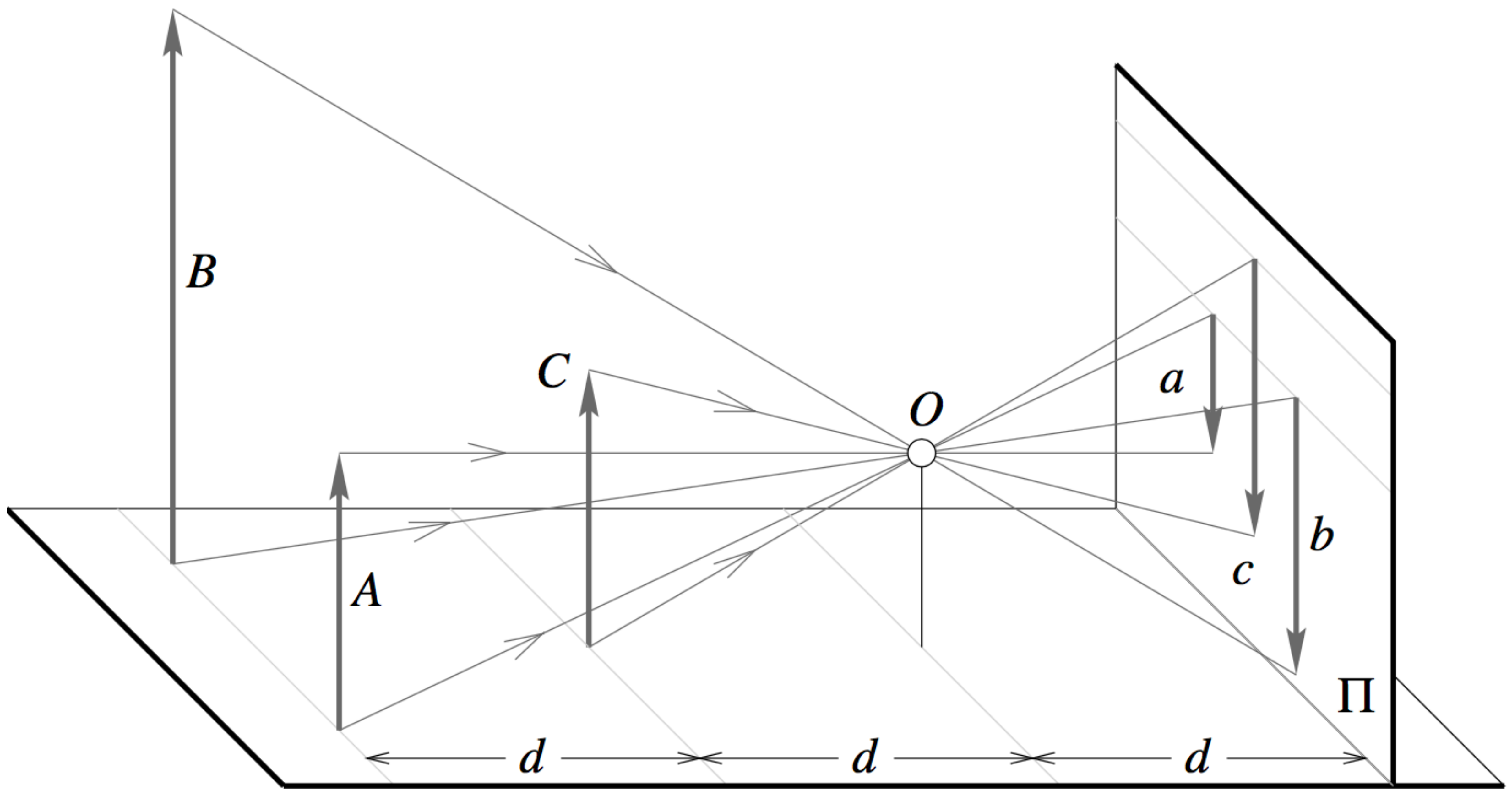



What happens if object moves towards the camera? Away from the camera?

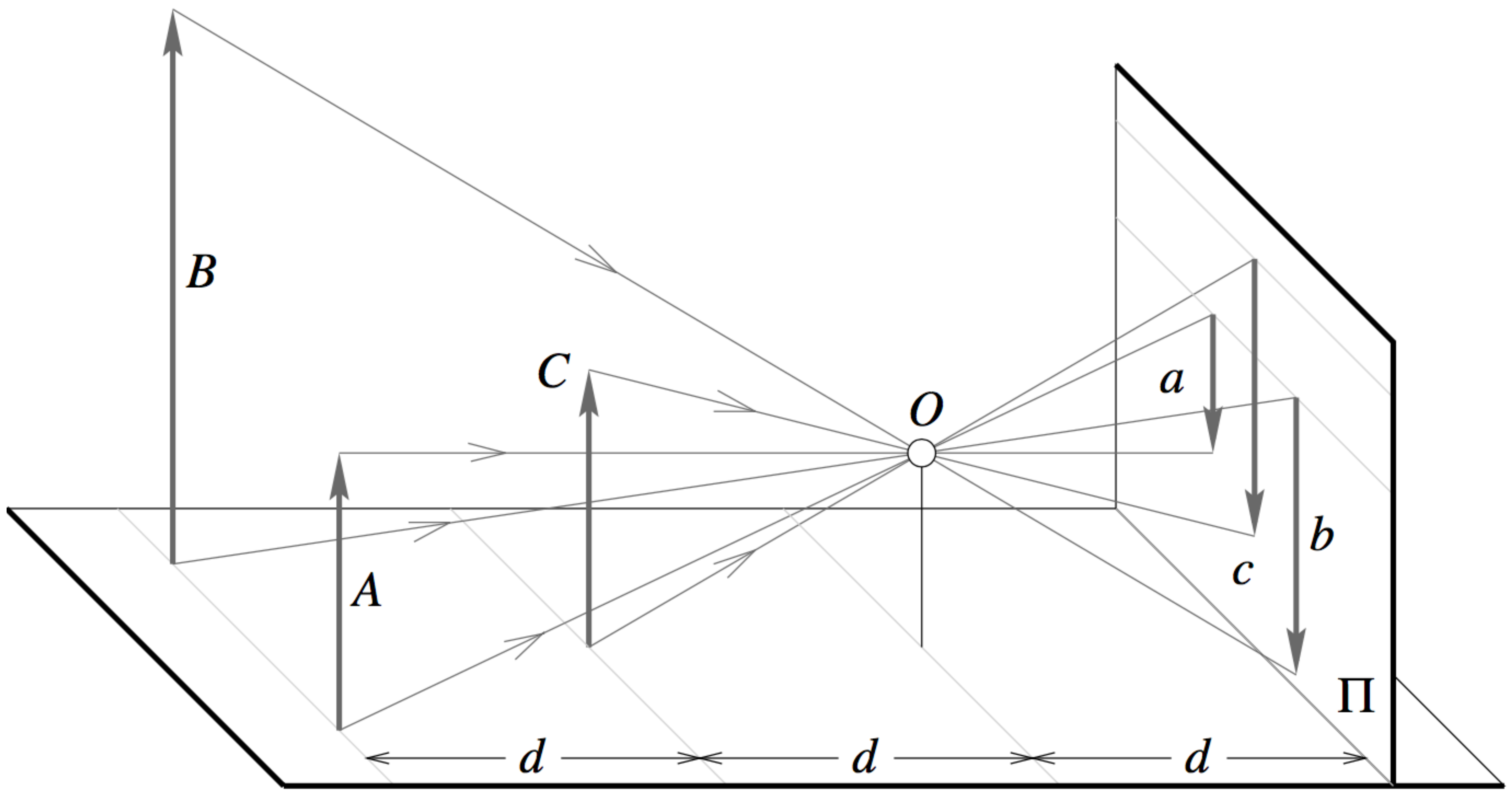
# **Perspective** Effects




Forsyth & Ponce (2nd ed.) Figure 1.3a

# **Perspective** Effects

**Far objects** appear **smaller** than close ones




Forsyth & Ponce (2nd ed.) Figure 1.3a

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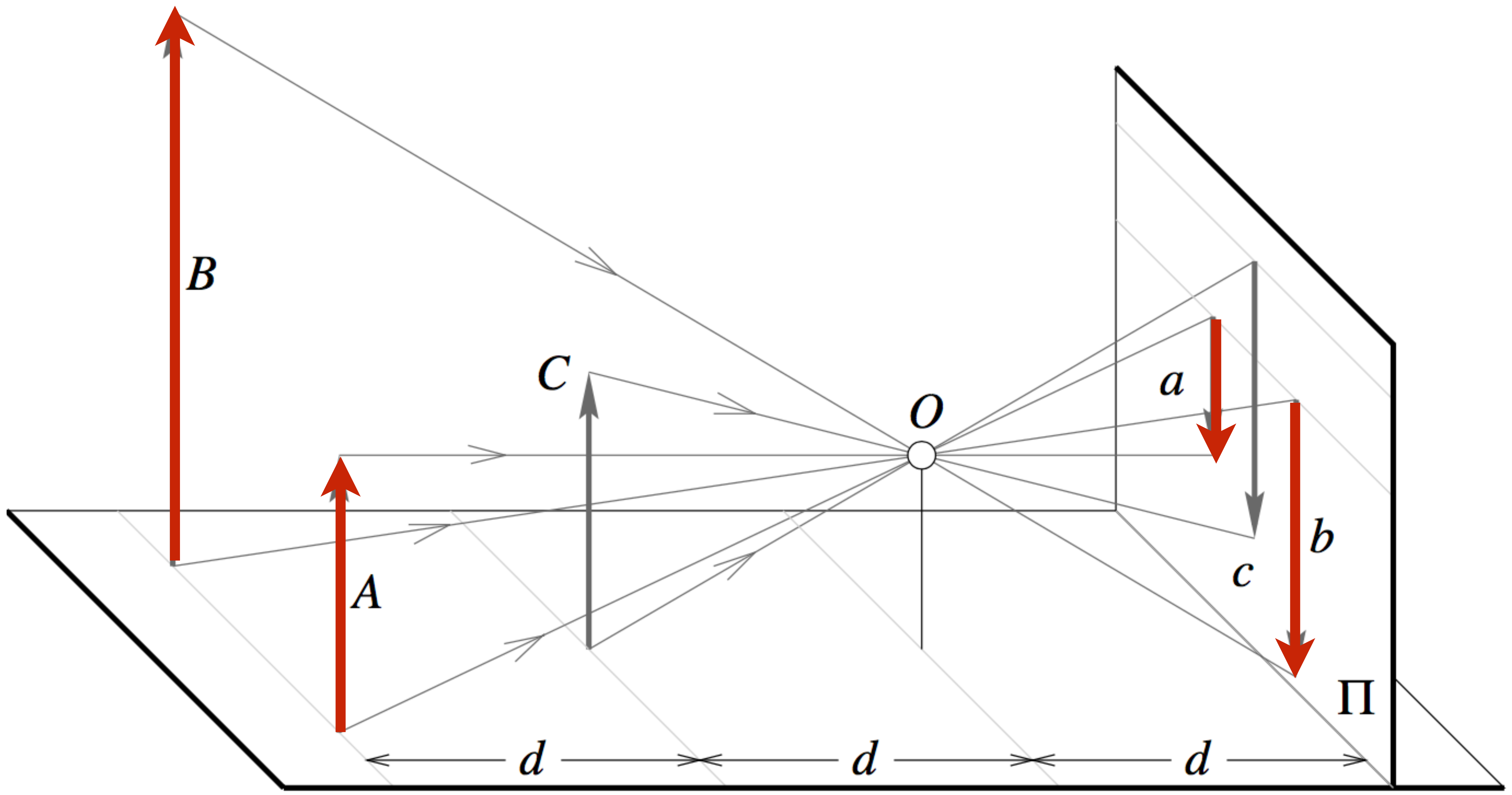


Forsyth & Ponce (2nd ed.) Figure 1.3a

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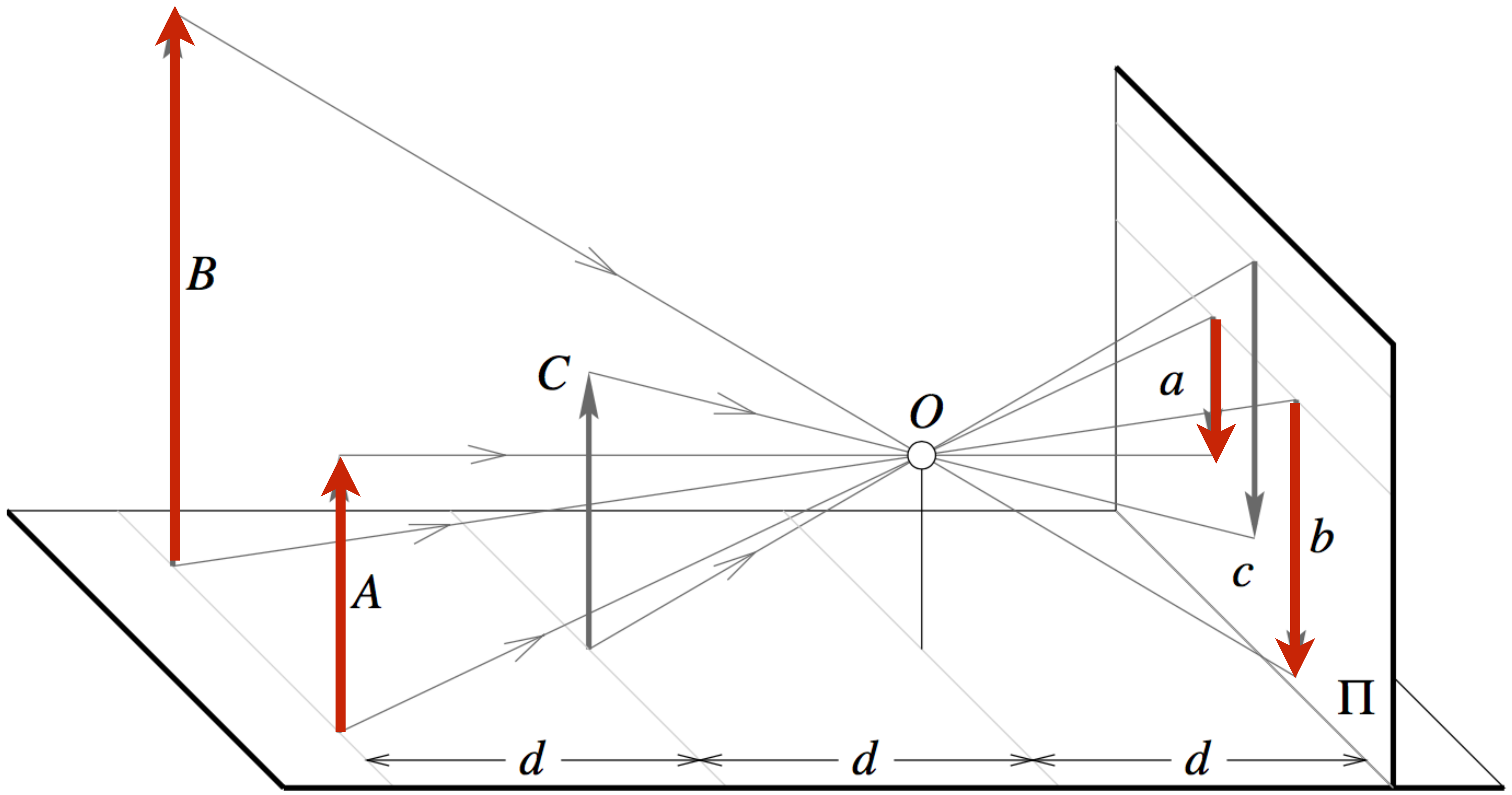


Forsyth & Ponce (2nd ed.) Figure 1.3a

Relative size of objects that are equally far from camera is preserved

(e.g., **A** is half size of **B**, so **a** will be half the size of **b** in the image plane)

# **Perspective** Effects

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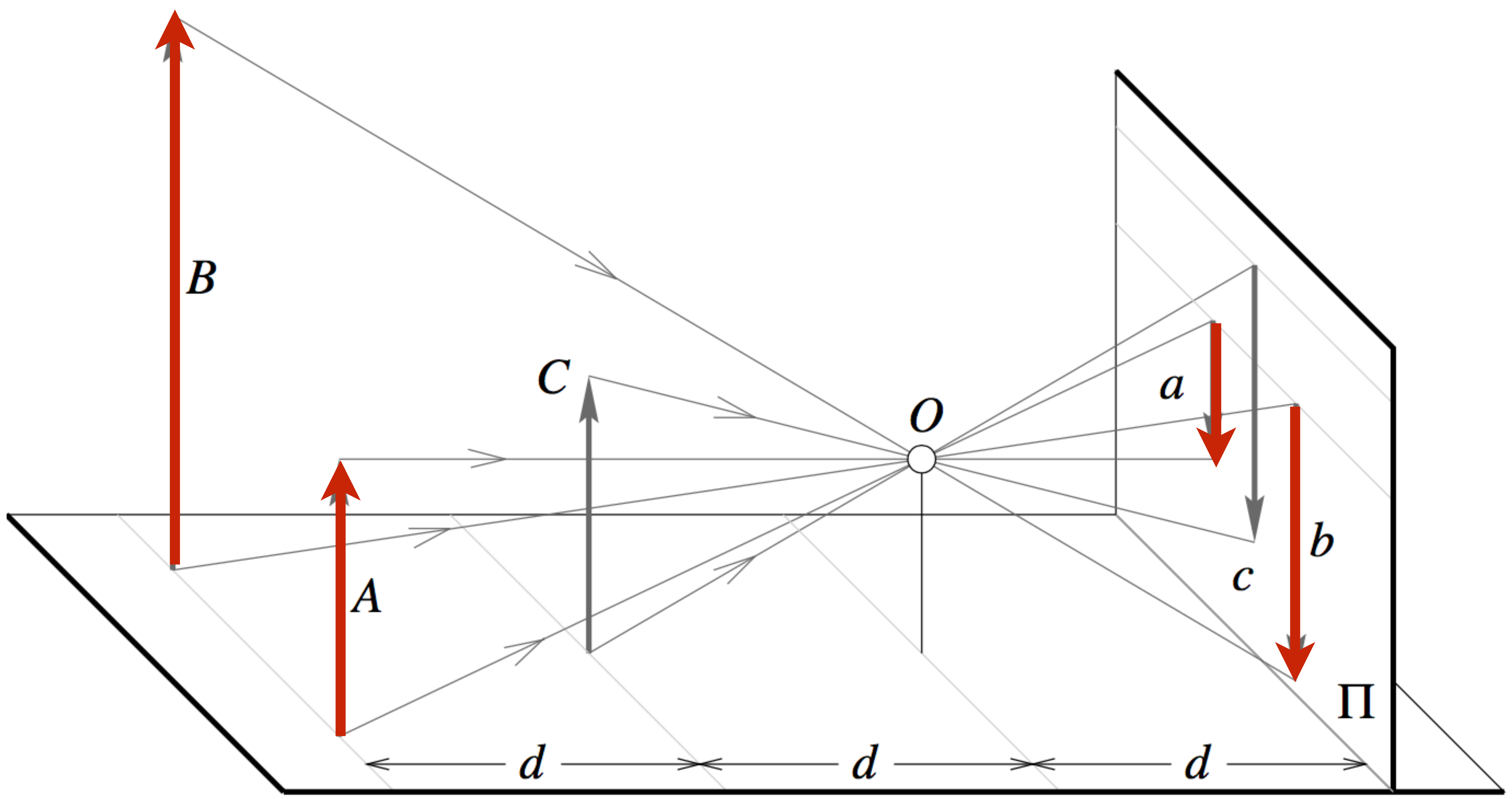
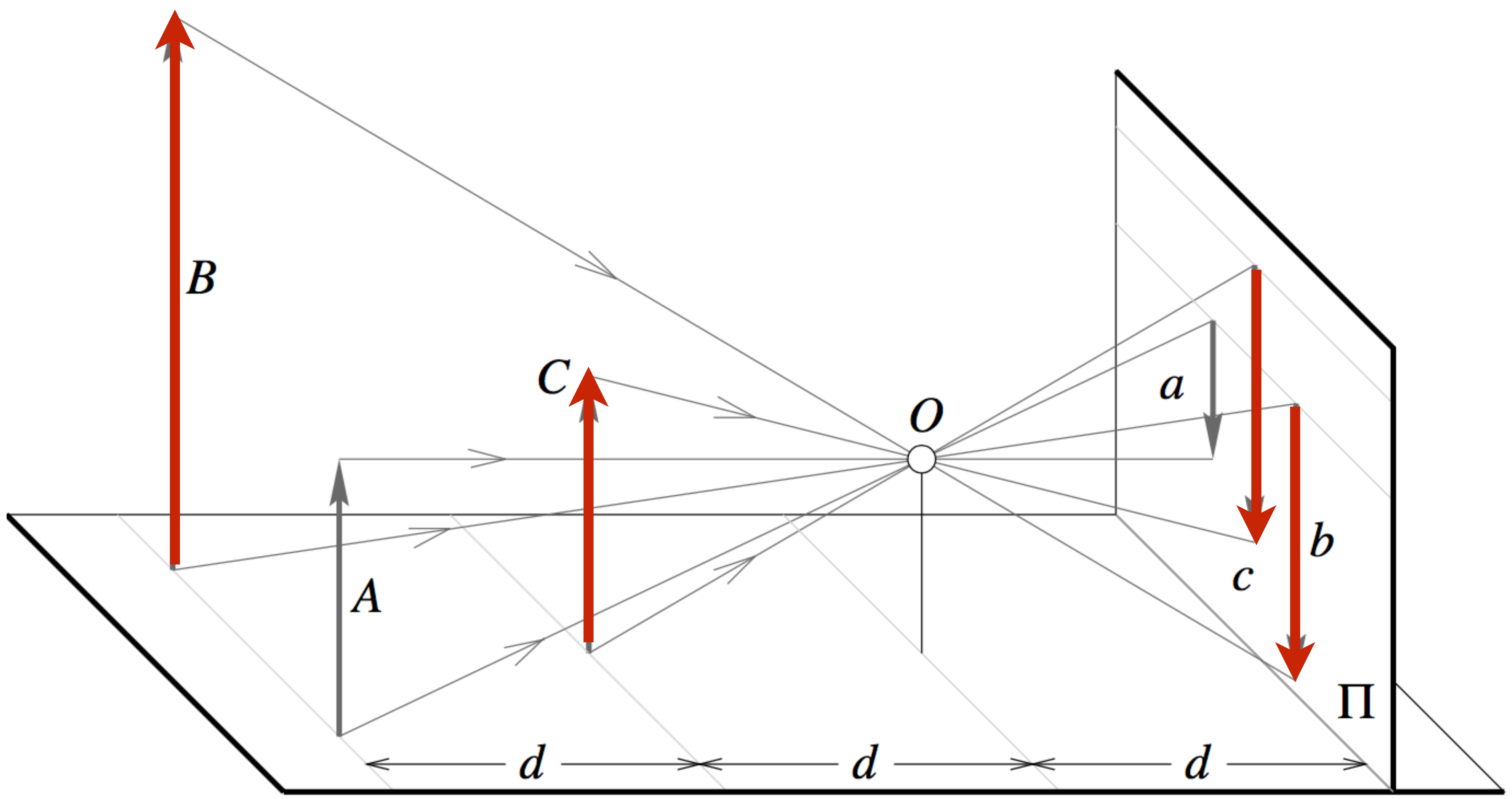



Forsyth & Ponce (2nd ed.) Figure 1.3a

Size is **inversely** proportions to distance

# **Perspective** Effects

**Far objects** appear **smaller** than close ones




Forsyth & Ponce (2nd ed.) Figure 1.3a

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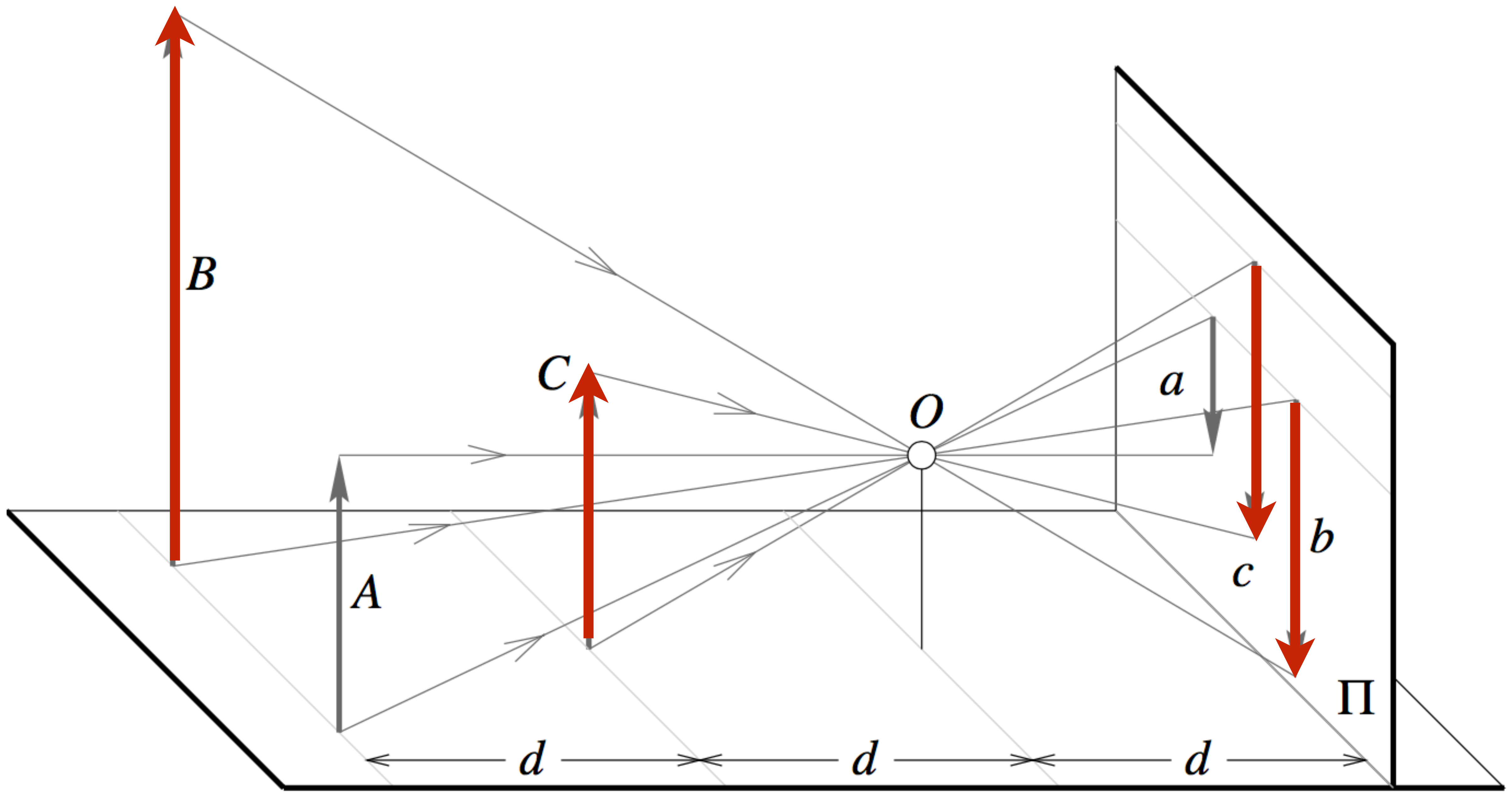
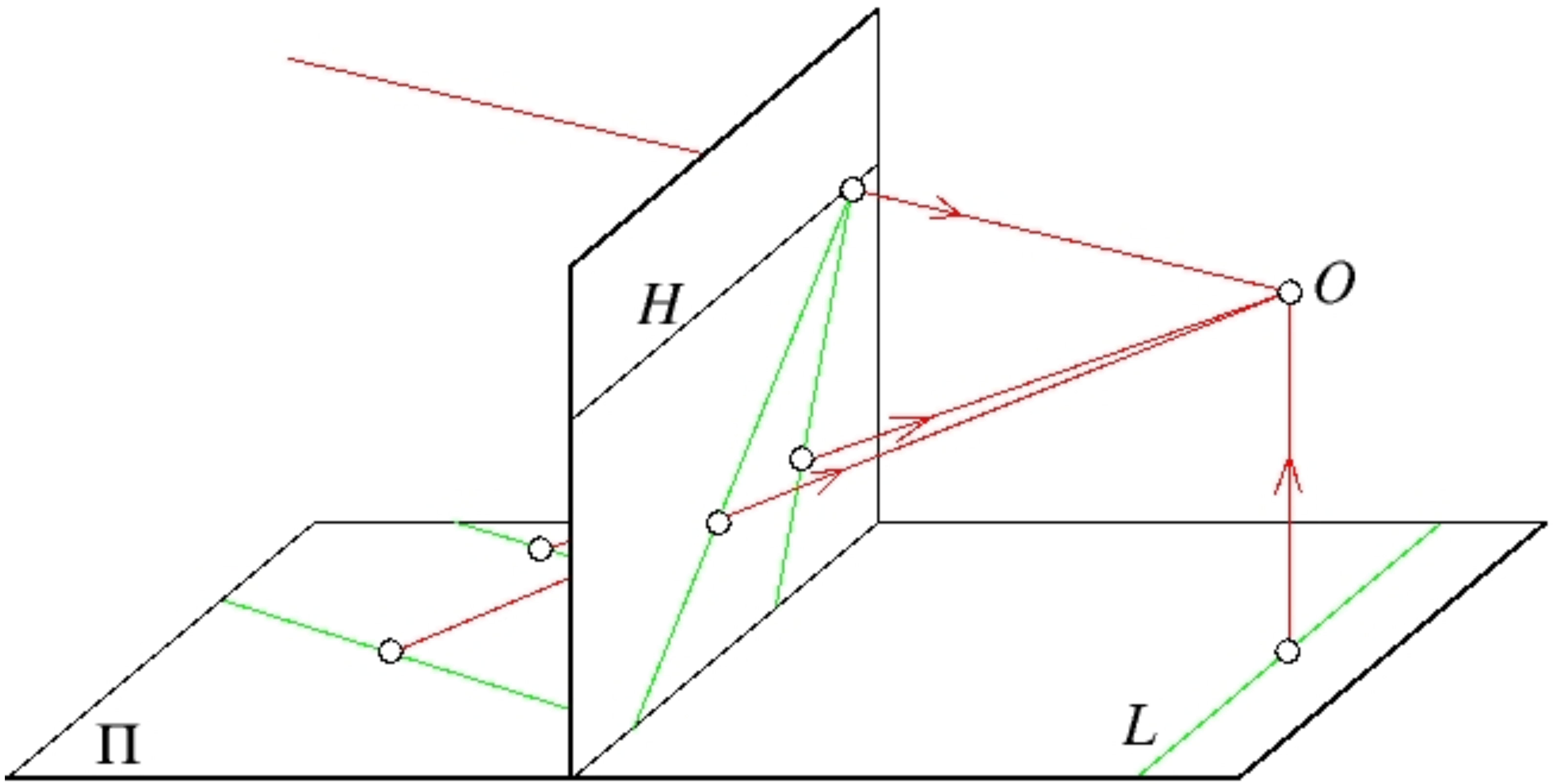



Forsyth & Ponce (2nd ed.) Figure 1.3a

(e.g., **C** is half size of **B** and half closer, resulting in **c** and **b** being same size in projection)

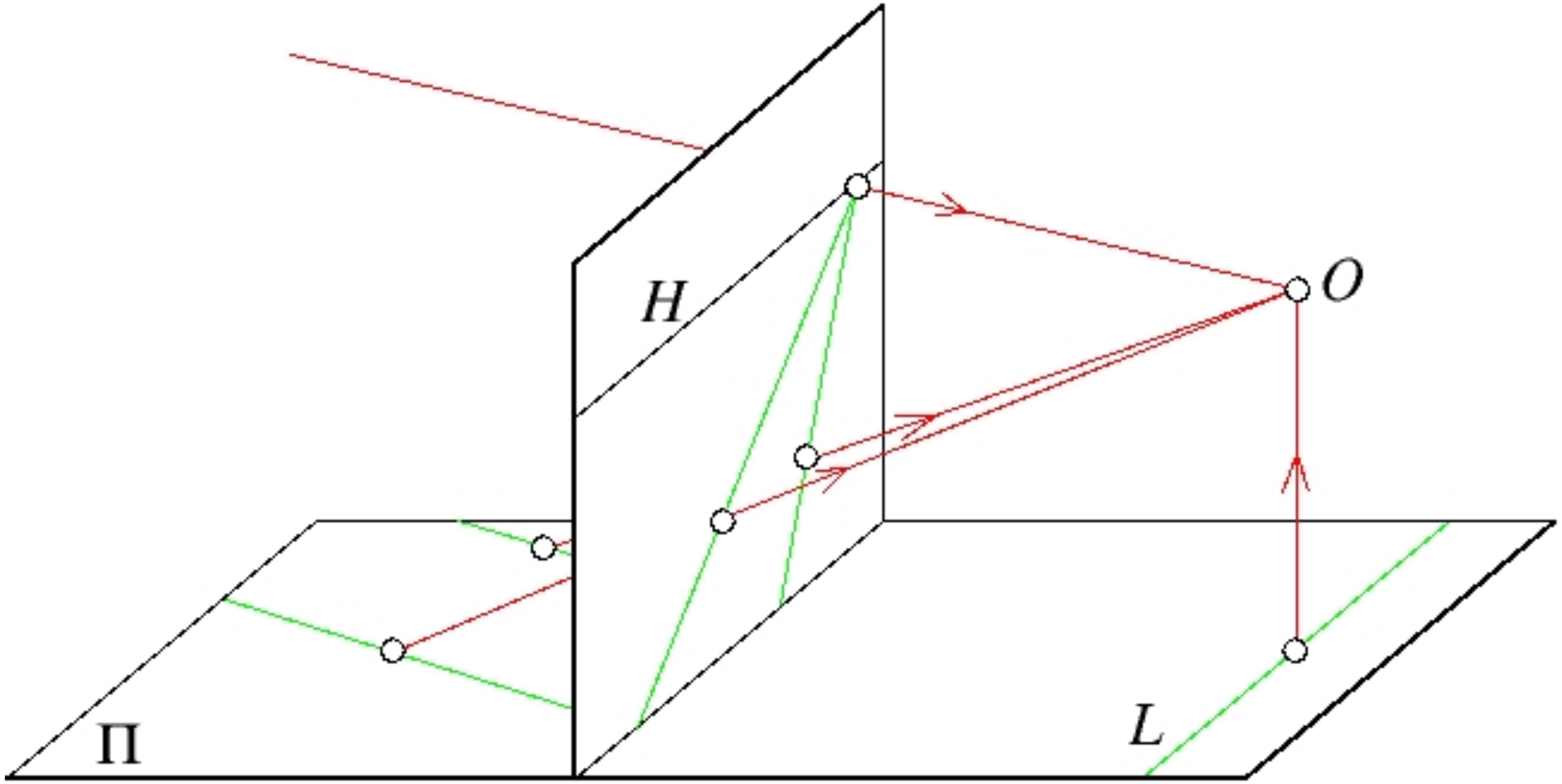
# **Perspective** Effects




Forsyth & Ponce (1st ed.) Figure 1.3b

# **Perspective** Effects

Parallel lines meet at a point (**vanishing point**)




Forsyth & Ponce (1st ed.) Figure 1.3b

# **Vanishing** Points

Each set of parallel lines meet at a different point

— the point is called **vanishing point**

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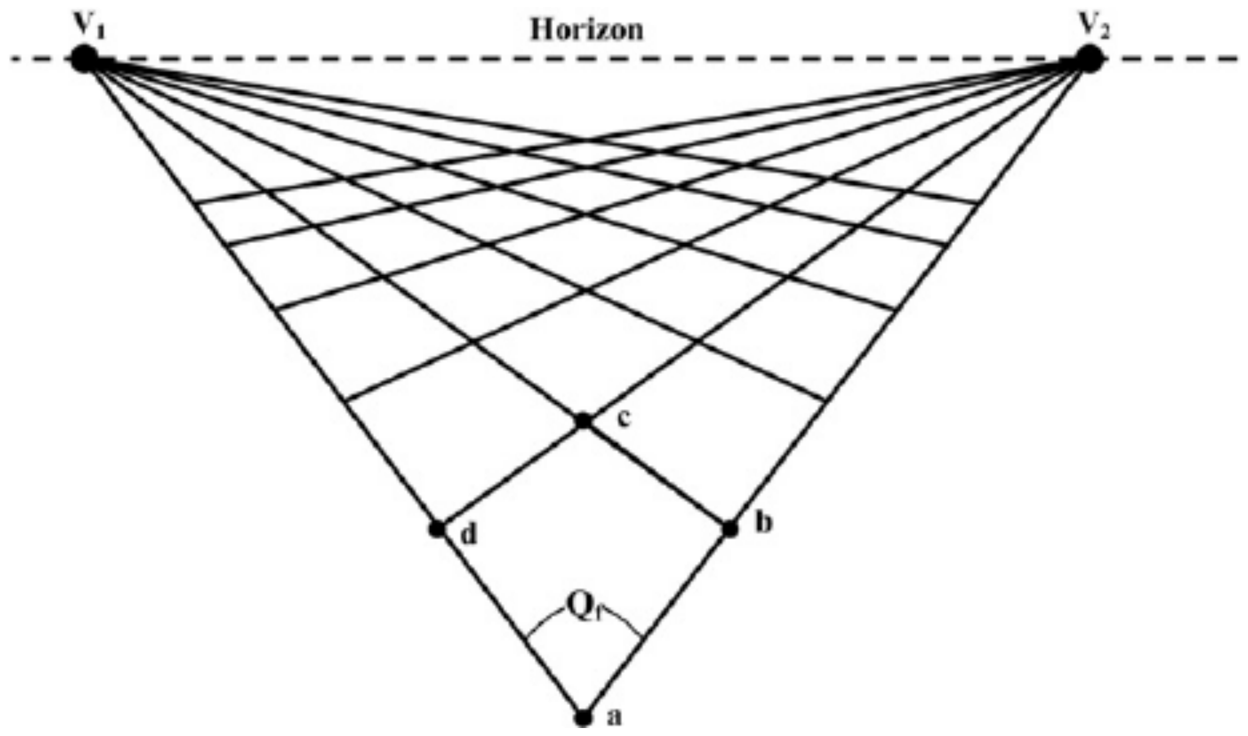
Sets of parallel lines on the same plane lead to **collinear** vanishing points

— the line is called a **horizon** for that plane



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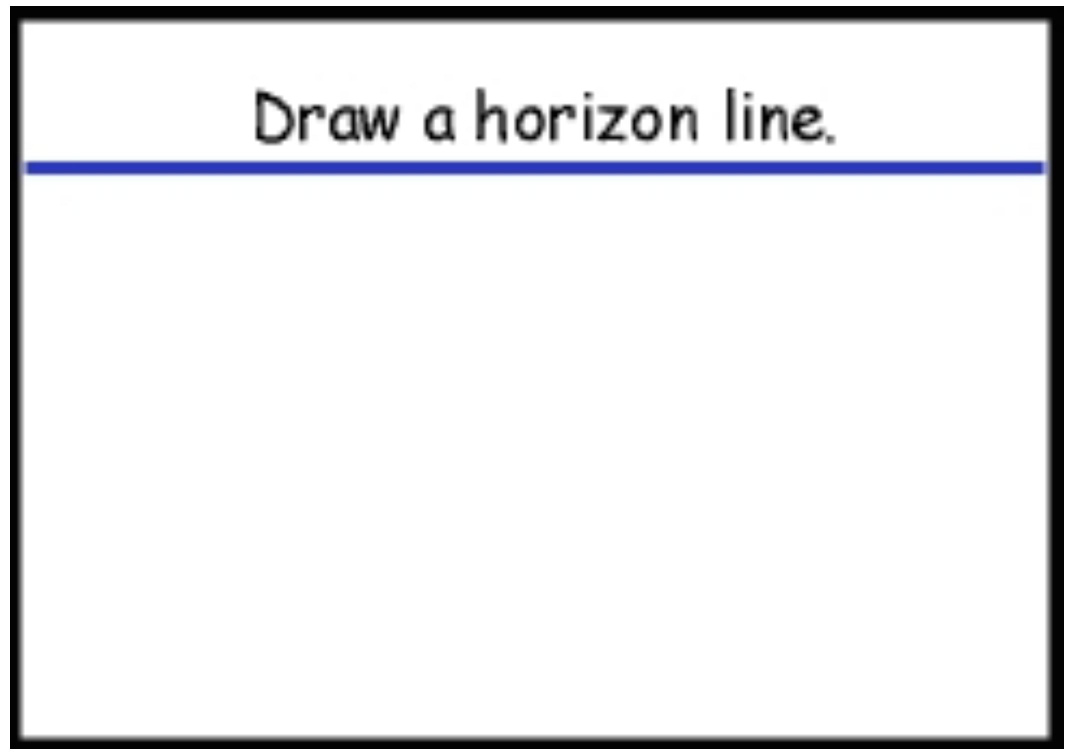


**Slide Credit**: David Jacobs

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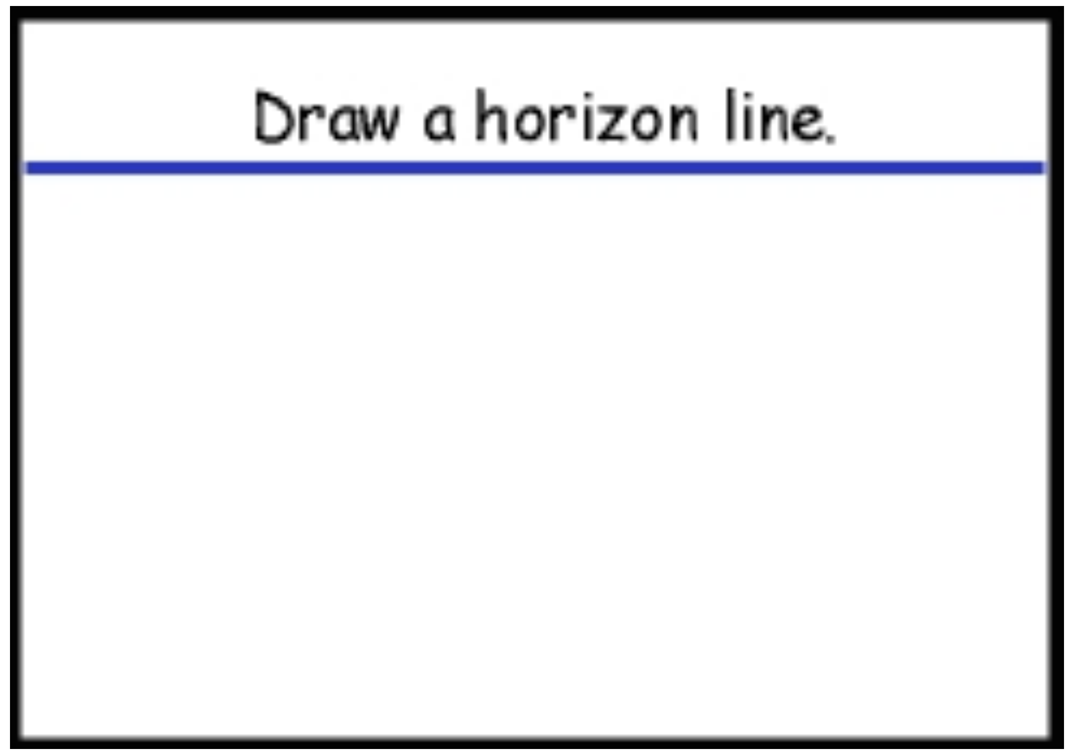


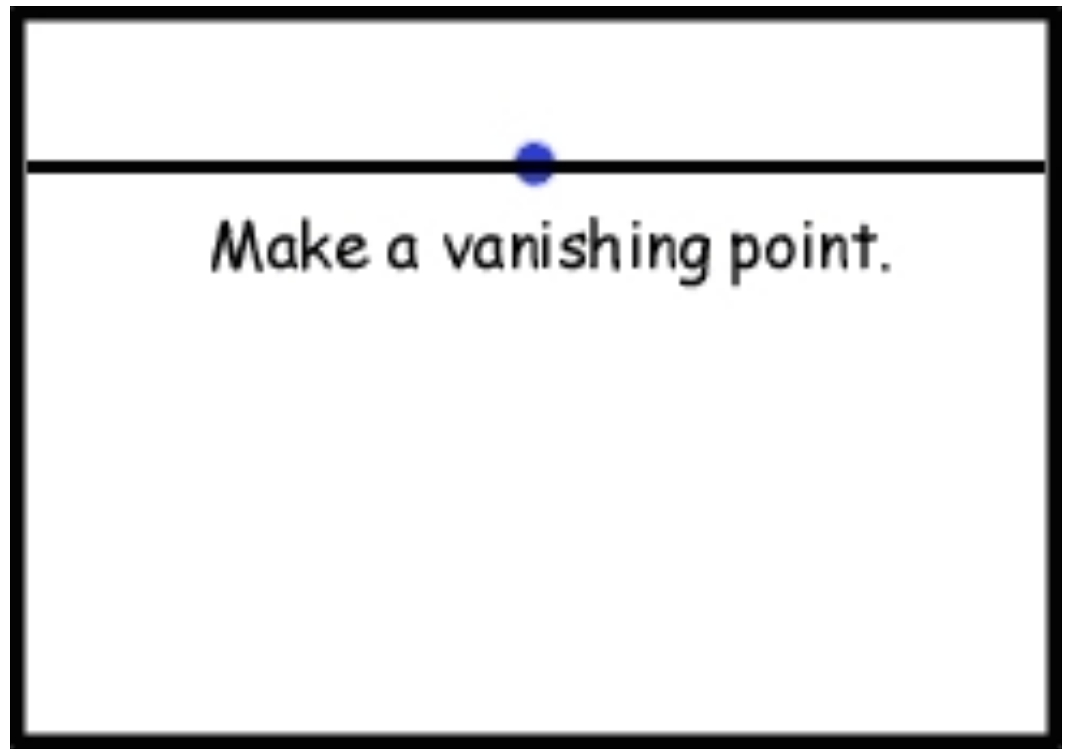


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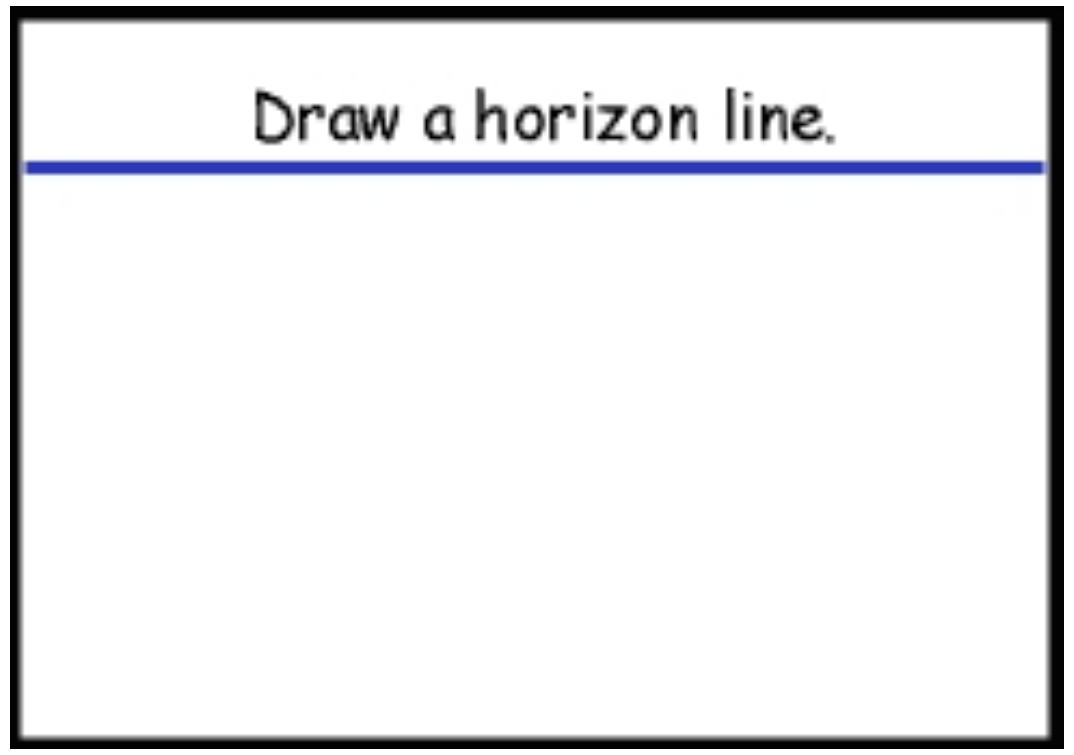


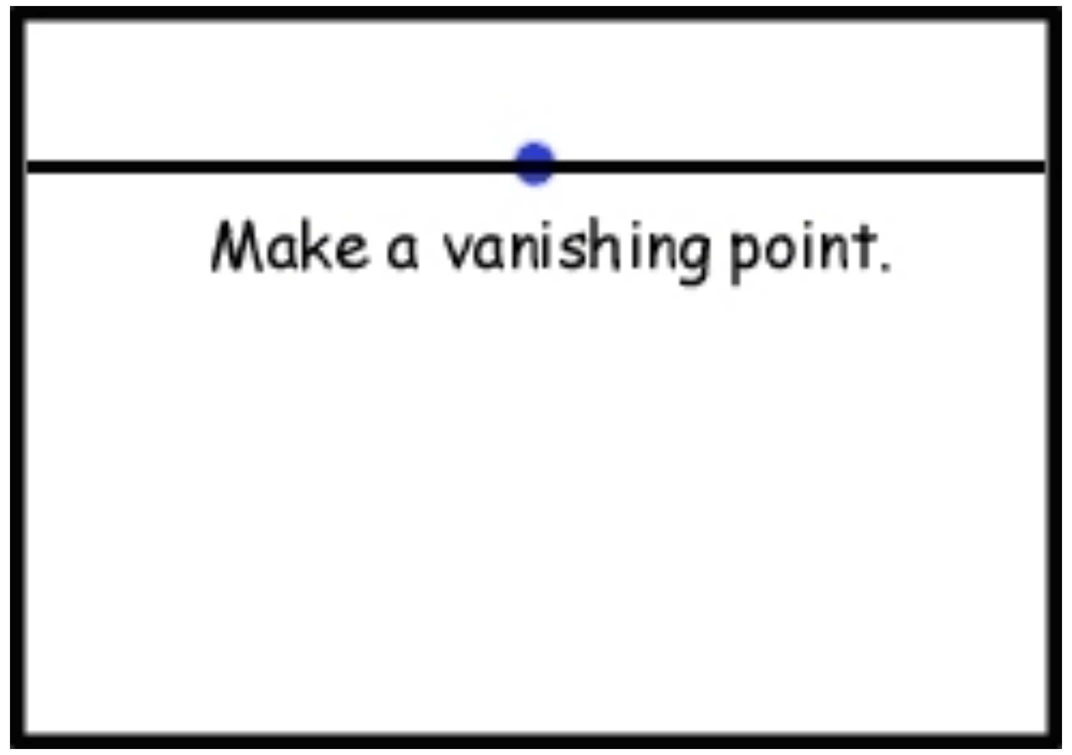


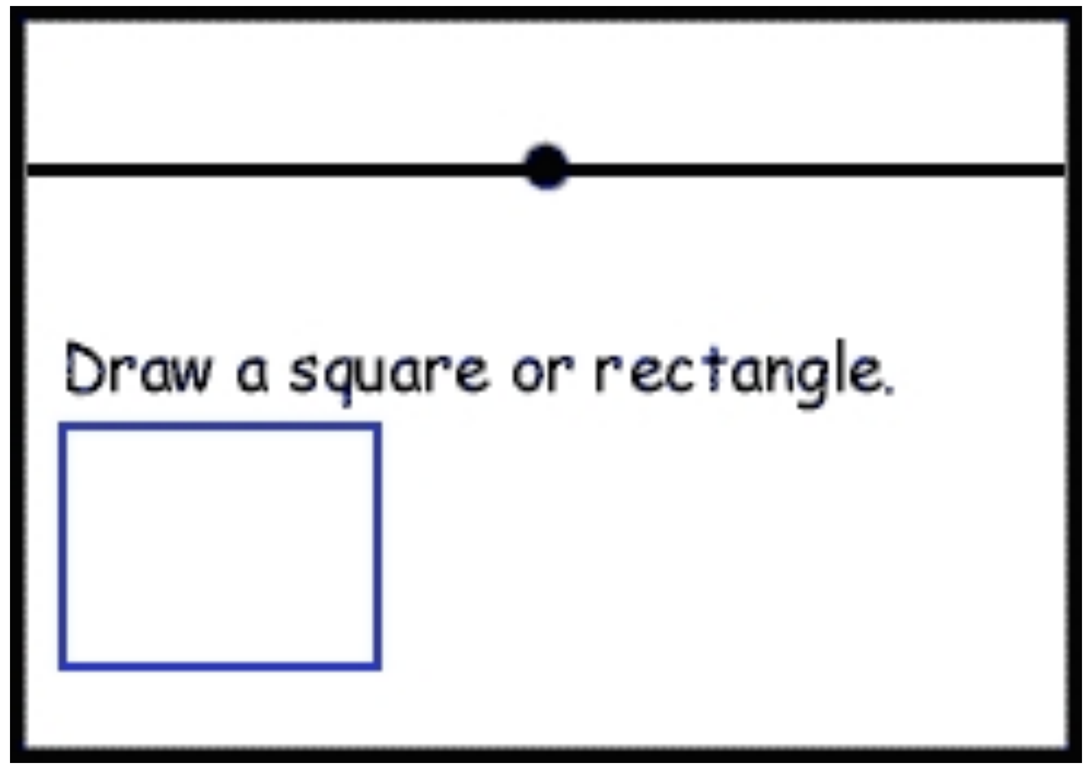


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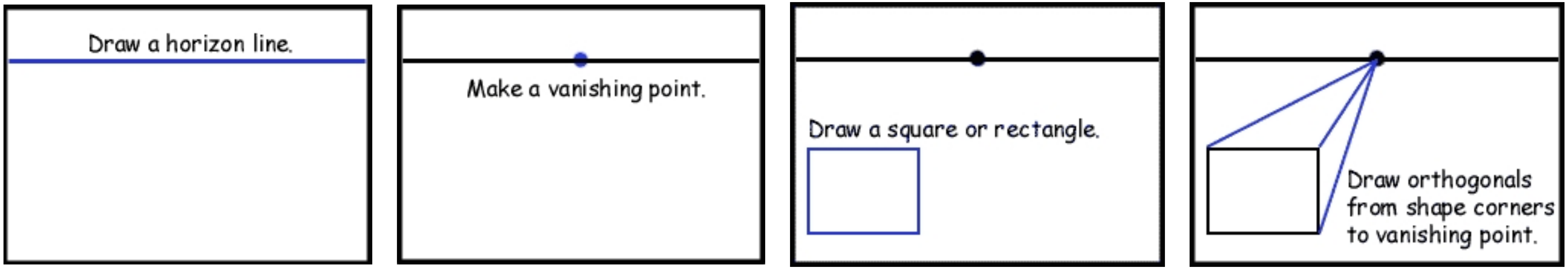


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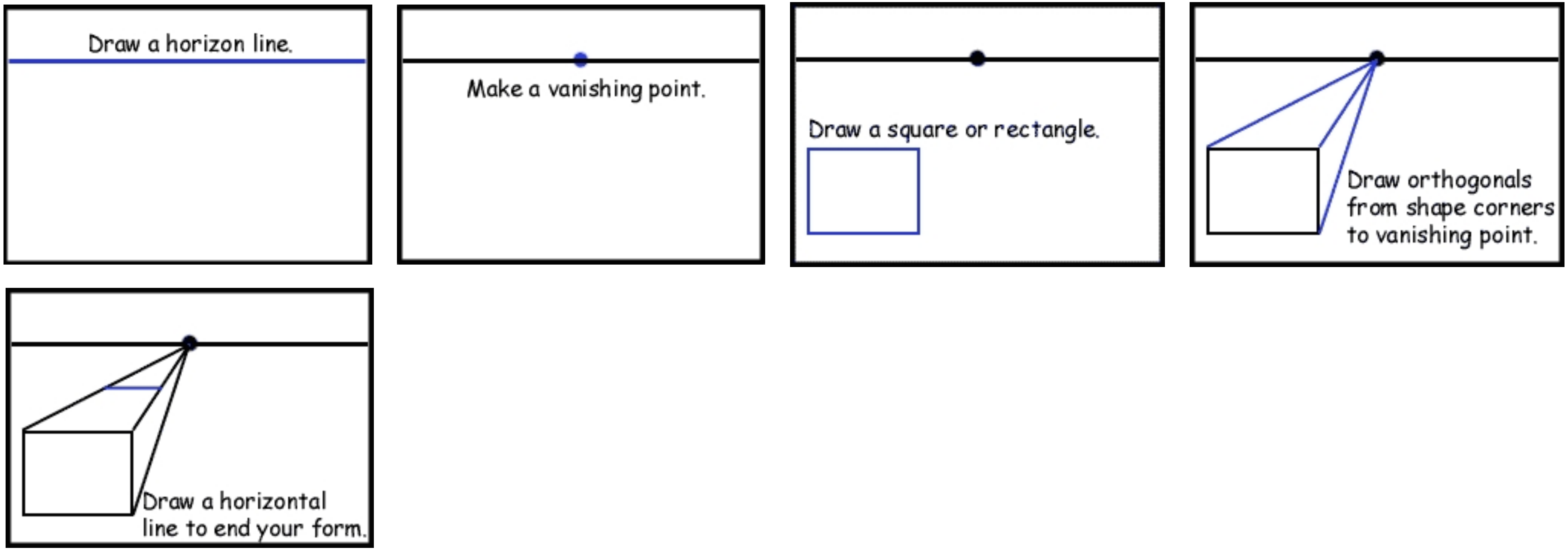


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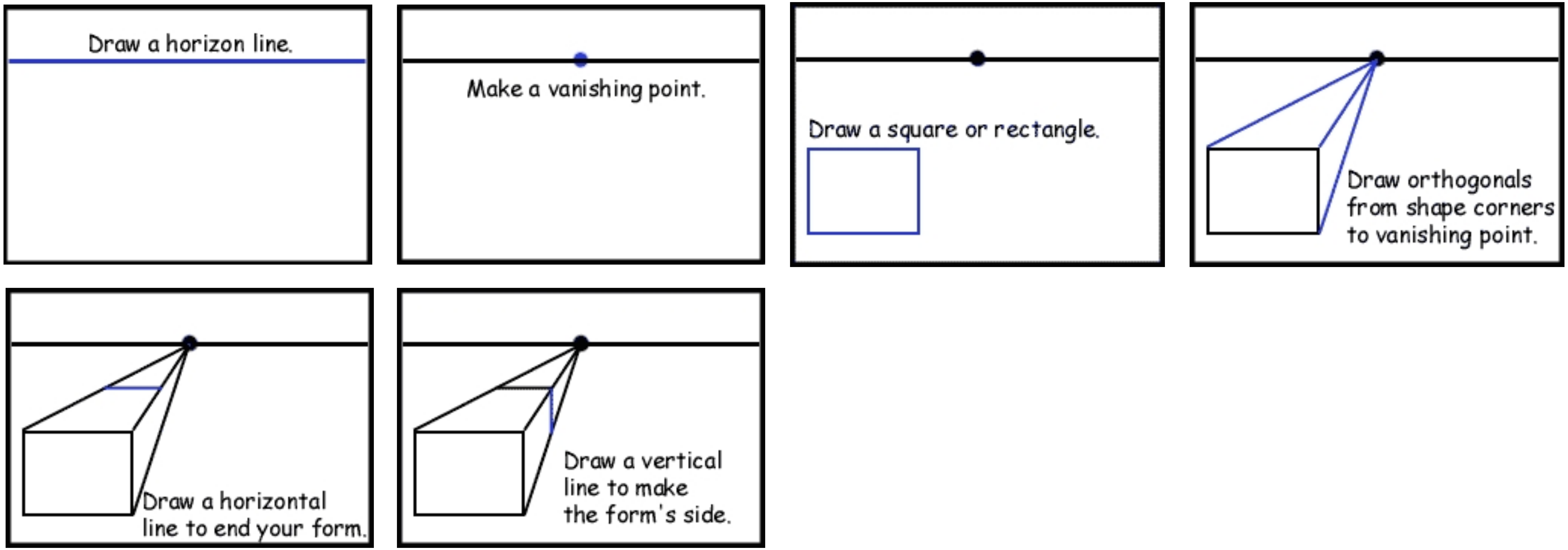


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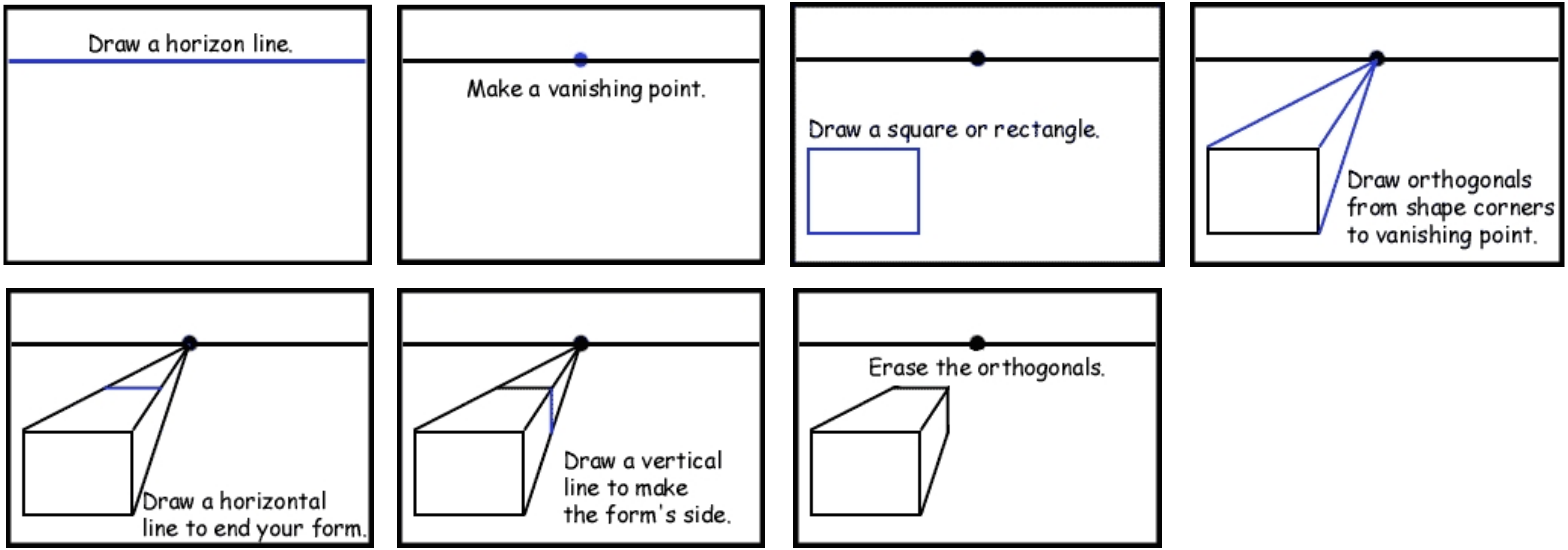


**Slide Credit**: David Jacobs

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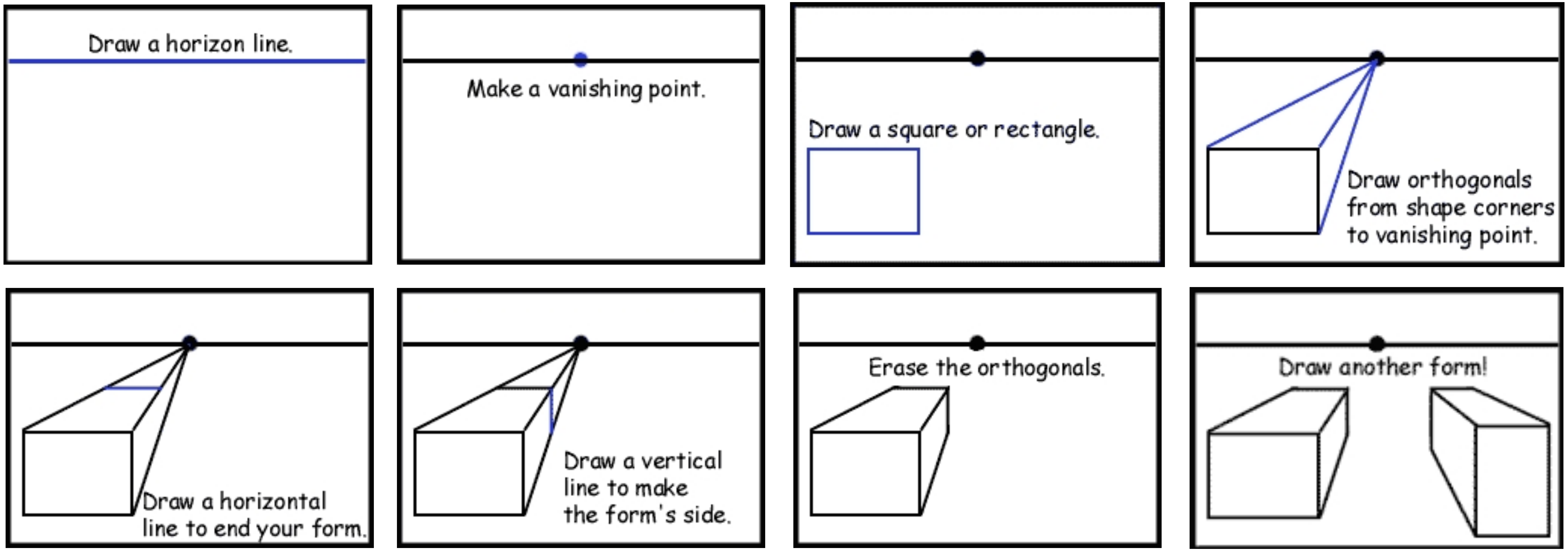


**Slide Credit**: David Jacobs

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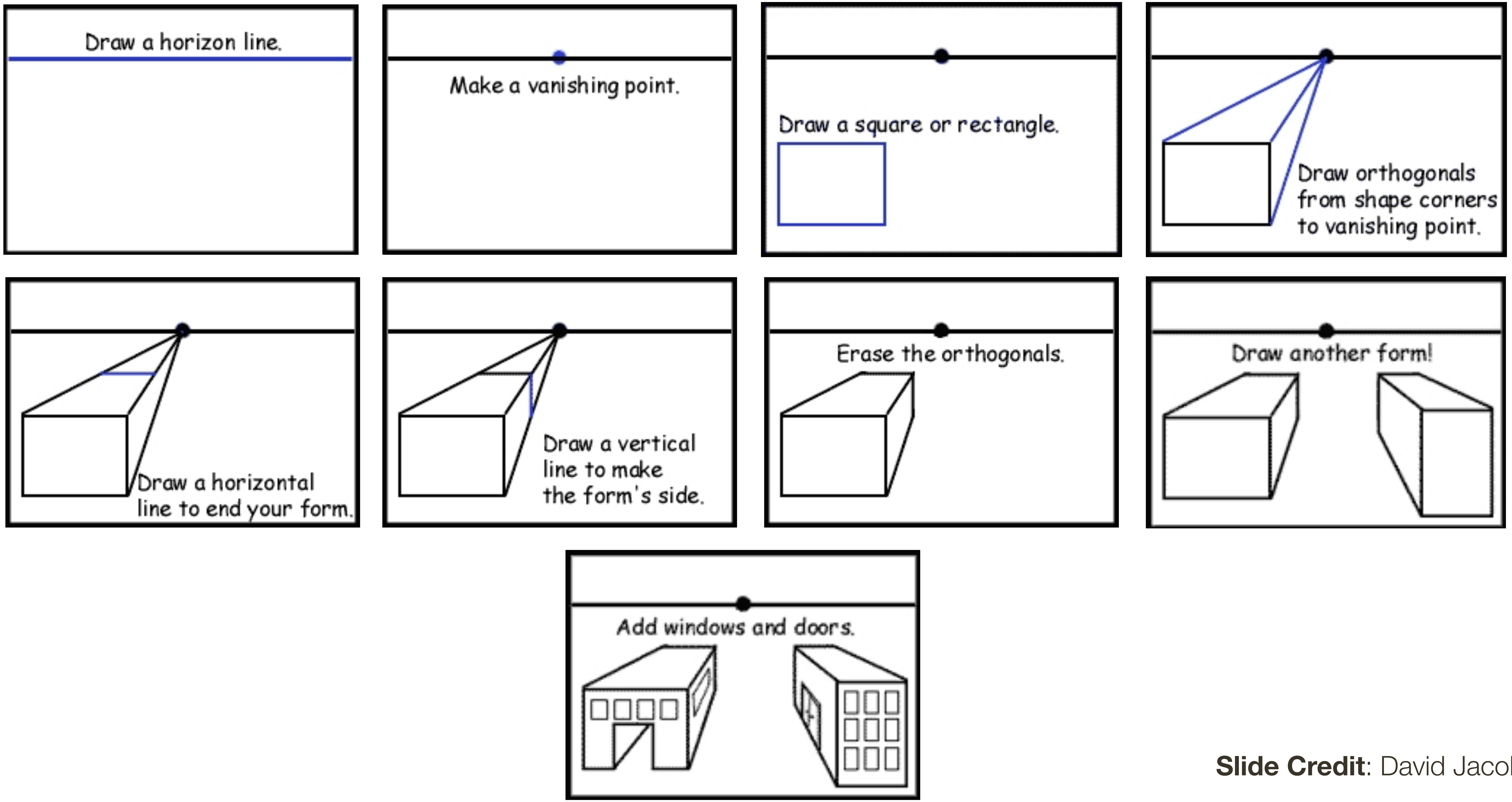


**Slide Credit**: David Jacobs

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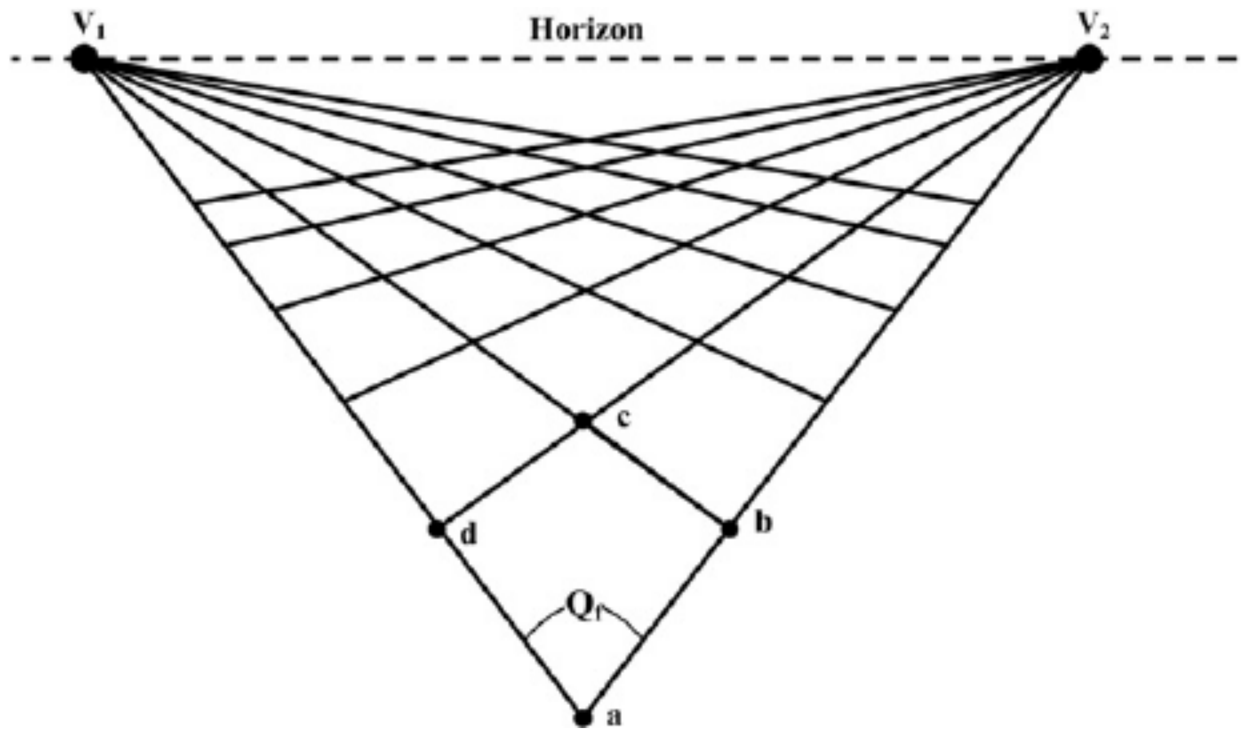
Good way to **spot fake images**

— scale and perspective do not work

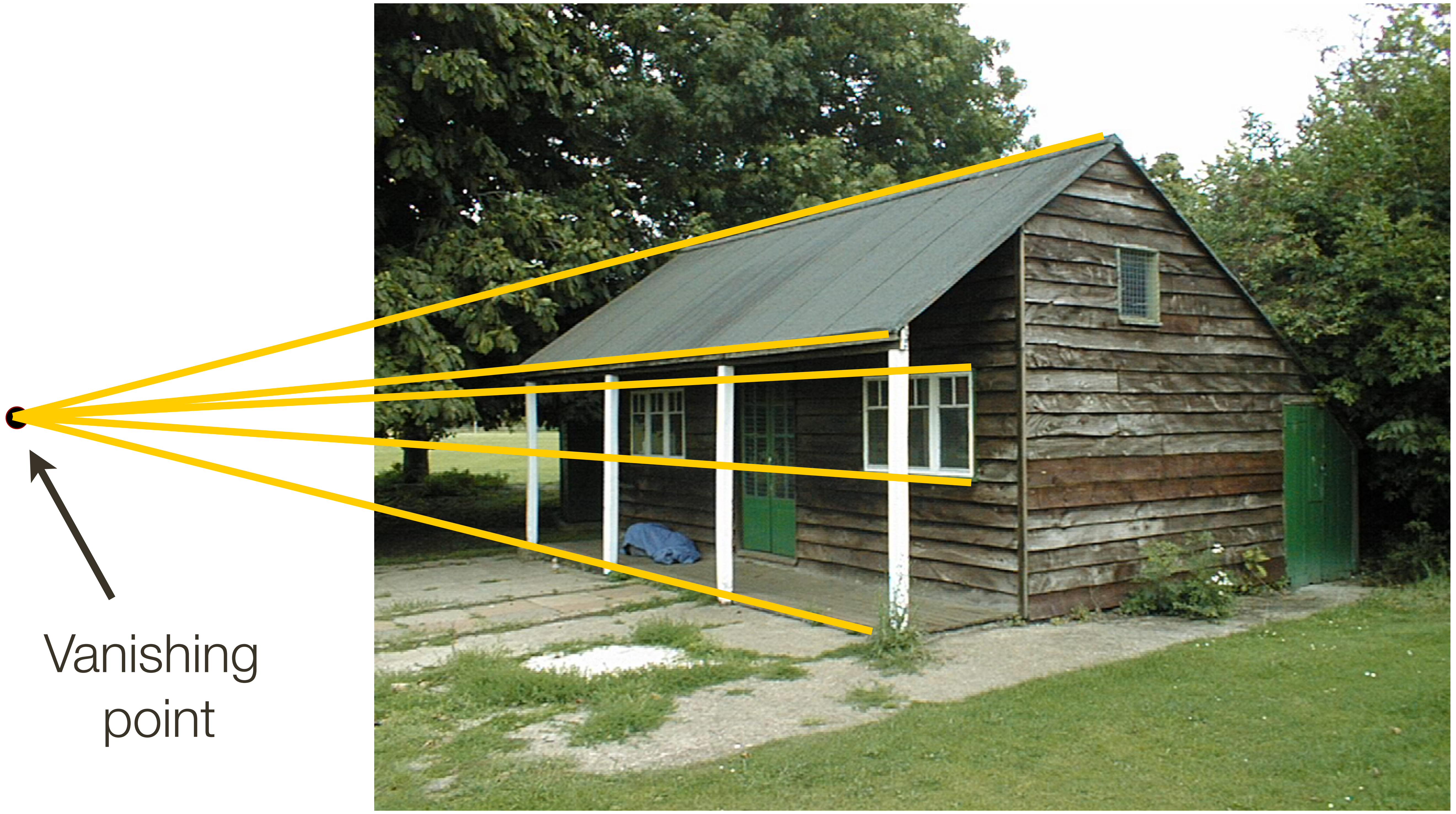
— vanishing points behave badly



# **Vanishing** Points

**Slide Credit**: Efros (Berkeley), photo from Criminisi

# **Vanishing** Points

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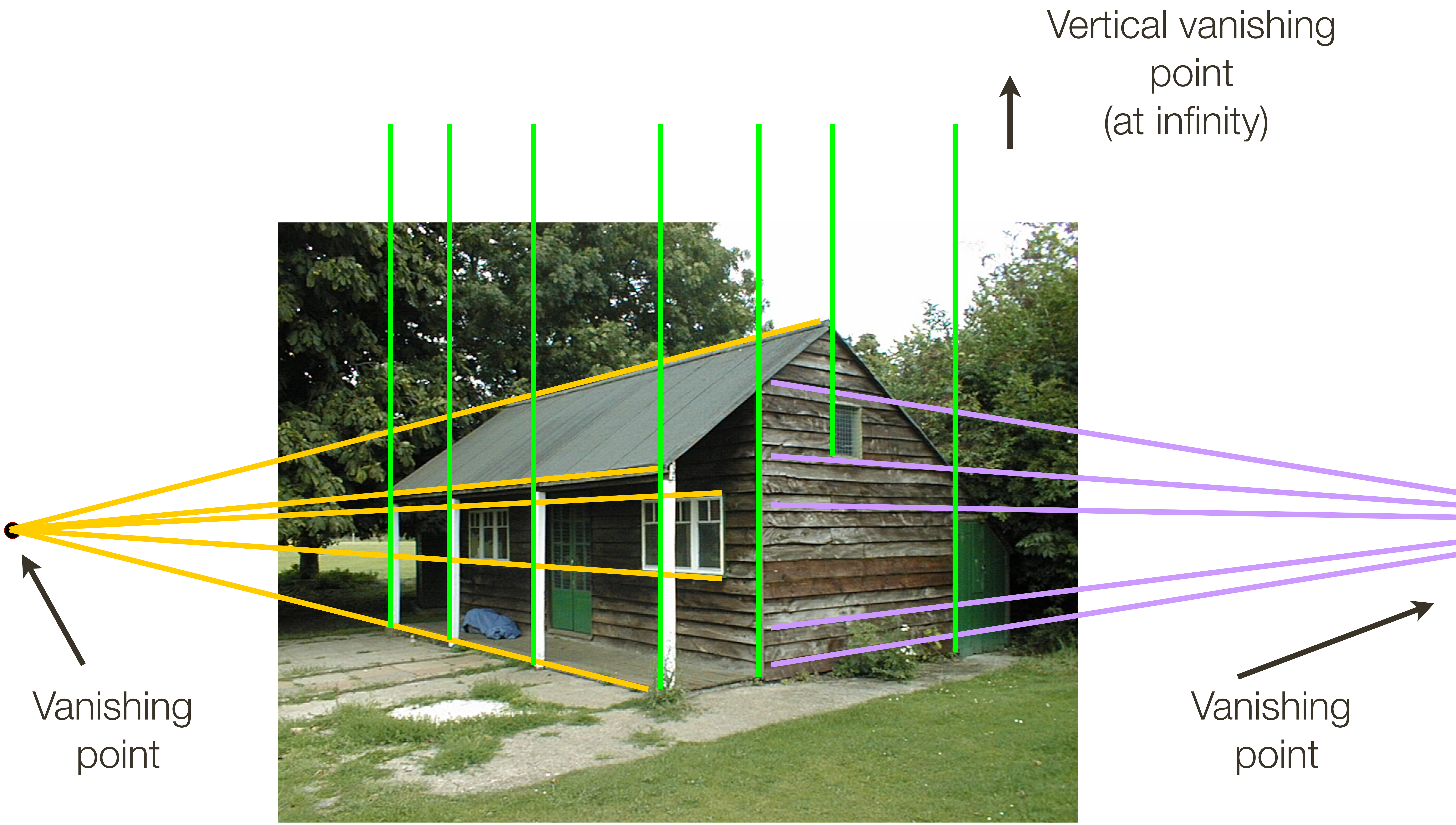


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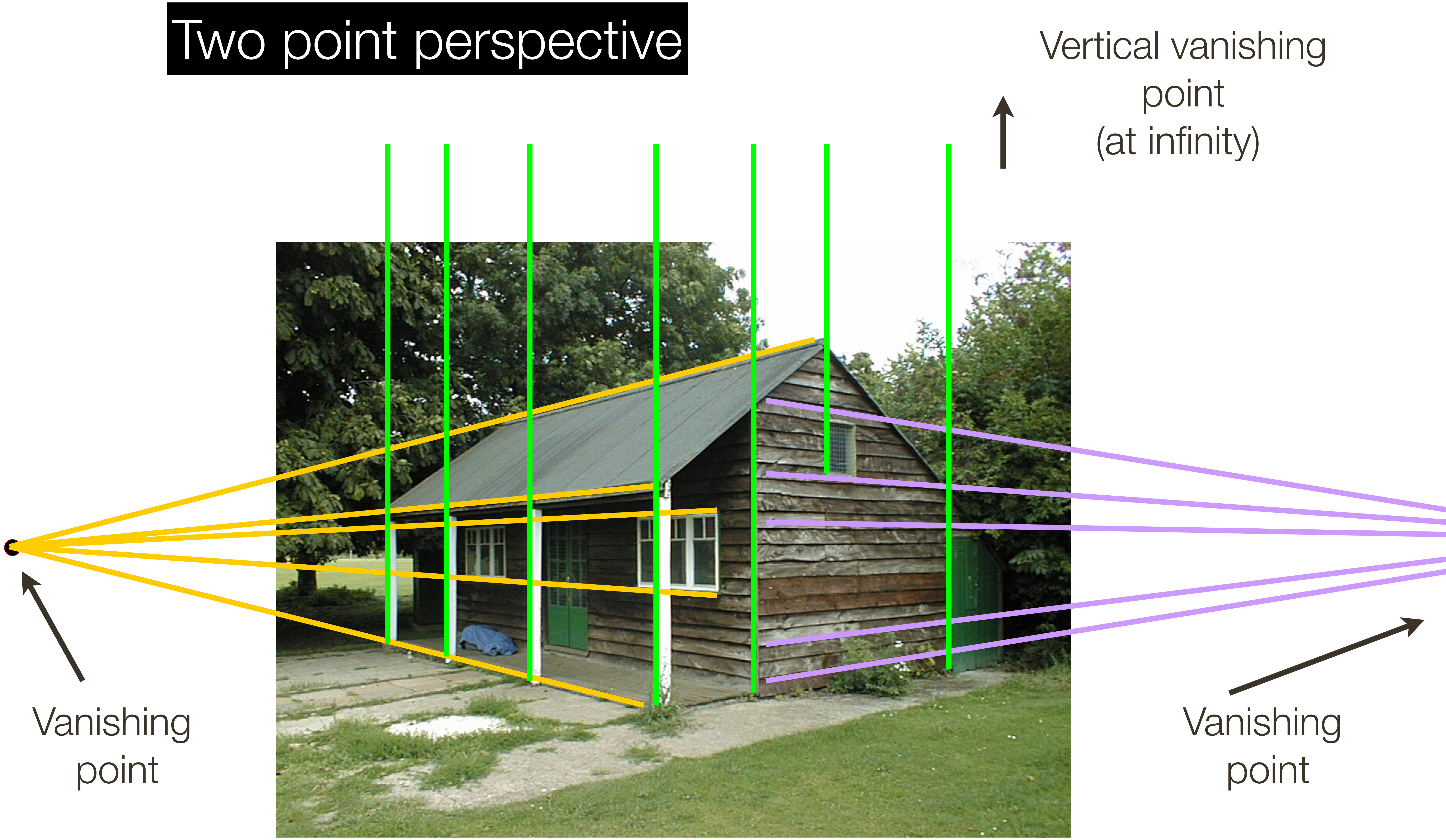
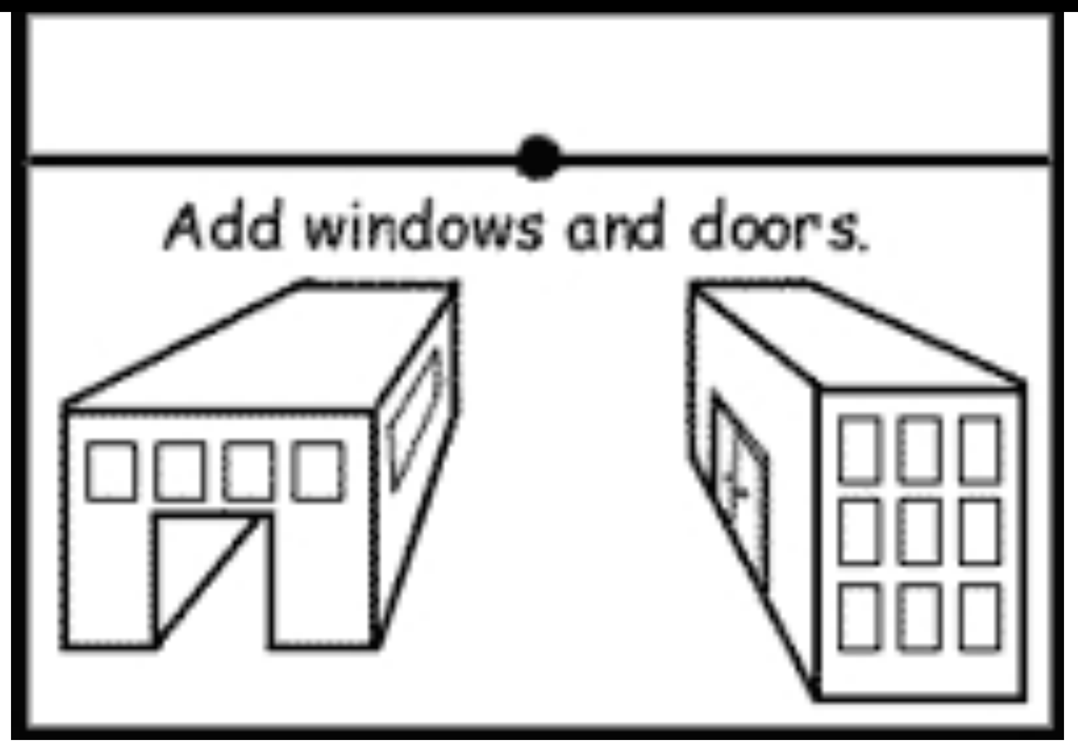


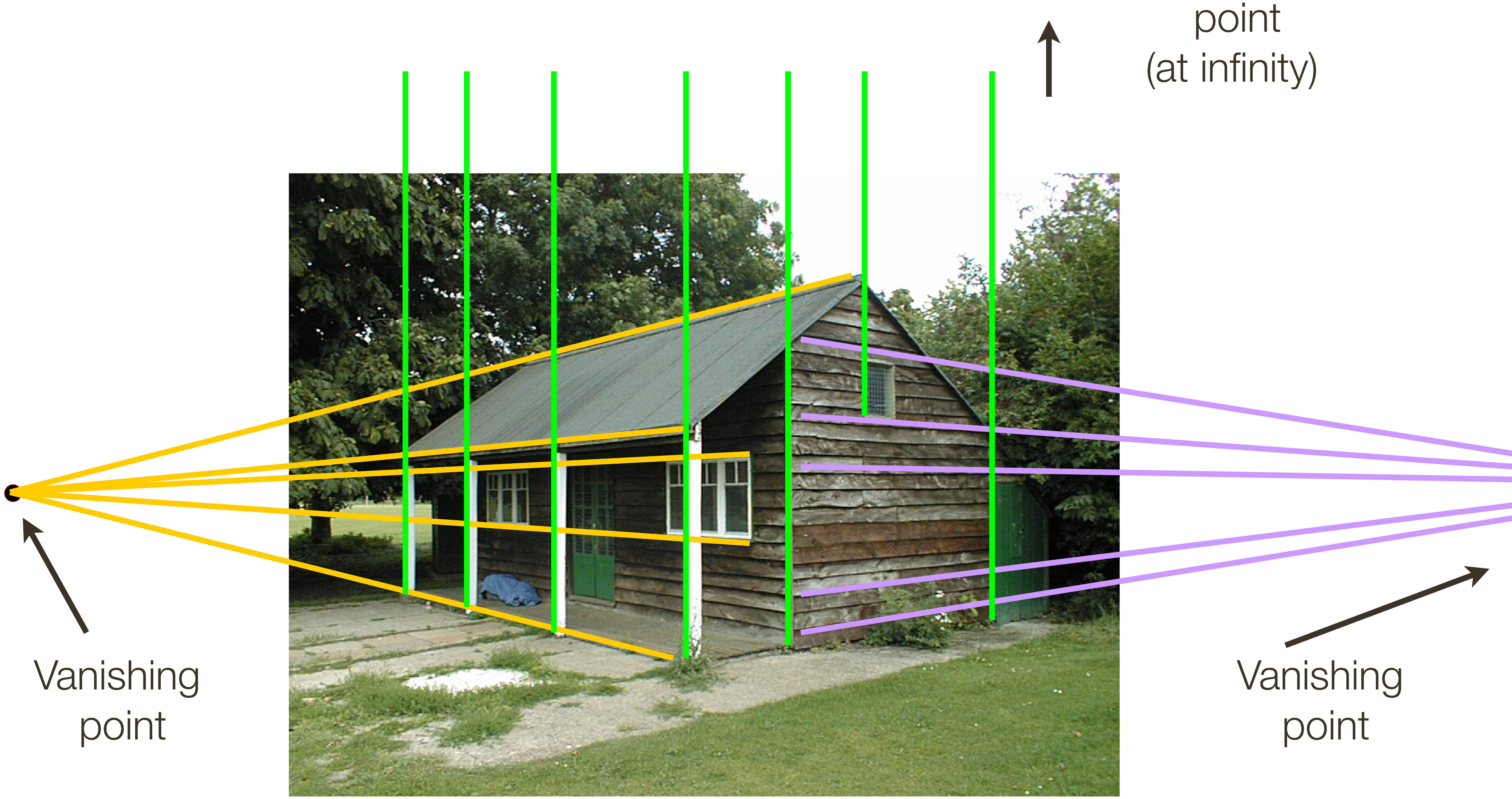

**Slide Credit**: Efros (Berkeley), photo from Criminisi

# **Vanishing** Points

One point perspective




Two point perspective




**Slide Credit**: Efros (Berkeley), photo from Criminisi

# Spotting fake images with **Vanishing** Points

Generated Image | Shadow Errors | Detected Shadow Errors | Vanishing Point Errors | Detected Perspective Errors

[Sarkar et al., 2023, Image from https://projective-geometry.github.io/ reproduced for educational purposes.]

# **Perspective** Aside

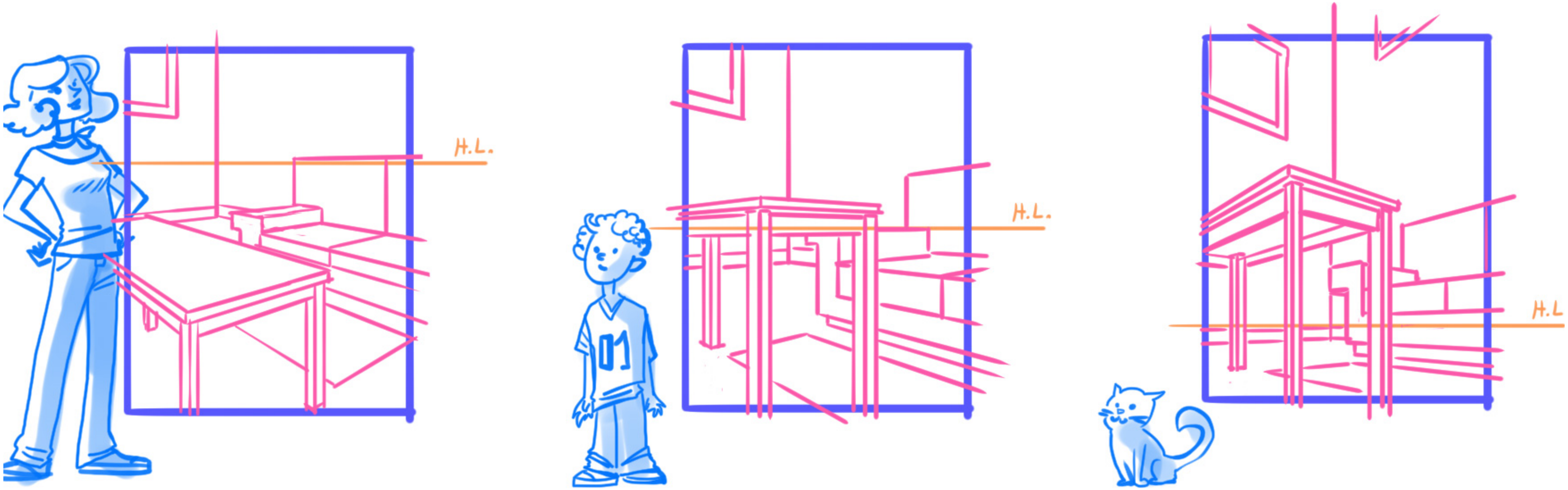
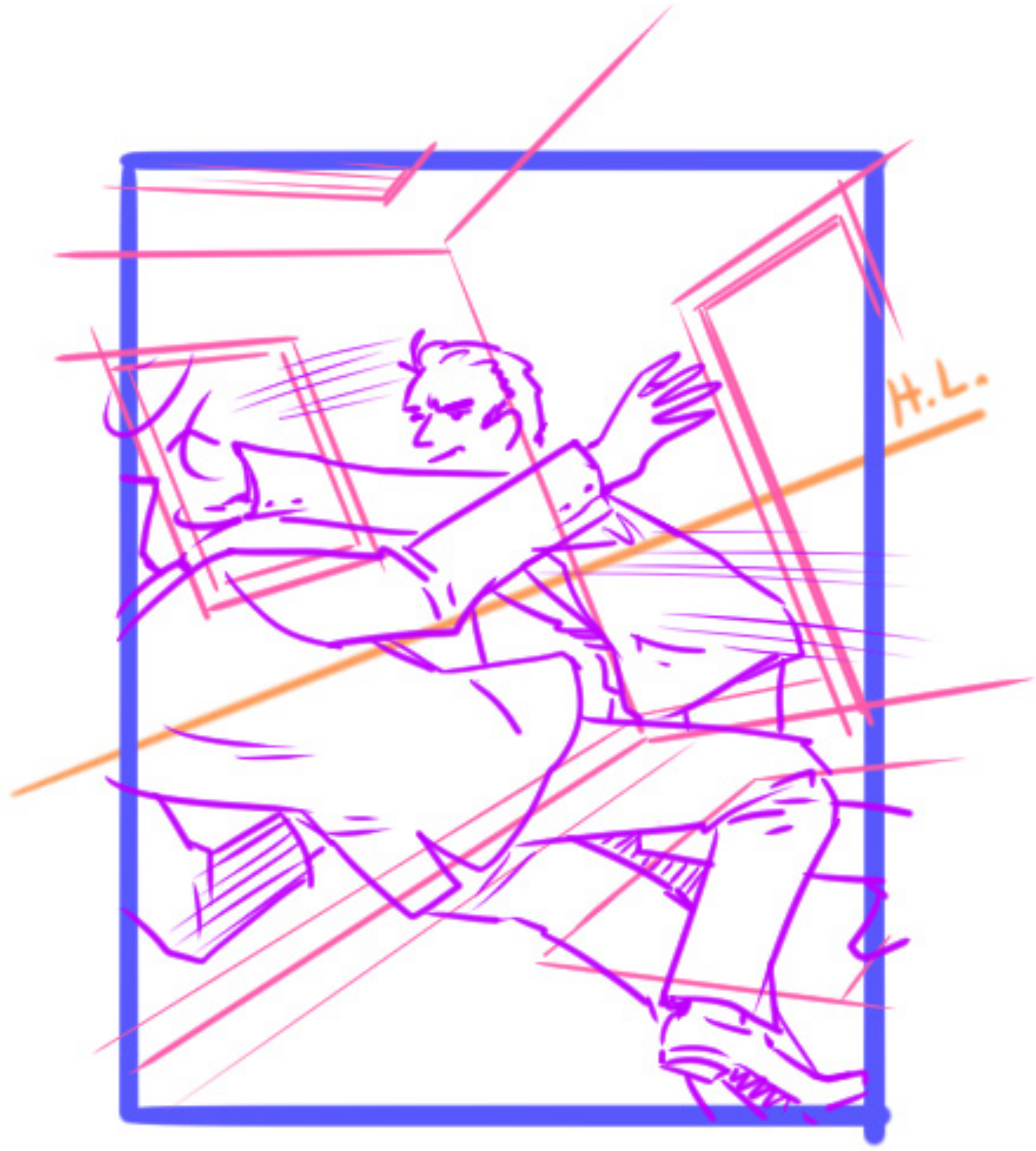



**Image credit**: http://www.martinacecilia.com/place-vanishing-points/

# **Perspective** Aside



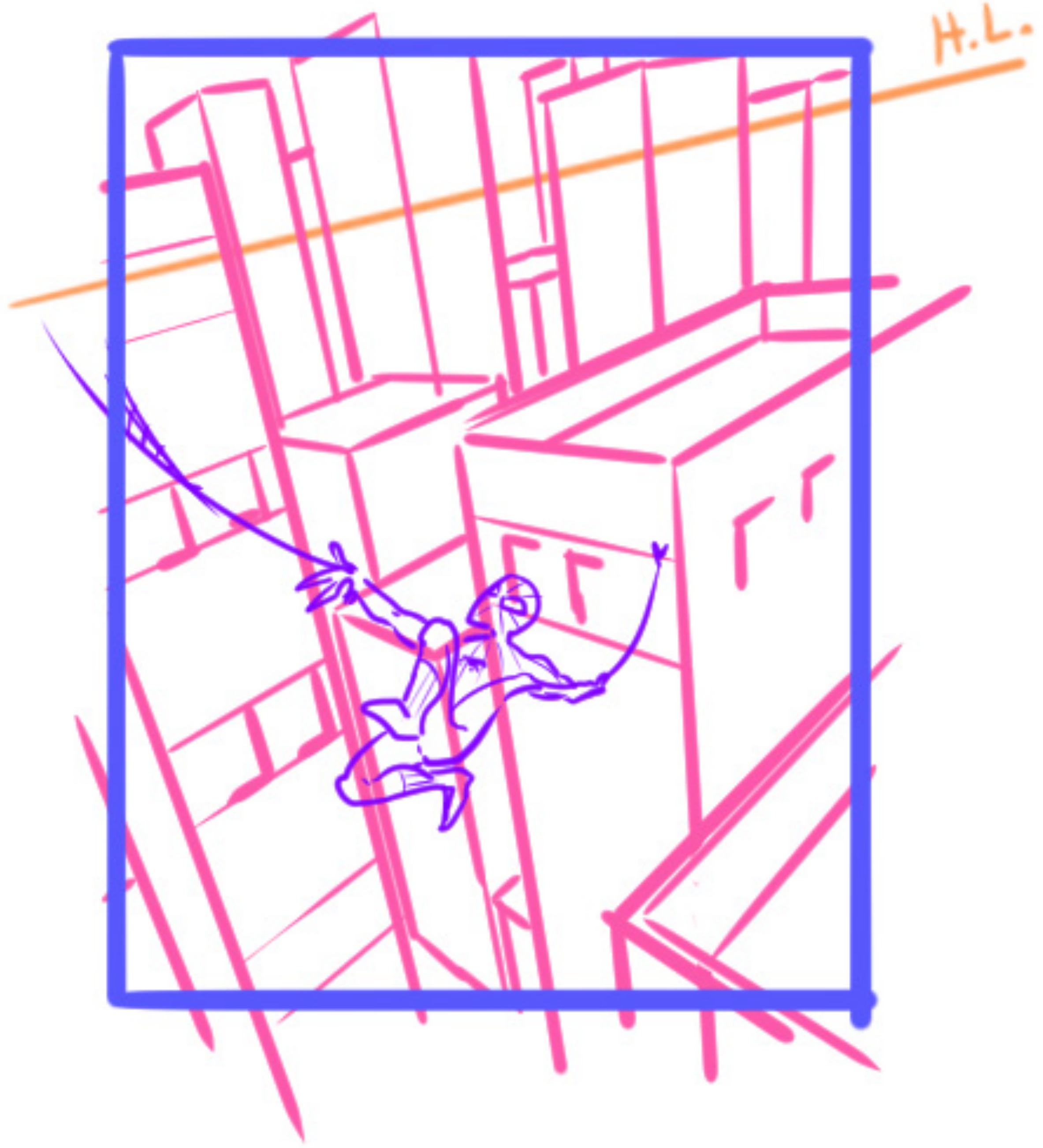




**Image credit**: http://www.martinacecilia.com/place-vanishing-points/

# **Properties** of Projection

- **Points** project to **points**
- **Lines** project to **lines**
- **Planes** project to the **whole** or **half** image
- Angles are **not** preserved

# **Properties** of Projection

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— **Planes** project to the **whole** or **half** image

— Angles are **not** preserved

**Degenerate** cases

— Line through focal point projects to a point

— Plane through focal point projects to a line

# **Projection** Illusion

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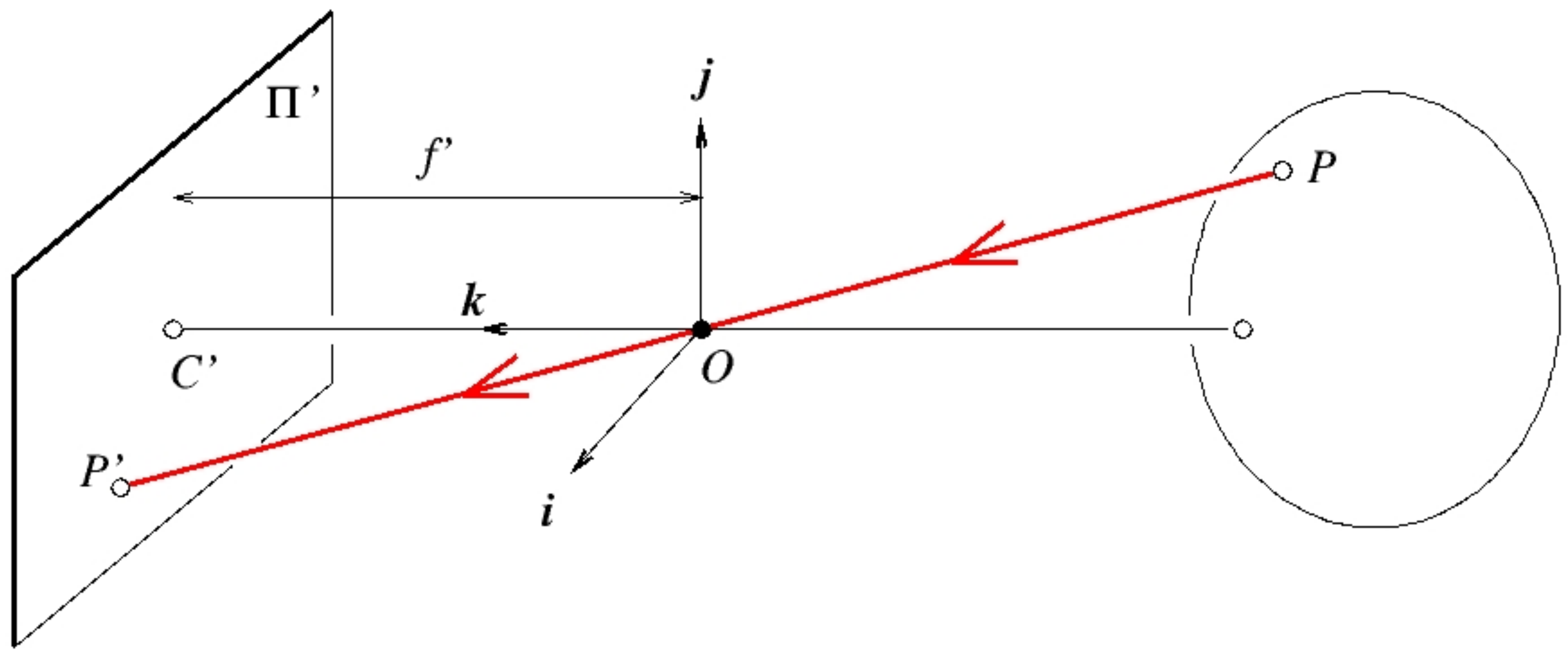
# **Perspective** Projection




Forsyth & Ponce (1st ed.) Figure 1.4

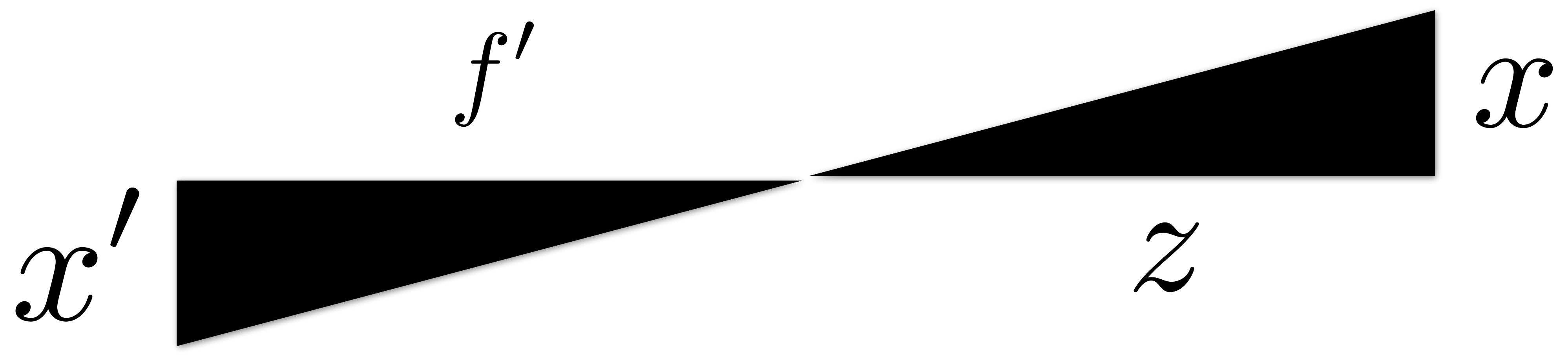
3D object point

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ projects to 2D image point $$P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$ where

$$\begin{aligned} x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z} \end{aligned}$$

**Note:** this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

# **Perspective** Projection: Proof

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Forsyth & Ponce (1st ed.) Figure 1.4

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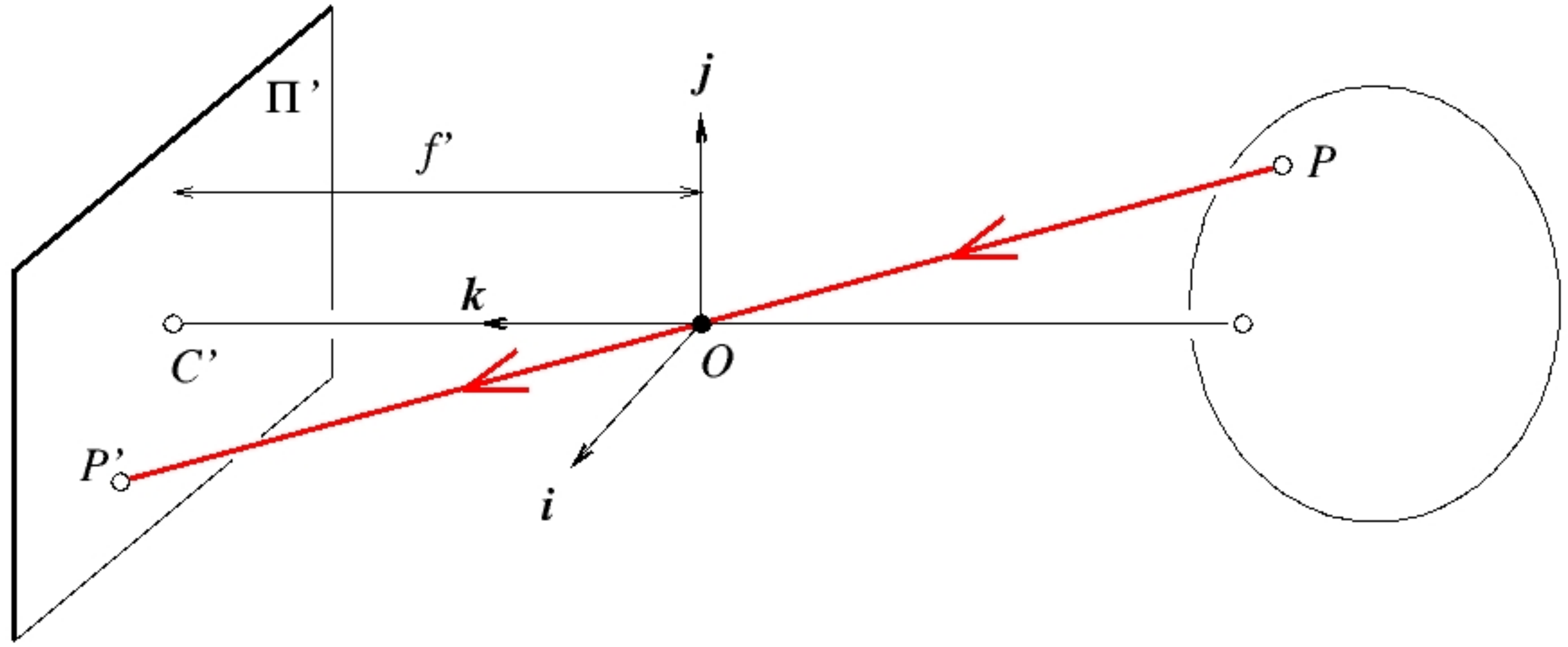
# **Aside:** Camera Matrix




Forsyth & Ponce (1st ed.) Figure 1.4

**Camera** Matrix

$$\mathbf{C} = \begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

3D object point

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$ projects to 2D image point $$P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$ where $$P' = \mathbf{C}P$$

**Note:** this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

# **Aside:** Camera Matrix

$$x' = f'\frac{x}{z}$$
$$y' = f'\frac{y}{z}$$

## **Camera** Matrix

$$\mathbf{C} = \begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$ where $P' = \mathbf{C}P$

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$$x' = f' \frac{x}{z}$$
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**Camera** Matrix

$$\mathbf{C} = \begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} f'x \\ f'y \\ z \end{bmatrix} = \begin{bmatrix} \frac{f'x}{z} \\ \frac{f'y}{z} \\ 1 \end{bmatrix}$$

$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$ where $P' = \mathbf{C}P$

# **Aside:** Camera Matrix

**Camera** Matrix

$$\mathbf{C} = \begin{bmatrix} f' & 0 & 0 & 0 \\ 0 & f' & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Pixels are squared / lens is perfectly symmetric

Sensor and pinhole perfectly aligned

Coordinate system centered at the pinhole

$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$ where $P' = \mathbf{C}P$

# **Aside:** Camera Matrix

**Camera** Matrix

$$\mathbf{C} = \begin{bmatrix} f'_x & 0 & 0 & 0 \\ 0 & f'_y & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

~~Pixels are squared / lens is perfectly symmetric~~

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# **Aside:** Camera Matrix

**Camera** Matrix

$$\mathbf{C} = \begin{bmatrix} f'_x & 0 & 0 & c_x \\ 0 & f'_y & 0 & c_y \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

~~Pixels are squared / lens is perfectly symmetric~~

~~Sensor and pinhole perfectly aligned~~

Coordinate system centered at the pinhole

$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$ where $P' = \mathbf{C}P$

# **Aside:** Camera Matrix

**Camera** Matrix

$$\mathbf{C} = \begin{bmatrix} f'_x & 0 & 0 & c_x \\ 0 & f'_y & 0 & c_y \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbb{R}_{4\times 4}$$

~~Pixels are squared / lens is perfectly symmetric~~

~~Sensor and pinhole perfectly aligned~~

~~Coordinate system centered at the pinhole~~

$$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$ projects to 2D image point $$P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$ where $$P' = \mathbf{C}P$$

# **Aside:** Camera Matrix

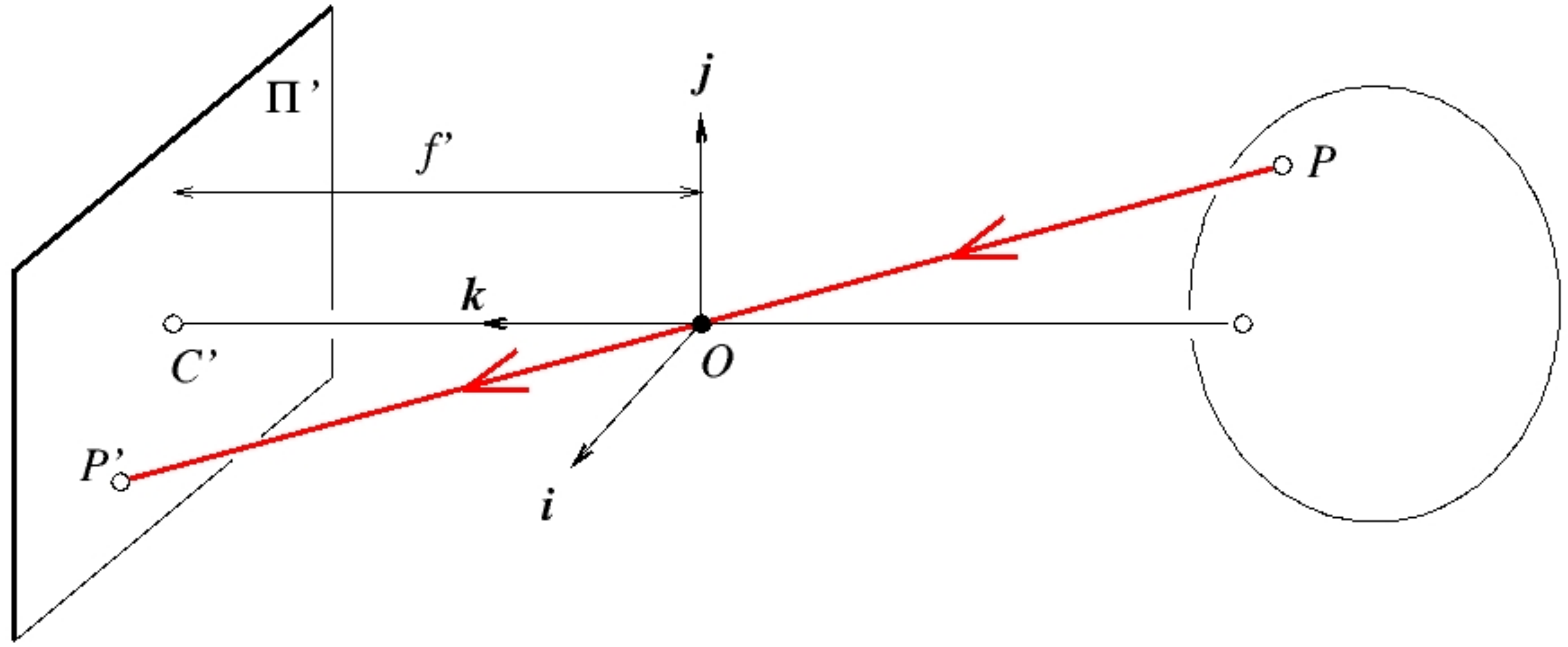
**Camera** Matrix

$$\mathbf{C} = \begin{bmatrix} f'_x & 0 & 0 & c_x \\ 0 & f'_y & 0 & c_y \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbb{R}_{4\times 4}$$

**Camera calibration** is the process of estimating parameters of the camera matrix based on set of 3D-2D correspondences (usually requires a pattern whos structure and size is known)

$P = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$ where $P' = \mathbf{C}P$

# **Perspective** Projection




3D object point

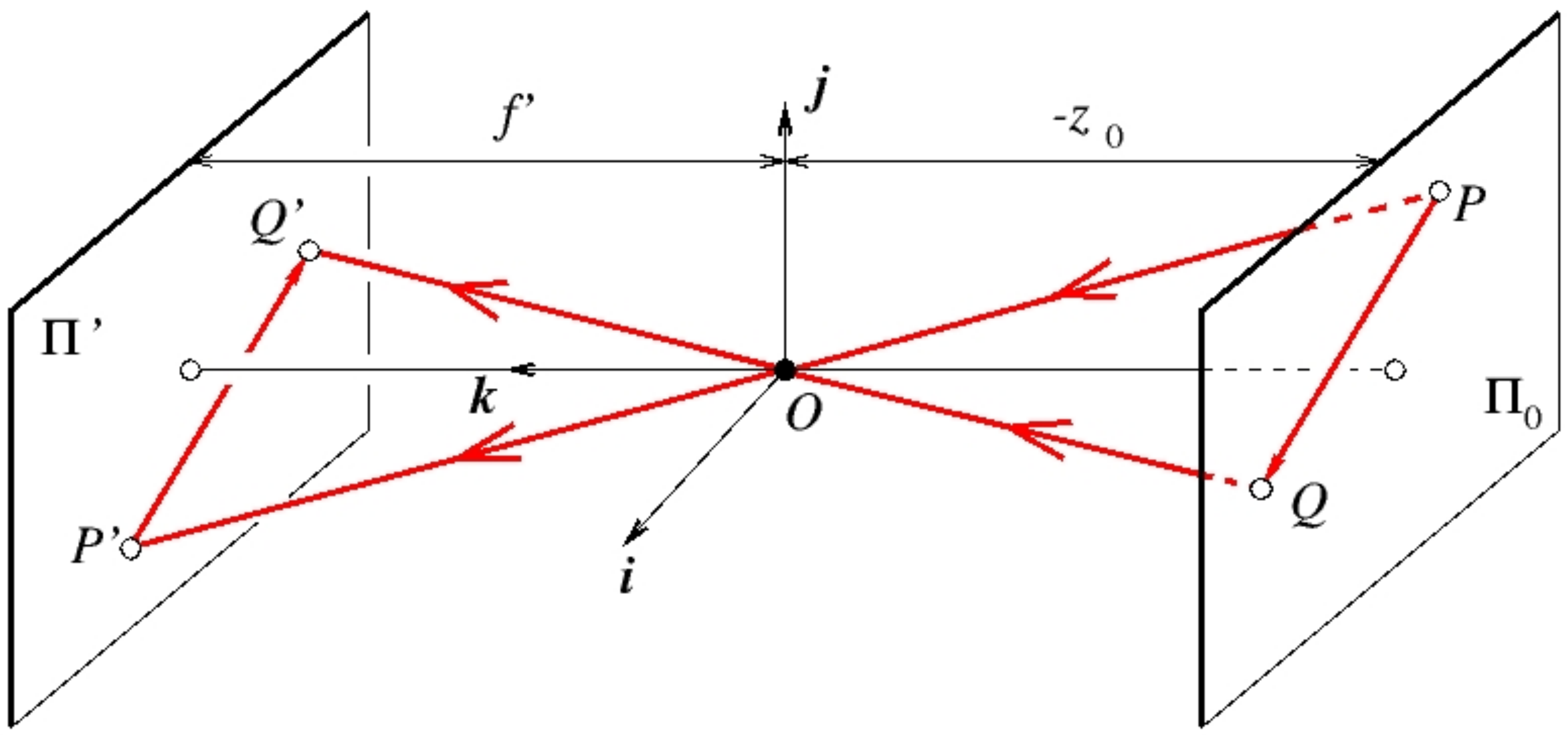
Forsyth & Ponce (1st ed.) Figure 1.4

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$ projects to 2D image point $$P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$ where

$$\begin{aligned} x' &= f' \frac{x}{z} \\ y' &= f' \frac{y}{z} \end{aligned}$$

**Note:** this assumes world coordinate frame at the optical center (pinhole) and aligned with the image plane, image coordinate frame aligned with the camera coordinate frame

# **Weak** Perspective

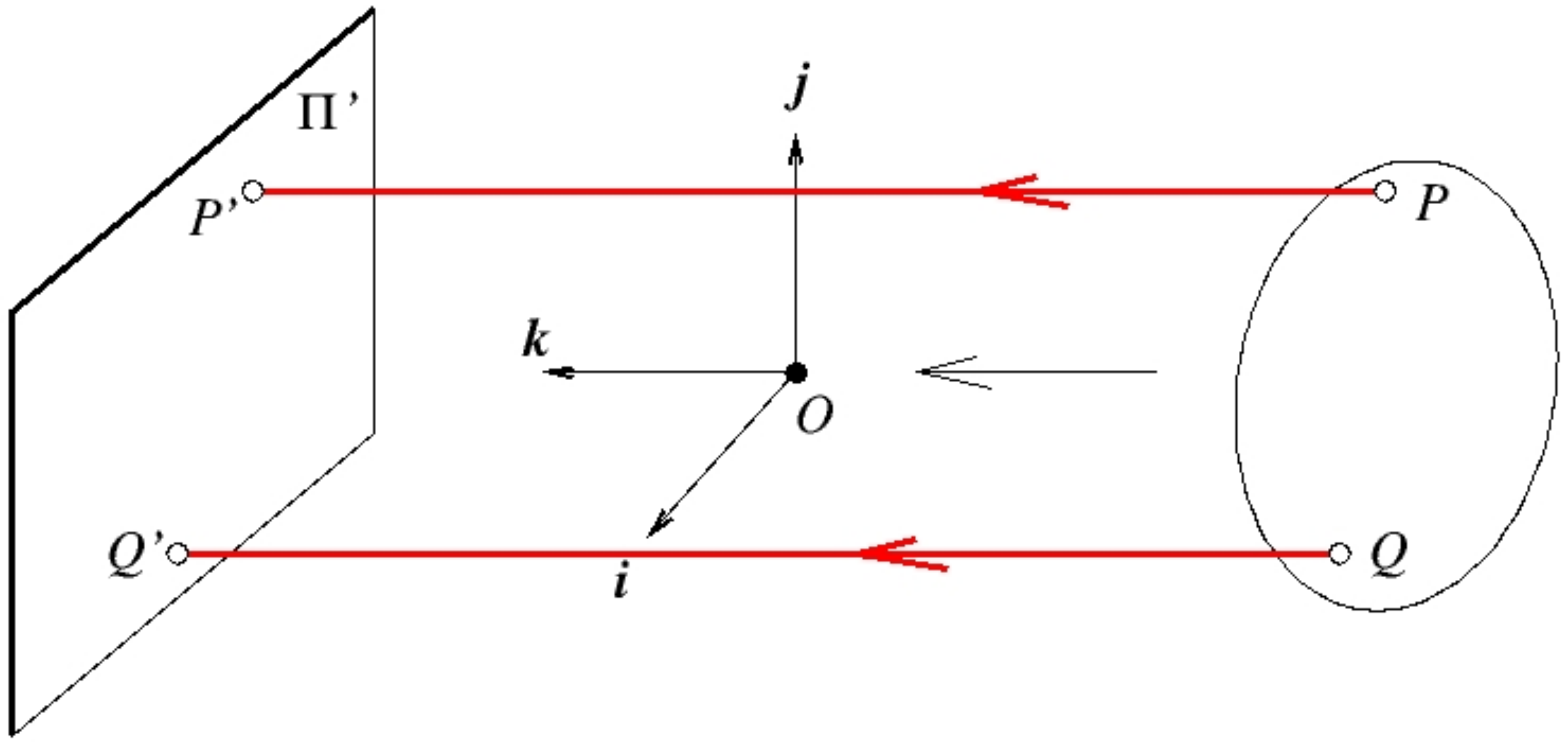



Forsyth & Ponce (1st ed.) Figure 1.5

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in $\Pi_0$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

where $\begin{array}{rcl} x' & = & mx \\ y' & = & my \end{array}$ and $m = \dfrac{f'}{z_0}$

# **Orthographic** Projection




Forsyth & Ponce (1st ed.) Figure 1.6

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

where $\begin{aligned} x' &= x \\ y' &= y \end{aligned}$

# Summary of **Projection Equations**

3D object point $P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ projects to 2D image point $P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$ where

| | | |
|---|---|---|
| Perspective | $x' = f' \frac{x}{z}$<br>$y' = f' \frac{y}{z}$ | |
| Weak Perspective | $x' = m\,x$<br>$y' = m\,y$ | $m = \frac{f'}{z_0}$ |
| Orthographic | $x' = x$<br>$y' = y$ | |

# **Projection Models**: Pros and Cons

**Weak perspective** (including orthographic) has simpler mathematics

— accurate when object is small and/or distant

— useful for recognition

**Perspective** is more accurate for real scenes

When **maximum accuracy** is required, it is necessary to model additional details of a particular camera

— use perspective projection with additional parameters (e.g., lens distortion)

# Why **Not** a Pinhole Camera?

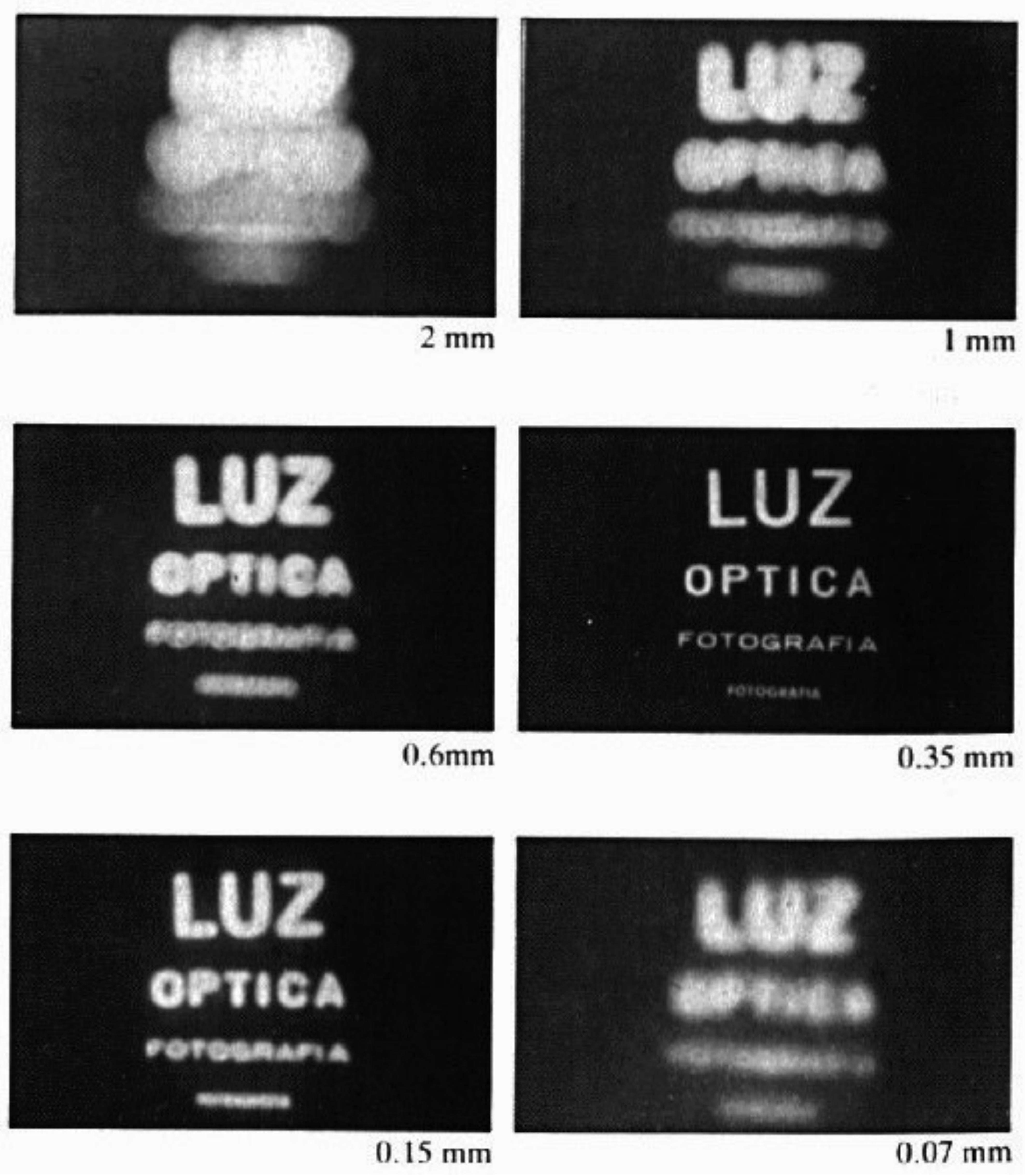



**Image Credit**: Credit: E. Hecht. “Optics,” Addison-Wesley, 1987

# Why **Not** a Pinhole Camera?

– If pinhole is **too big** then many directions are averaged, blurring the image

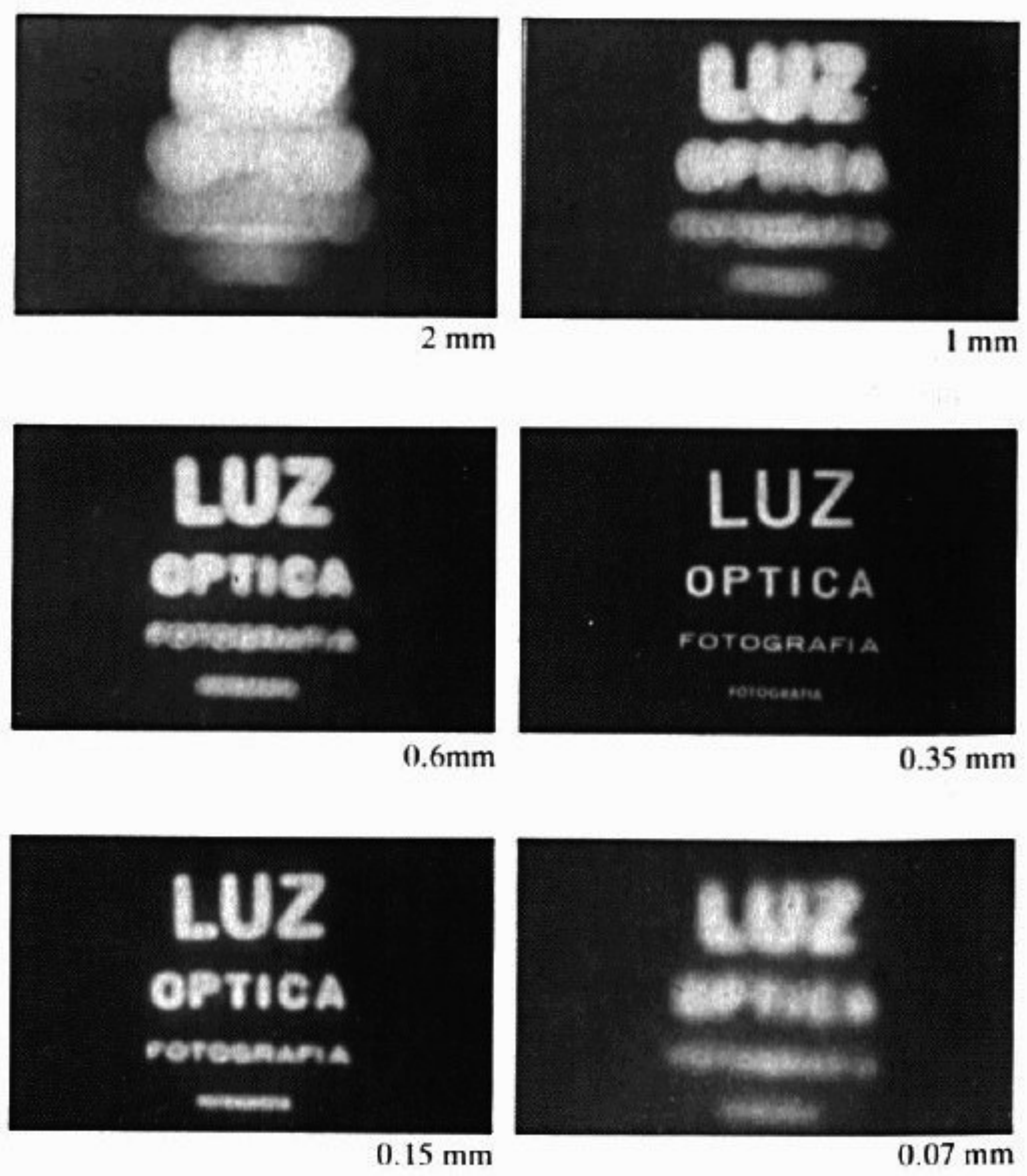



**Image Credit**: Credit: E. Hecht. "Optics," Addison-Wesley, 1987

# Why **Not** a Pinhole Camera?

– If pinhole is **too big** then many directions are averaged, blurring the image

– If pinhole is **too small** then diffraction becomes a factor, also blurring the image

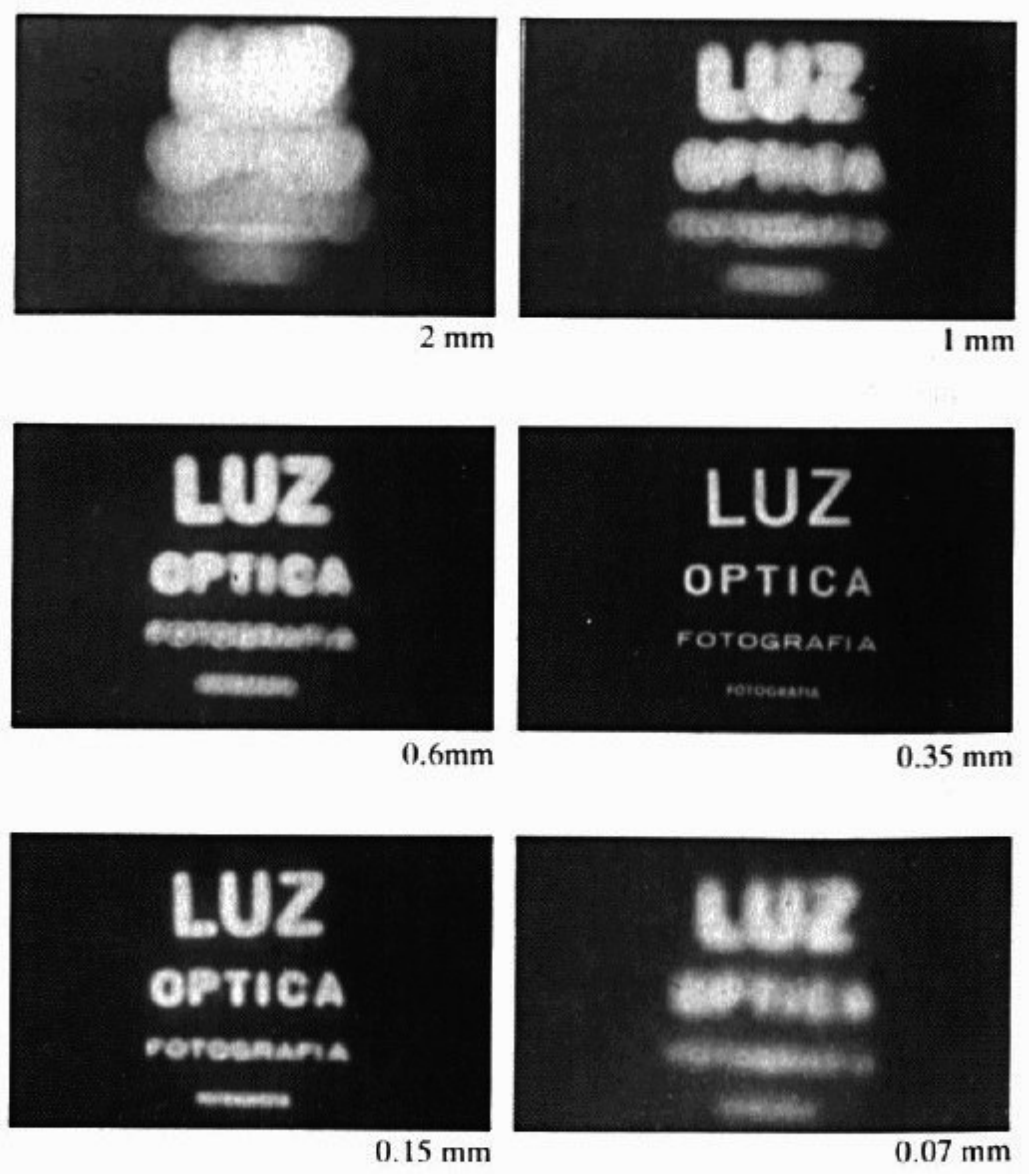


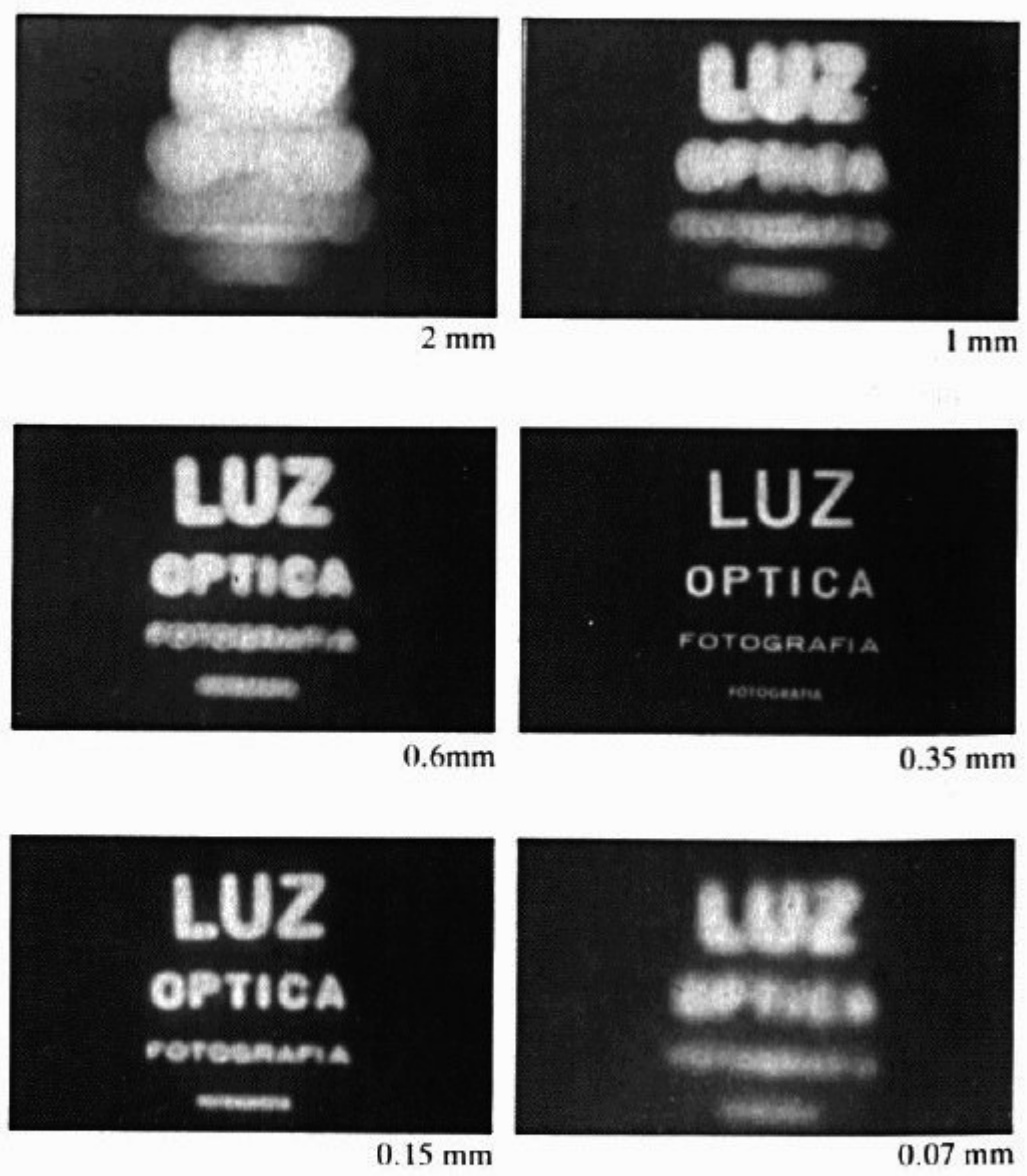

**Image Credit**: Credit: E. Hecht. "Optics," Addison-Wesley, 1987

# Why **Not** a Pinhole Camera?

– If pinhole is **too big** then many directions are averaged, blurring the image

– If pinhole is **too small** then diffraction becomes a factor, also blurring the image

– Generally, pinhole cameras are **dark**, because only a very small set of rays from a particular scene point hits the image plane

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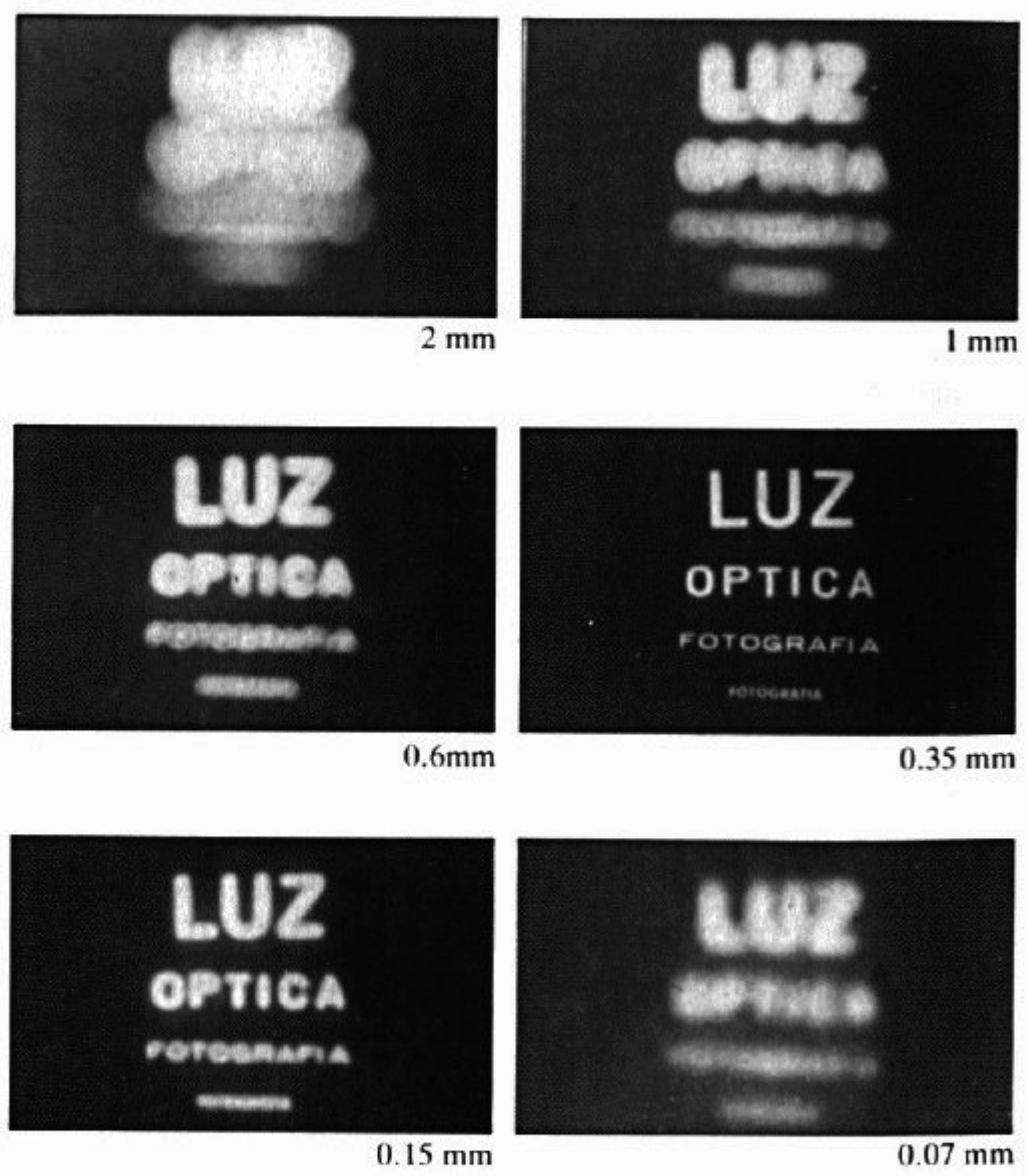
# Why **Not** a Pinhole Camera?

– If pinhole is **too big** then many directions are averaged, blurring the image

– If pinhole is **too small** then diffraction becomes a factor, also blurring the image

– Generally, pinhole cameras are **dark**, because only a very small set of rays from a particular scene point hits the image plane

– Pinhole cameras are **slow**, because only a very small amount of light from a particular scene point hits the image plane per unit time

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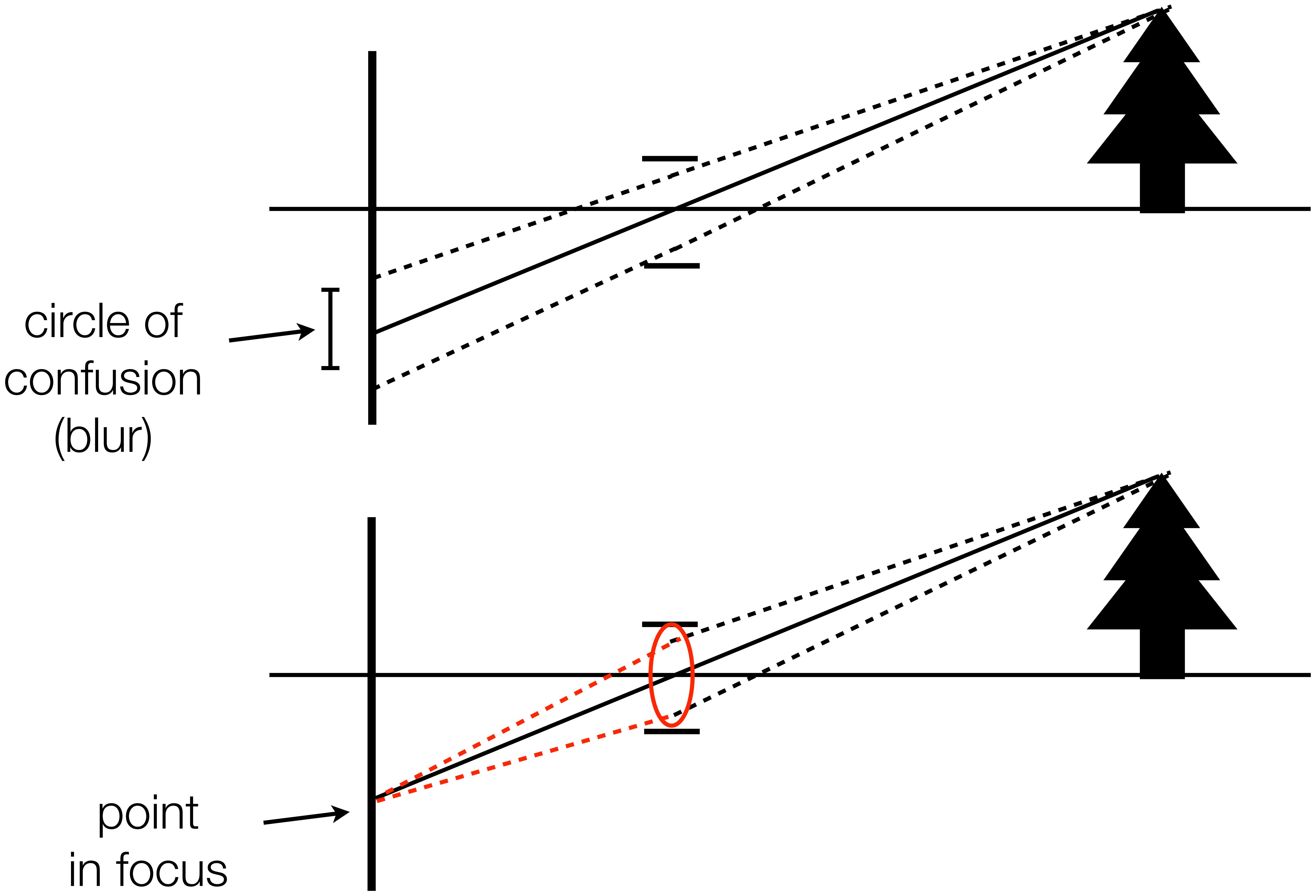
# Reason for **Lenses**

A real camera must have a finite aperture to get enough light, but this causes blur in the image




**Solution**: use a **lens** to focus light onto the image plane

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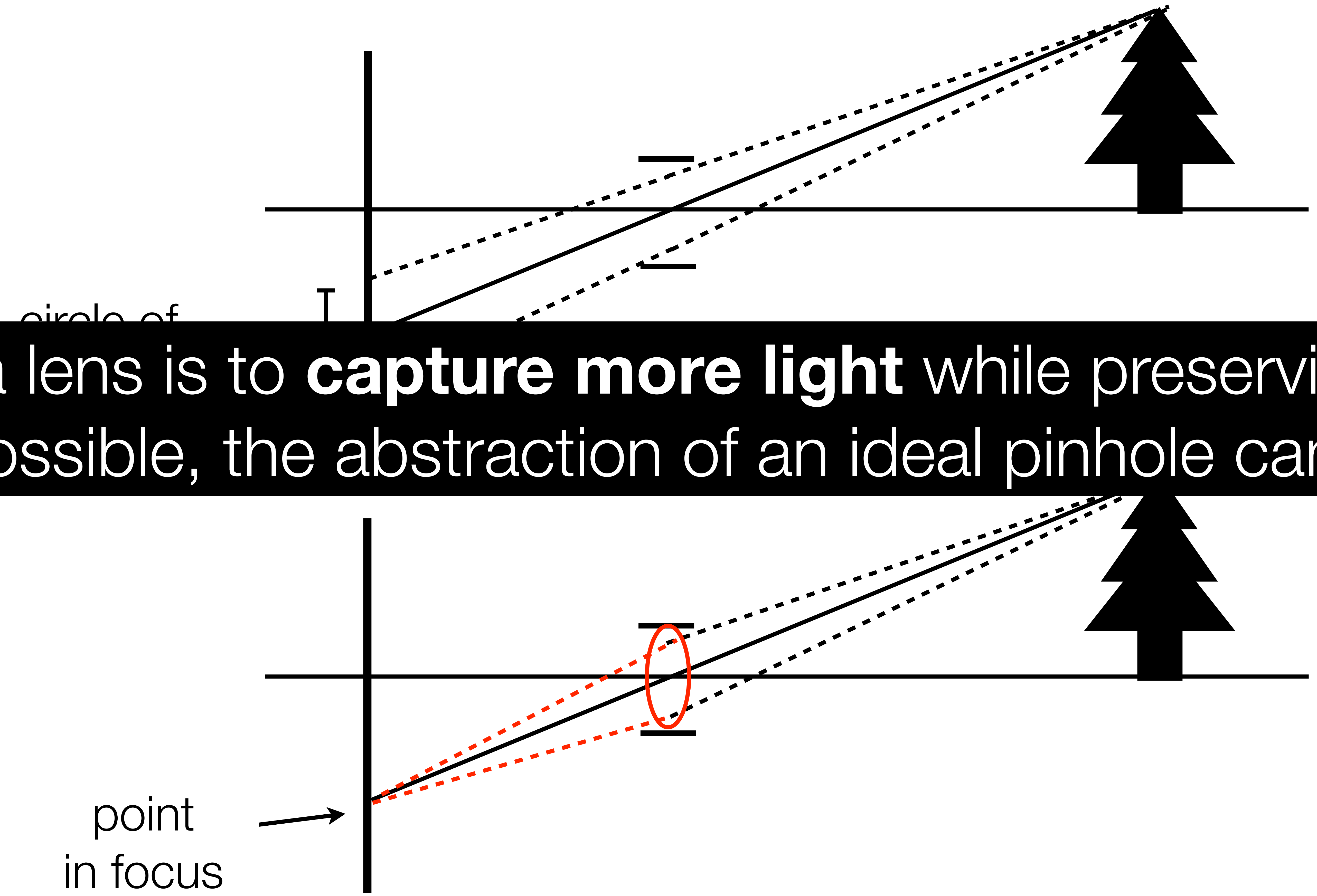
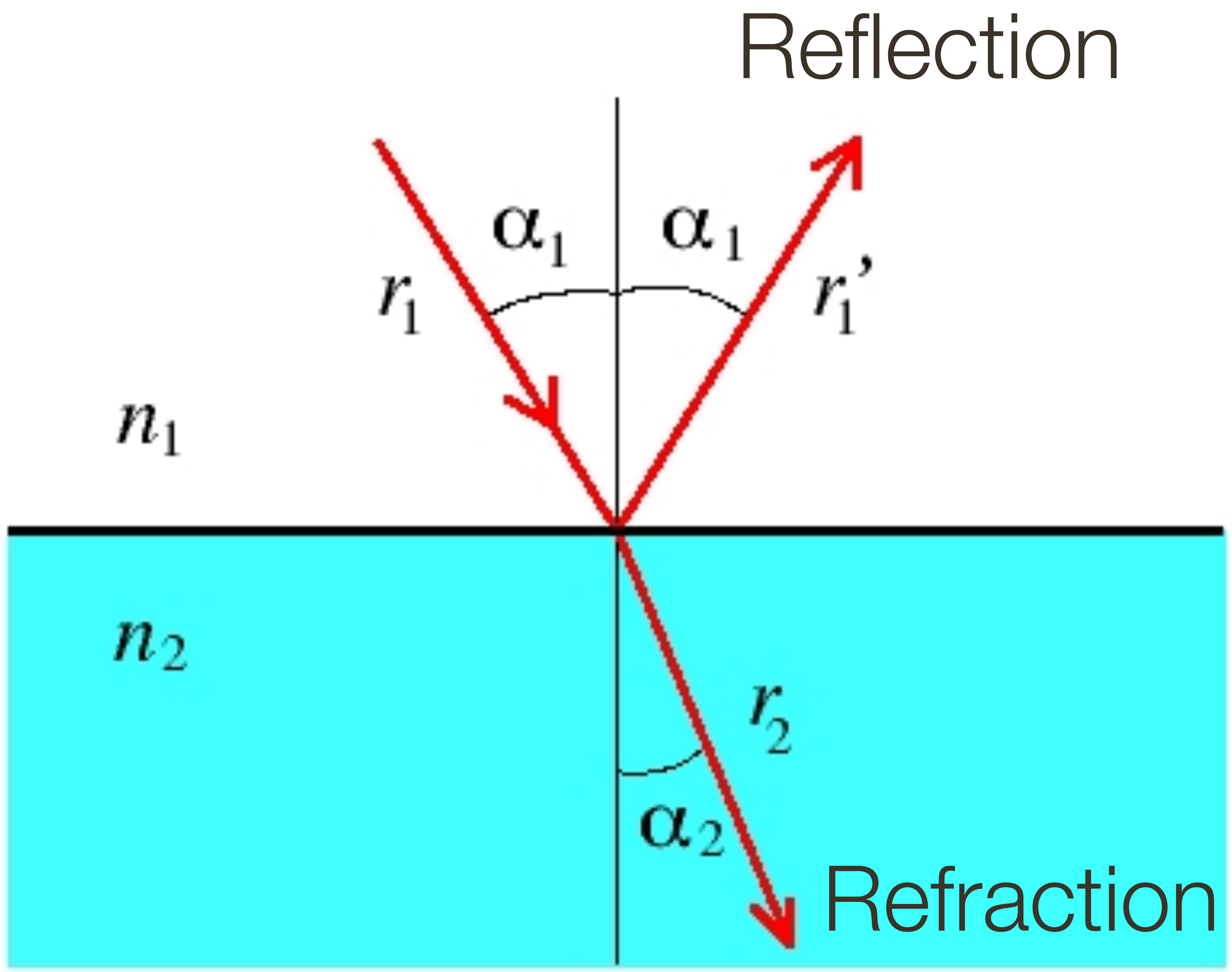
A real camera must have a finite aperture to get enough light, but this causes blur in the image




The role of a lens is to **capture more light** while preserving, as much as possible, the abstraction of an ideal pinhole camera.

**Solution**: use a **lens** to focus light onto the image plane

# **Snell's** Law



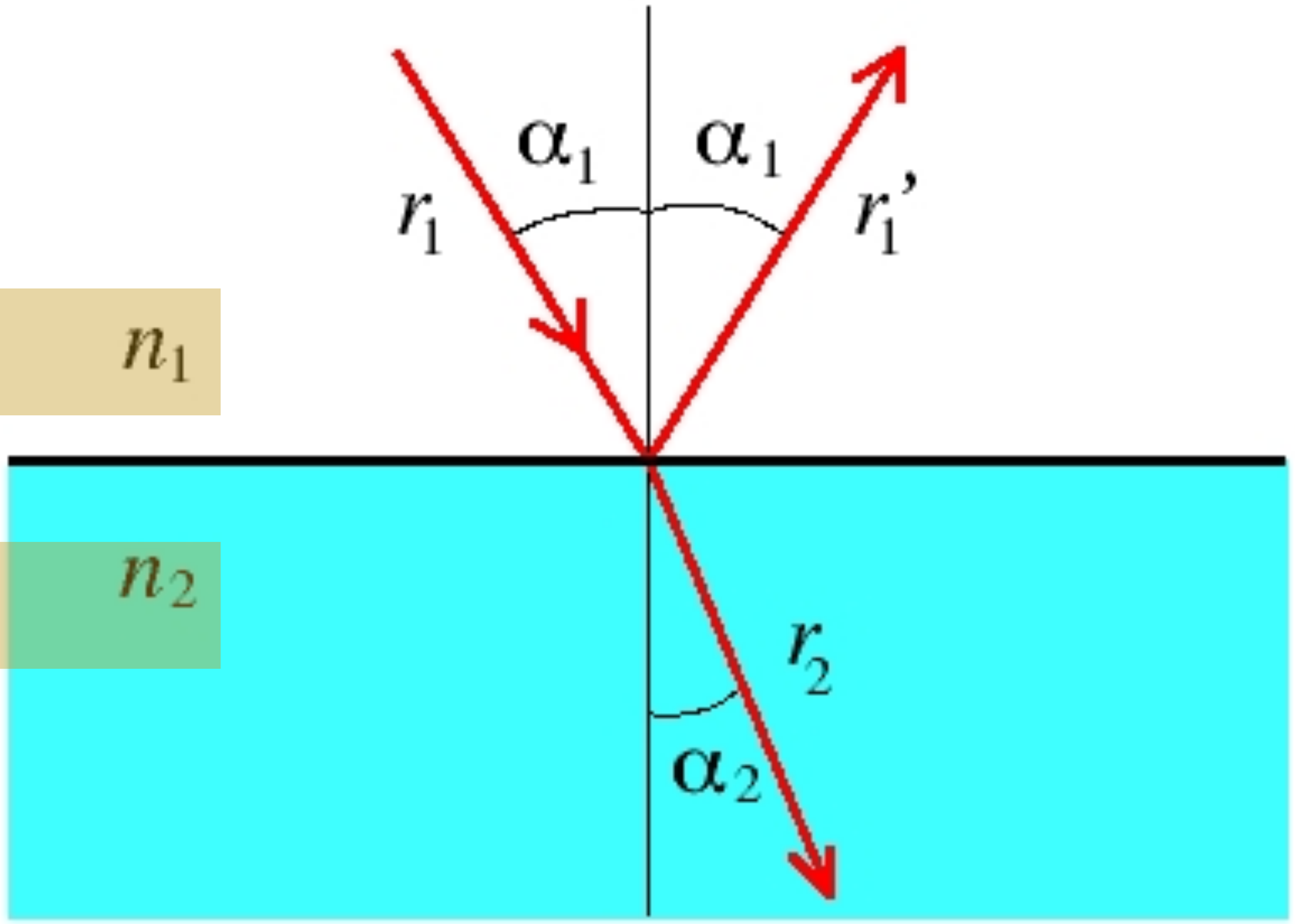

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

# **Snell's** Law

$n_1$

Index of **refraction**

$n_2$




$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

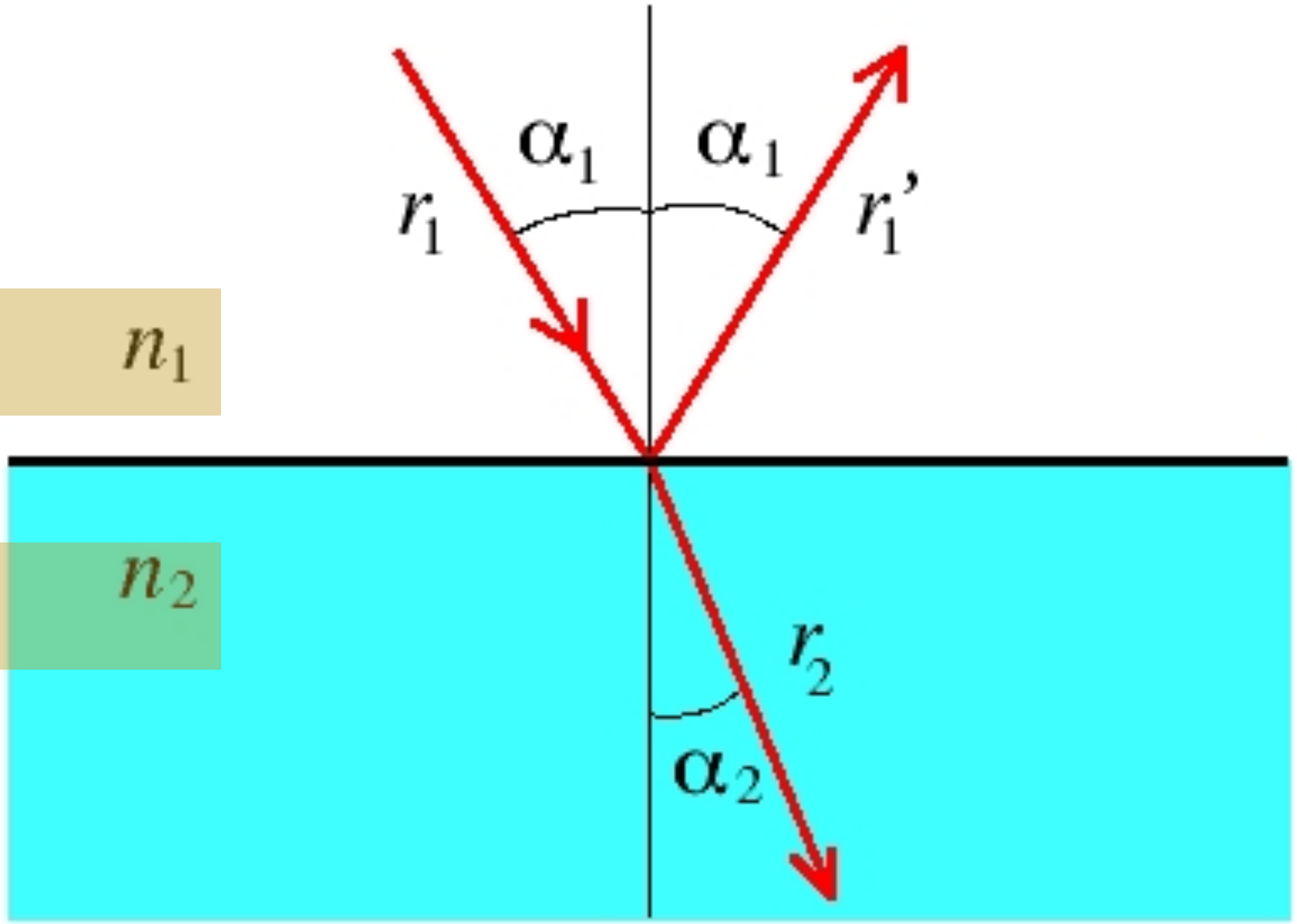
# **Snell's** Law

**Exercise:** Would it make sense to make the lens from material who's index of refraction equals to air? Why?

$n_1$

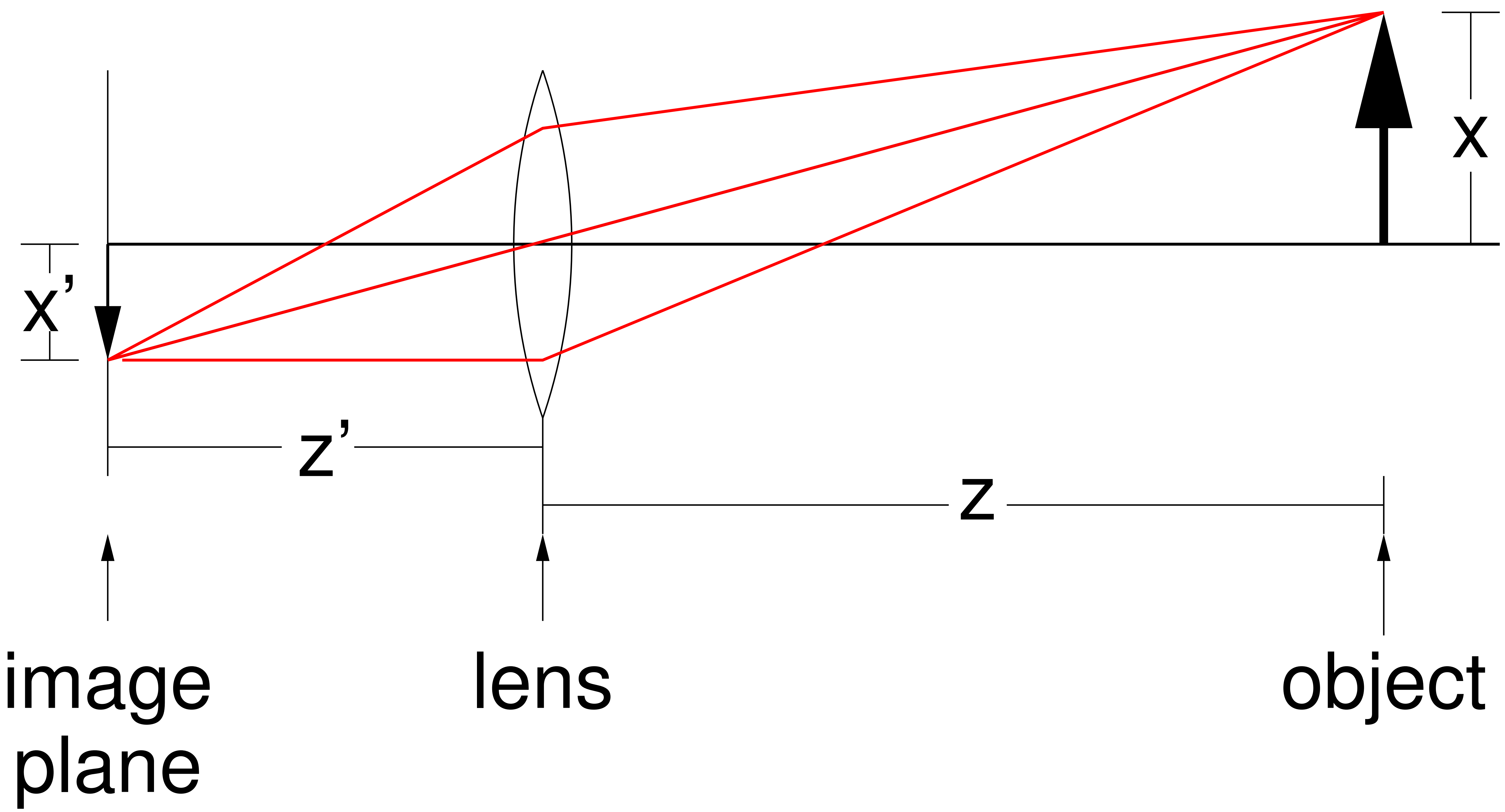
Index of **refraction**

$n_2$

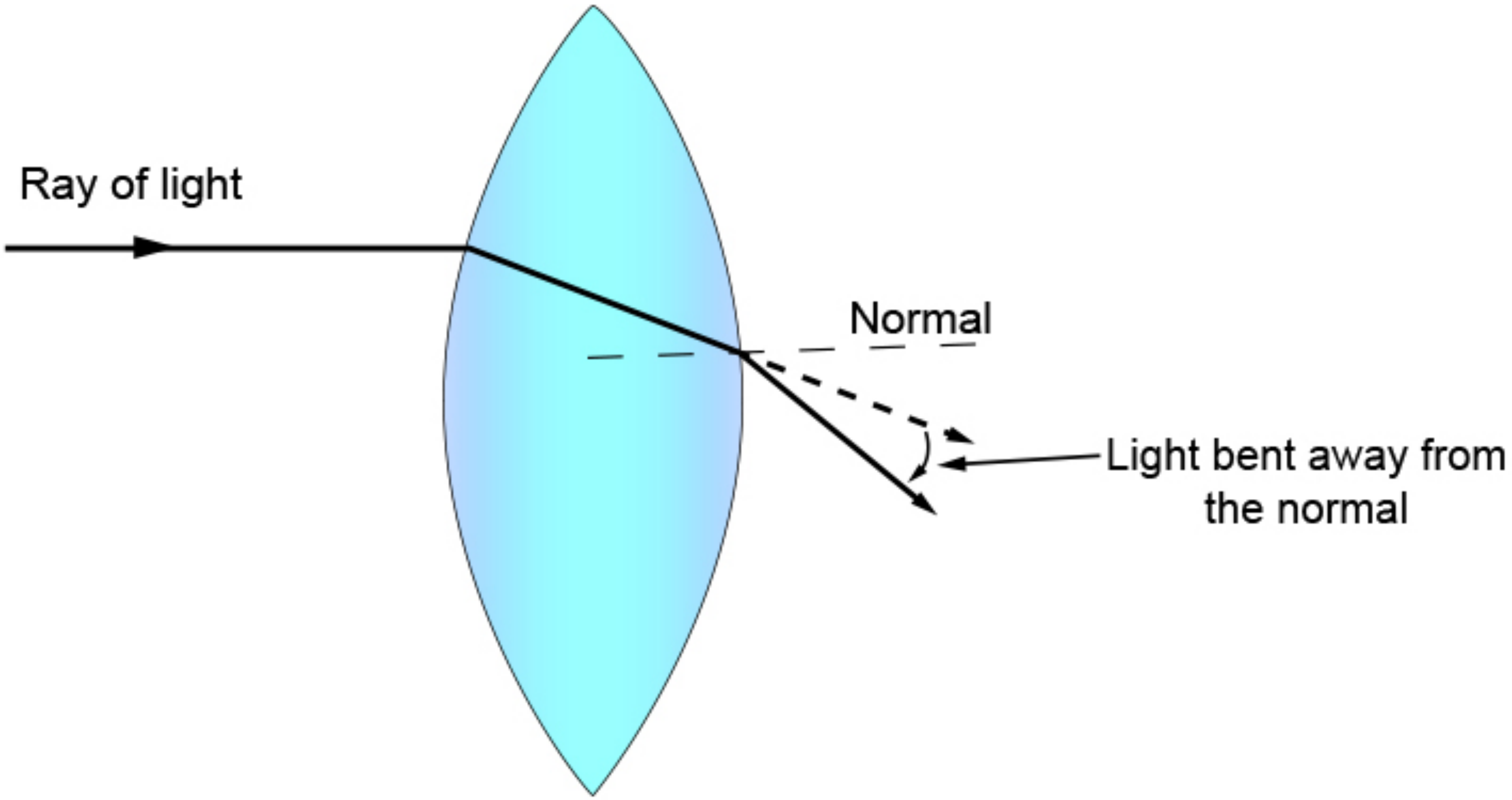



$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

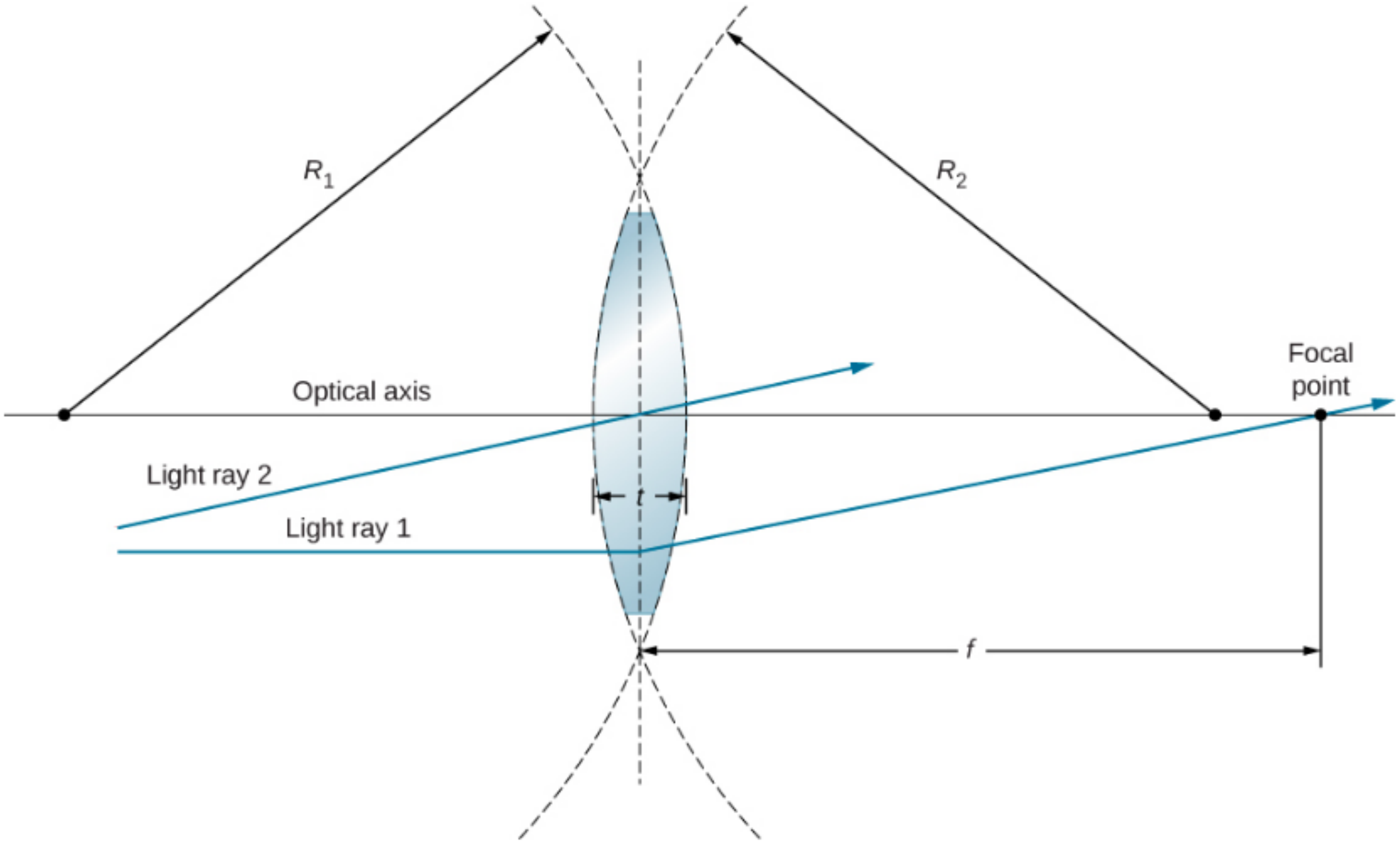
# Pinhole Model **with Lens**



# **General** Lens

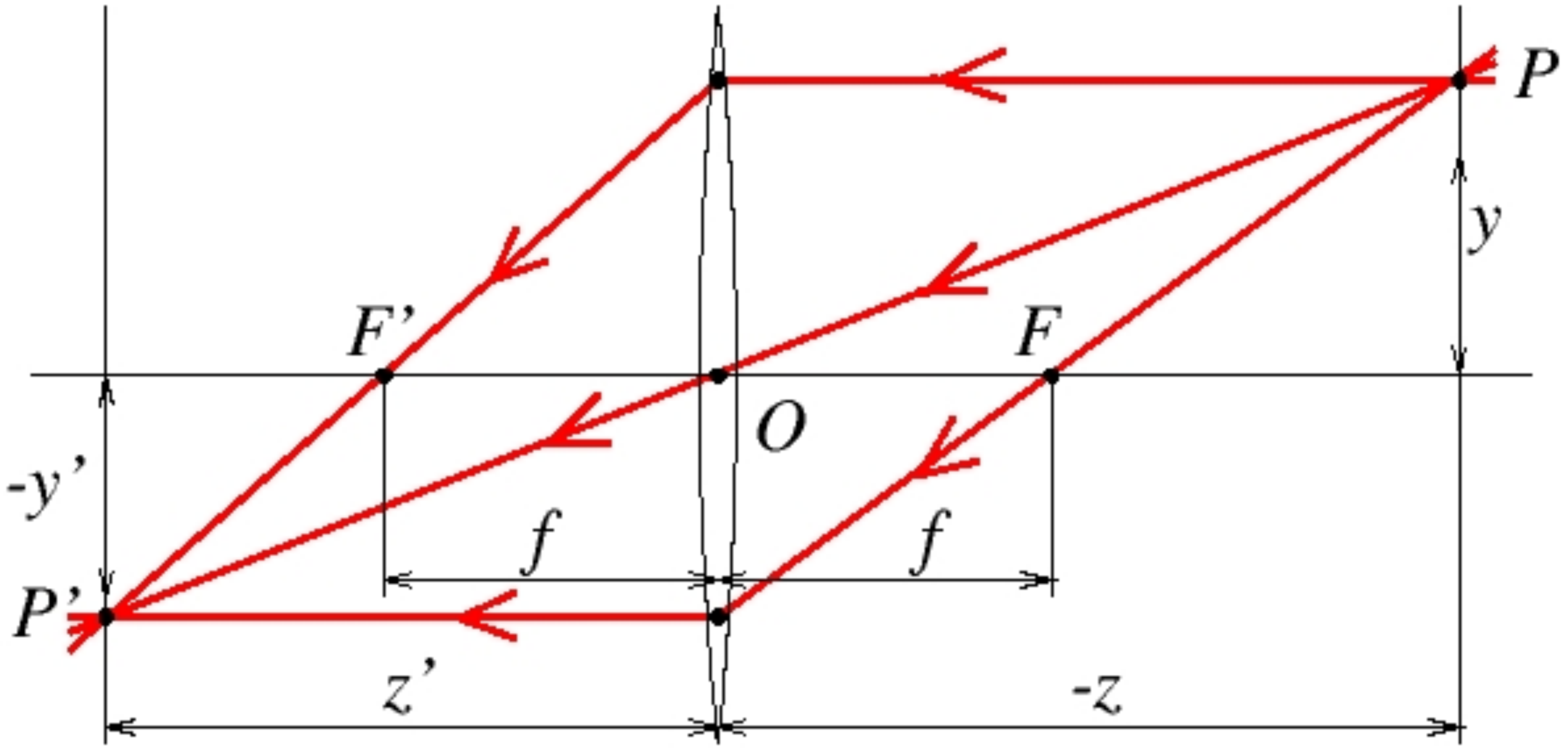


# **Thin** Lens




https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Map%3A_University_Physics_III_-_Optics_and_Modern_Physics_(OpenStax)/02%3A_Geometric_Optics_and_Image_Formation/2.05%3A_Thin_Lenses

# **Thin Lens** Equation

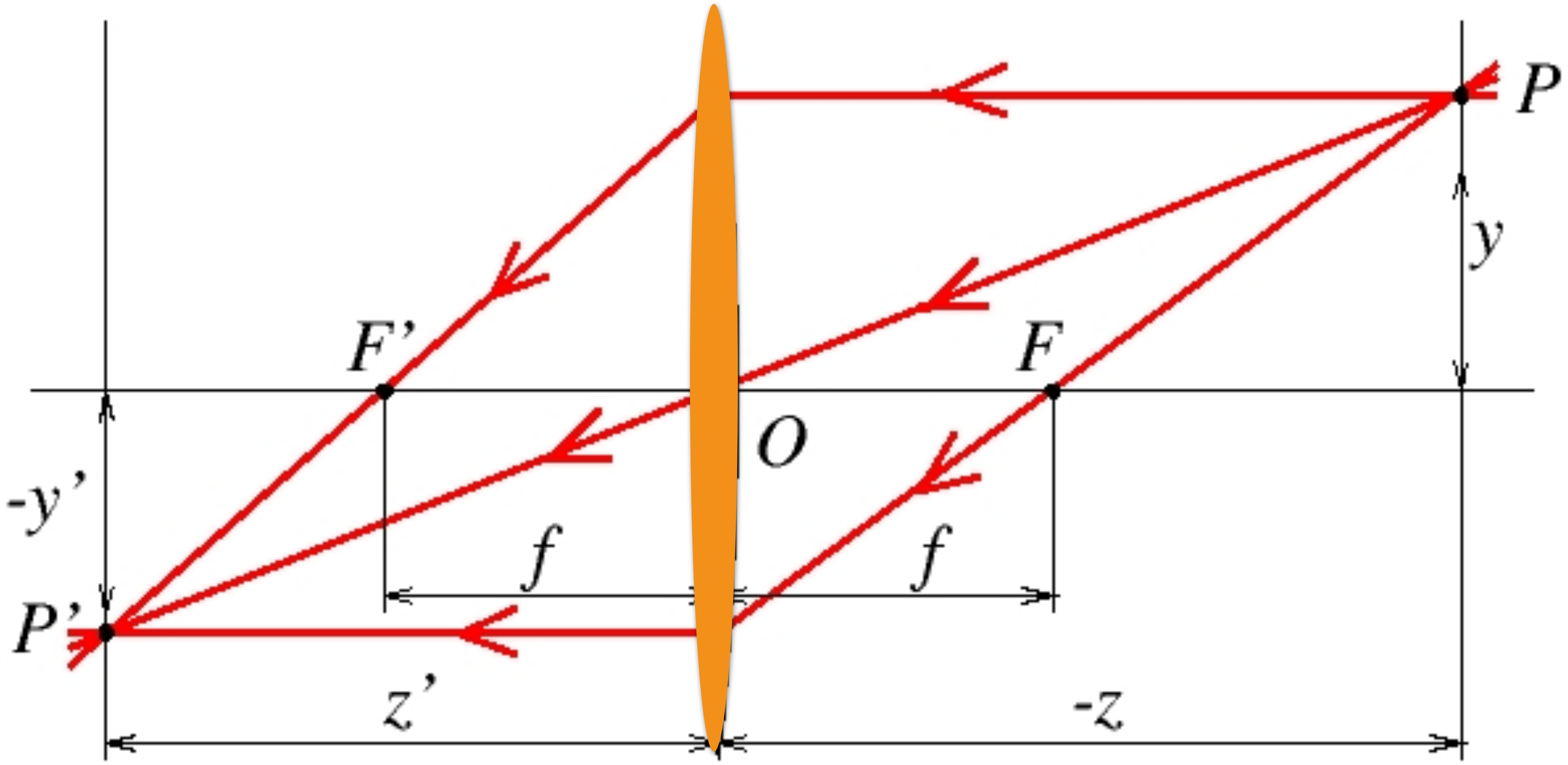



Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

# **Thin Lens** Equation

**Focal Length**: Property of the lens (geometry and refraction index)

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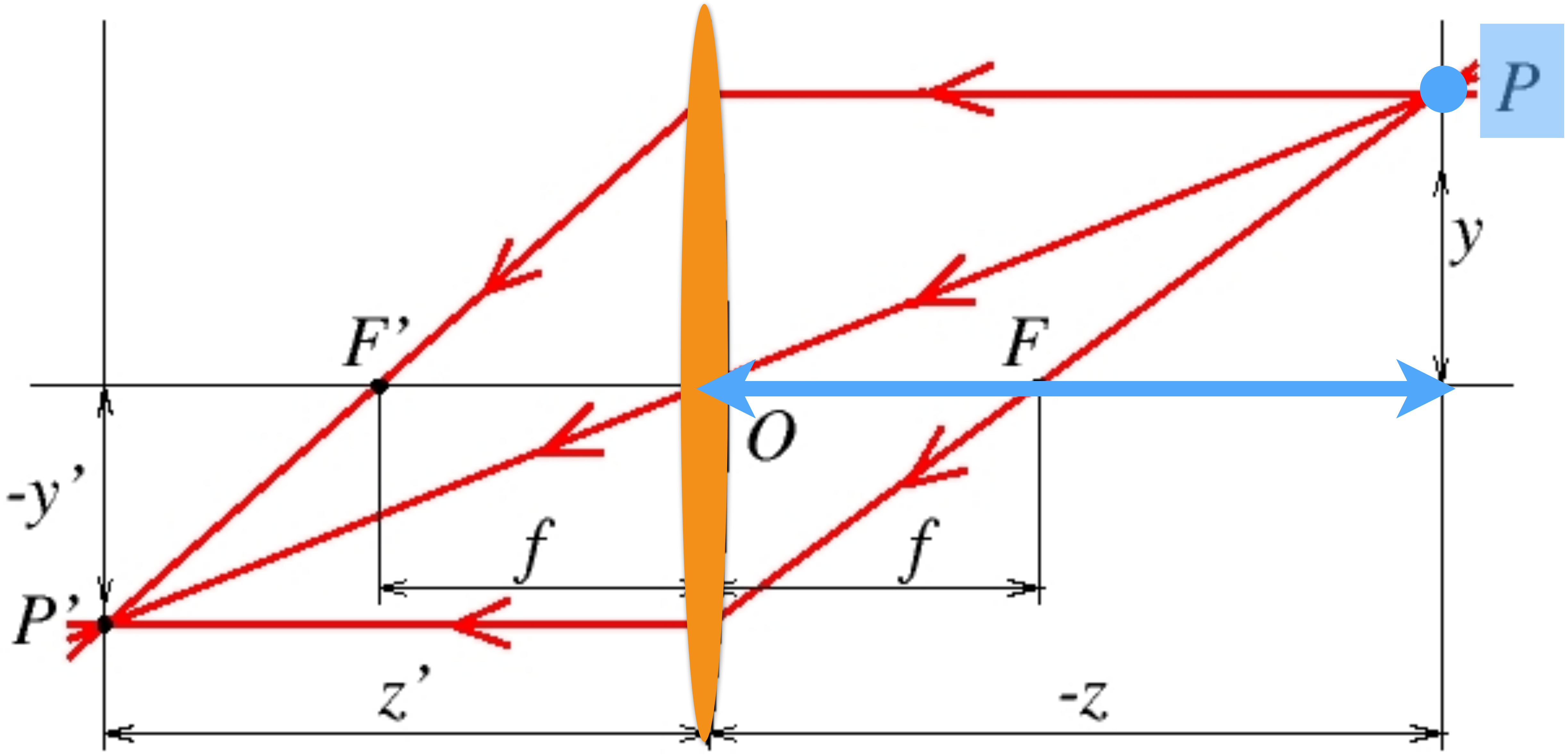



Depth of the point (P) in the world

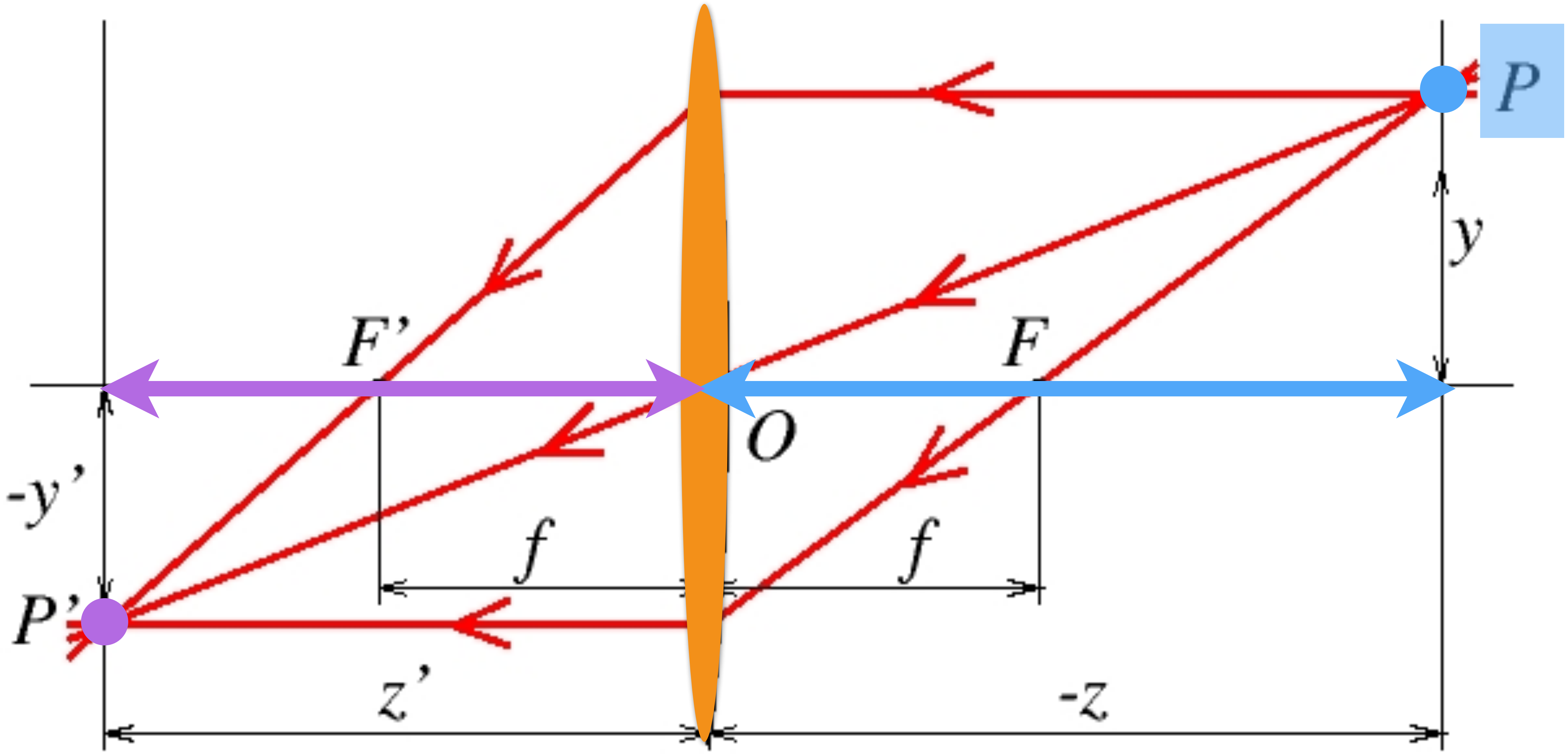
Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

# **Thin Lens** Equation

**Focal Length**: Property of the lens (geometry and refraction index)

Location of the imaging plane where the projection of this point (P) will be in focus

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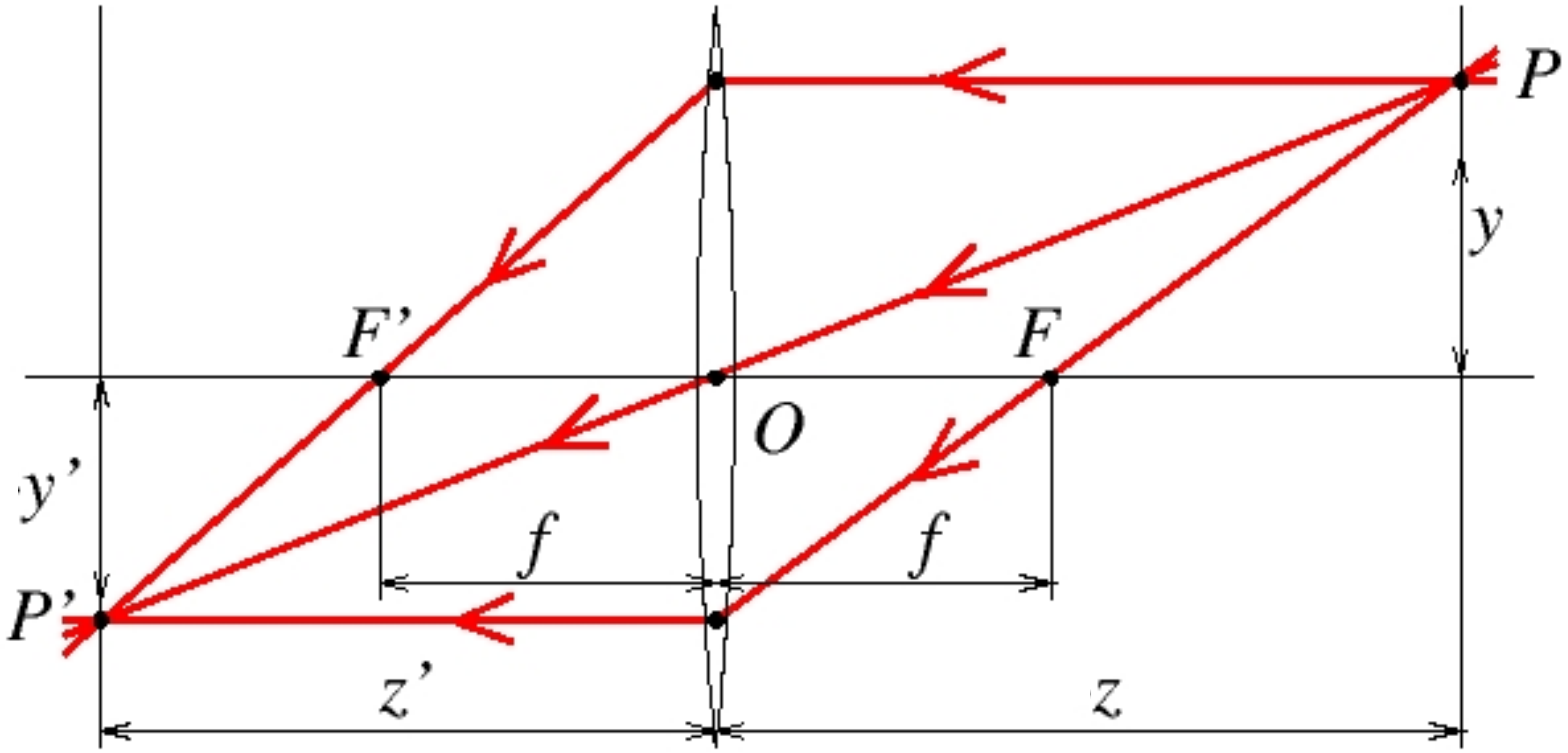
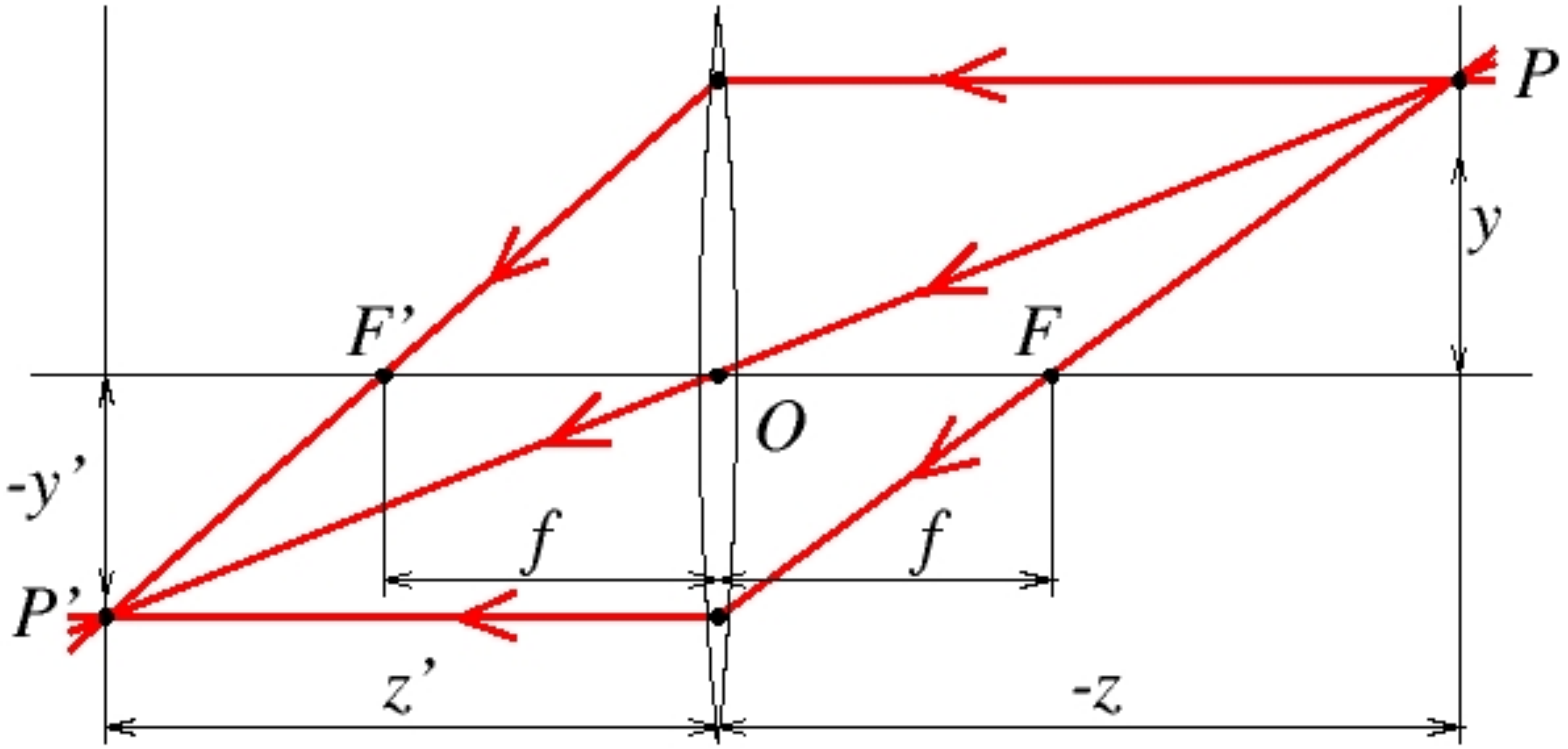



Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} + \frac{1}{z} = \frac{1}{f}$$

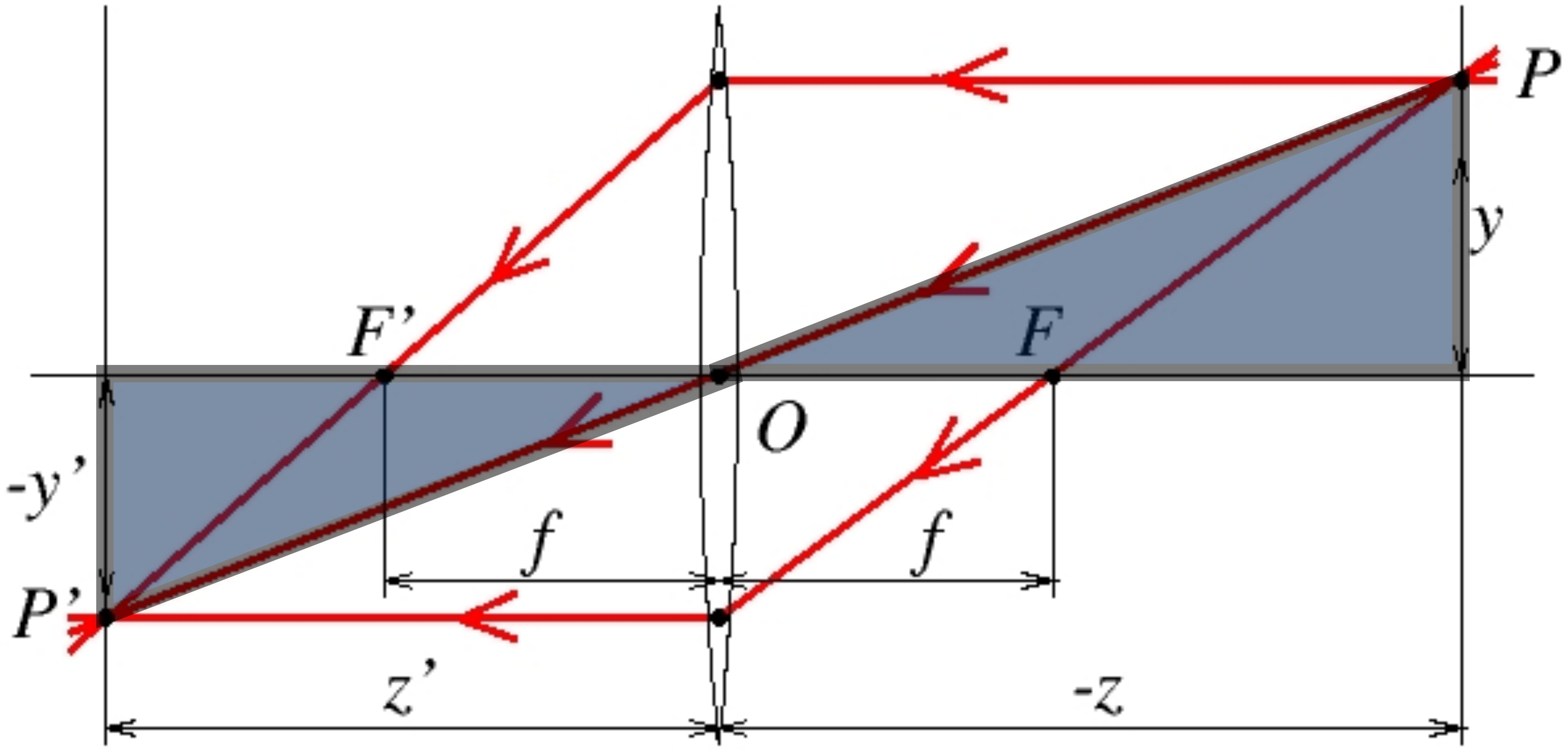
# **Thin Lens** Equation




Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

# **Thin Lens** Equation: Derivation



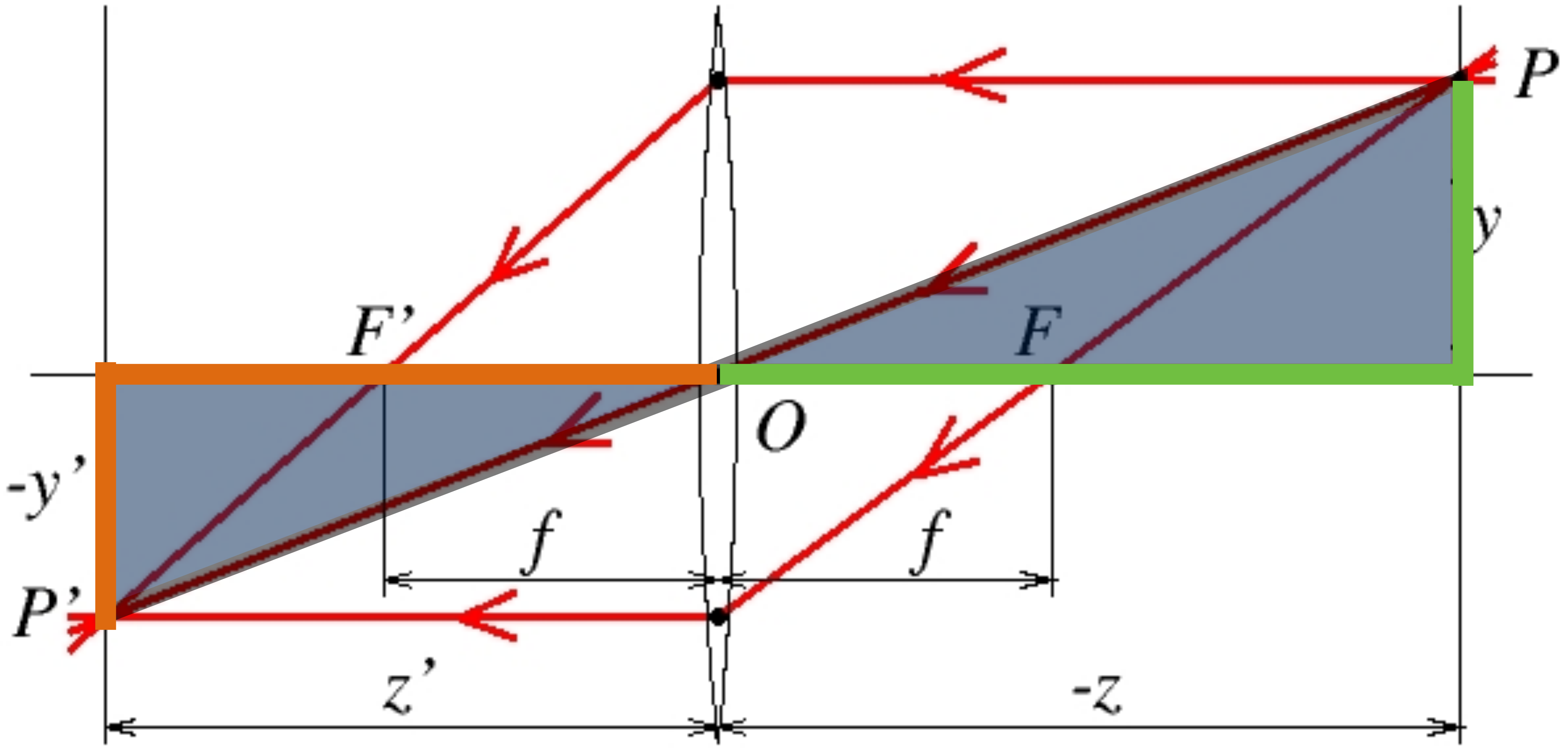

Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

# **Thin Lens** Equation: Derivation

$$\frac{y}{-z} = \frac{-y'}{z'}$$

$$\frac{y}{y'} = \frac{z}{z'}$$




Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

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$$\frac{y}{-z} = \frac{-y'}{z'}$$

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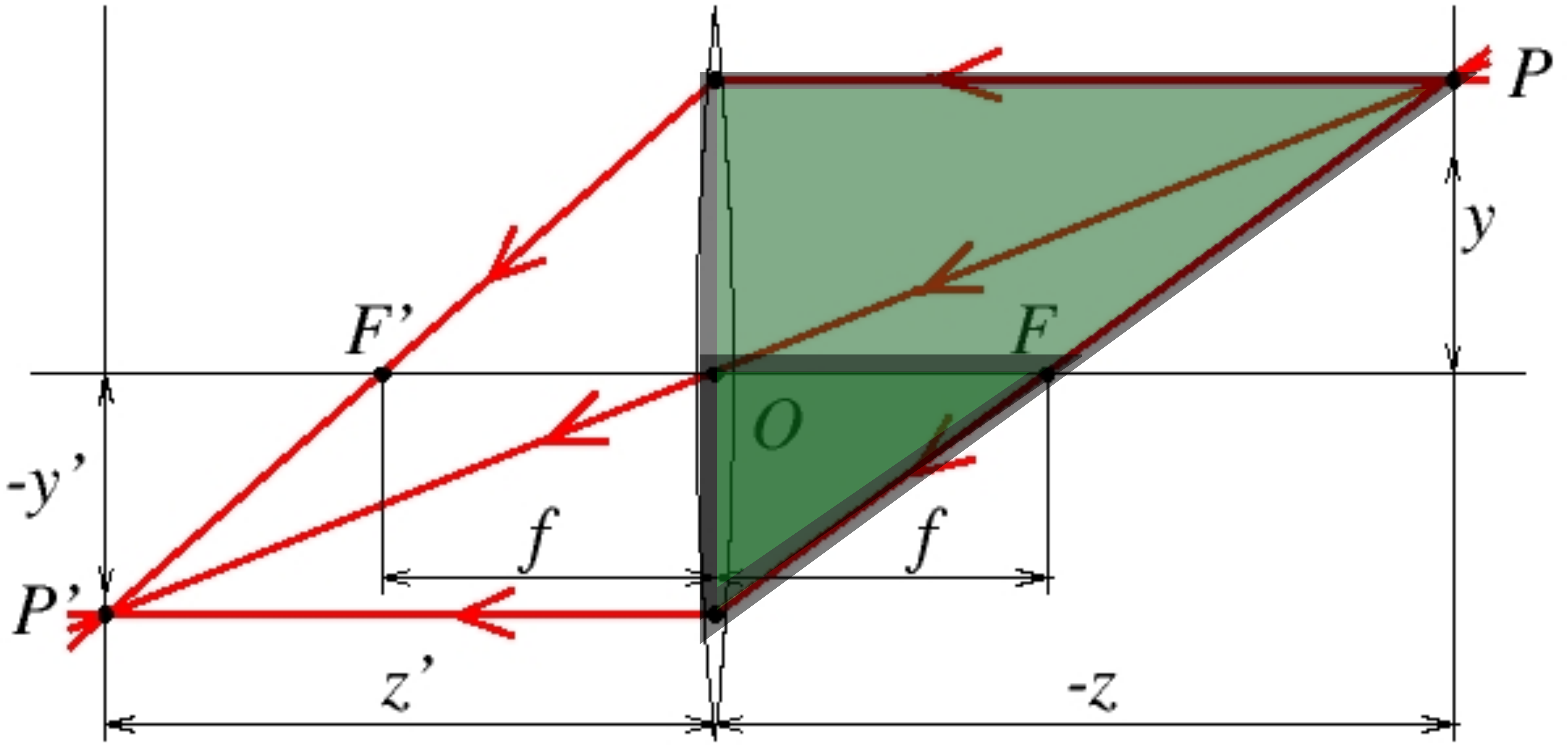


Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

# **Thin Lens** Equation: Derivation

$$\frac{y}{-z} = \frac{-y'}{z'}$$

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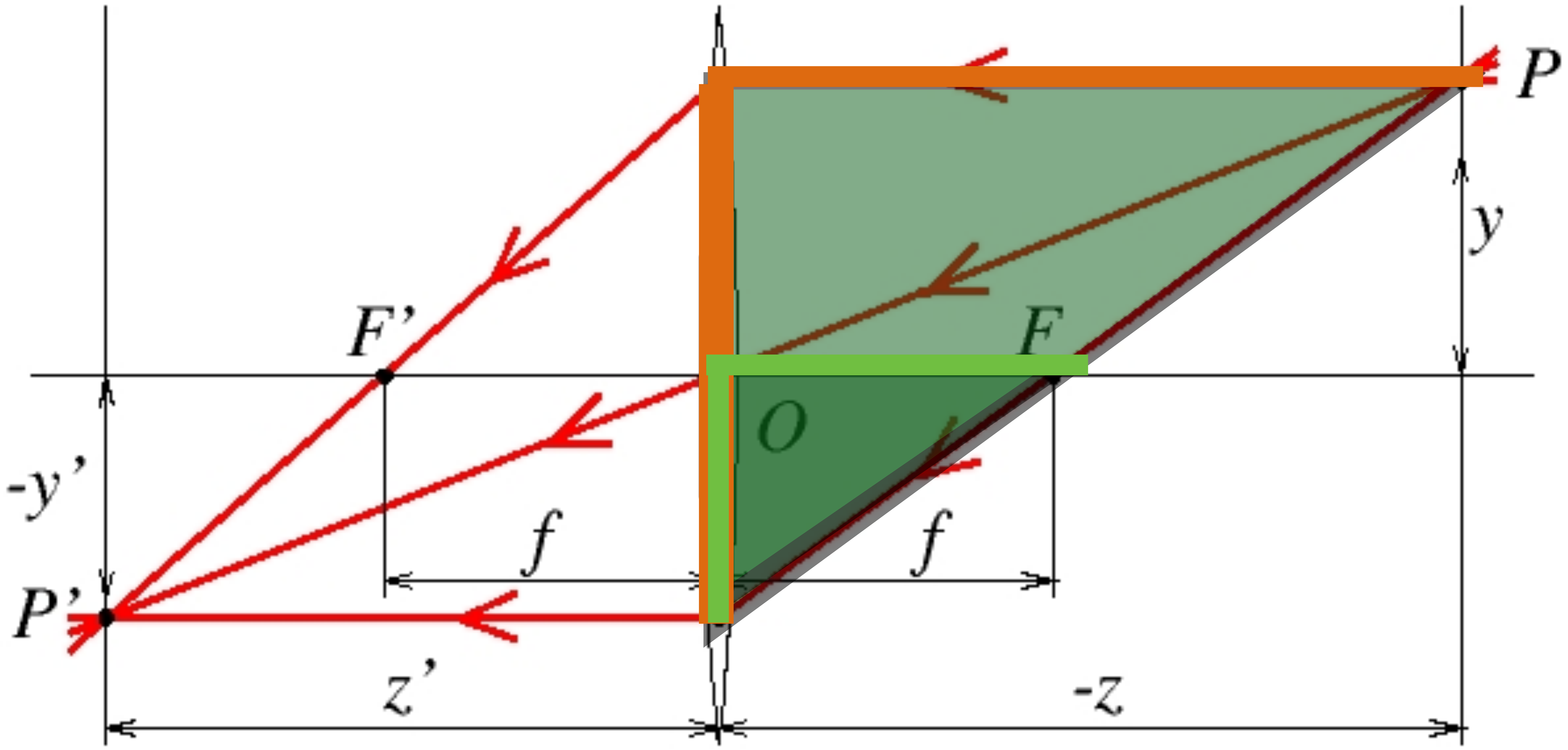


Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{-y'}{f} = \frac{y - y'}{-z}$$

$$\frac{1}{f} = \frac{y - y'}{zy'}$$

$$= \frac{y}{zy'} - \frac{y'}{zy'}$$

$$= \frac{y}{zy'} - \frac{1}{z}$$

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

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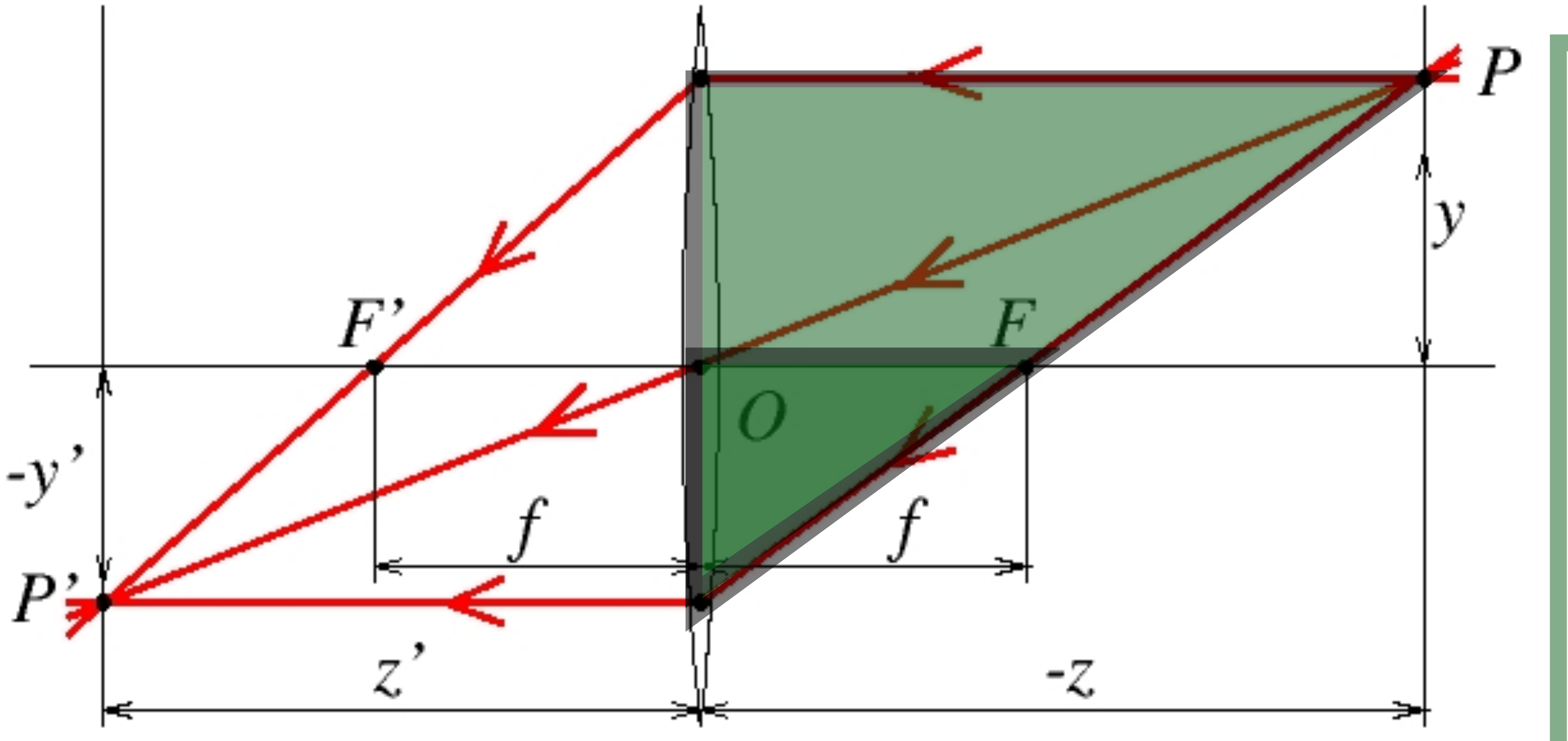



Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{-y'}{f} = \frac{y - y'}{-z}$$

$$\frac{1}{f} = \frac{y - y'}{zy'}$$

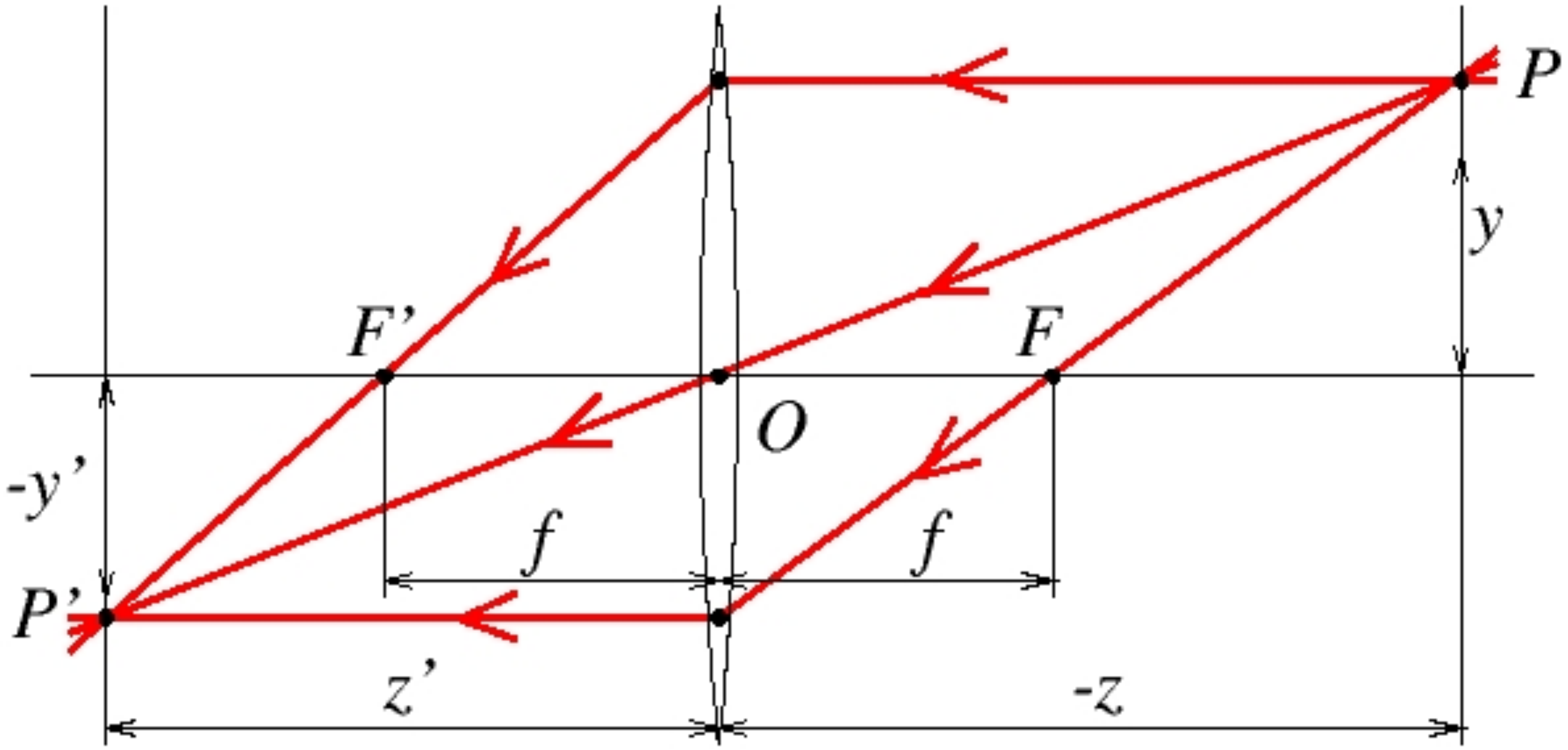
$$= \frac{y}{zy'} - \frac{y'}{zy'}$$

$$= \frac{y}{zy'} - \frac{1}{z}$$

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Substitute: $\frac{1}{f} = \frac{1}{z}\frac{z}{z'} - \frac{1}{z}$

# **Possible Uses** of Thin Lens Abstraction

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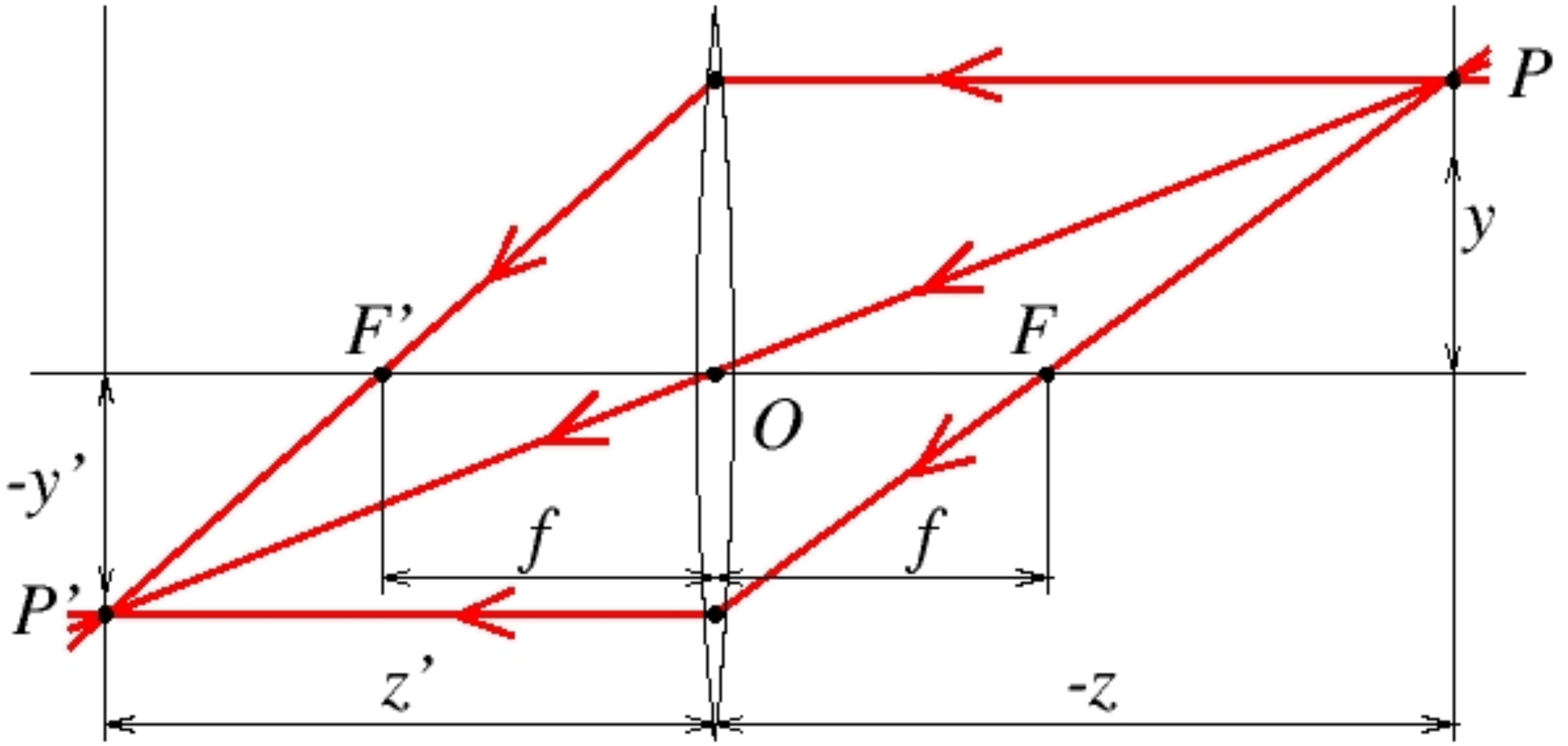
$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

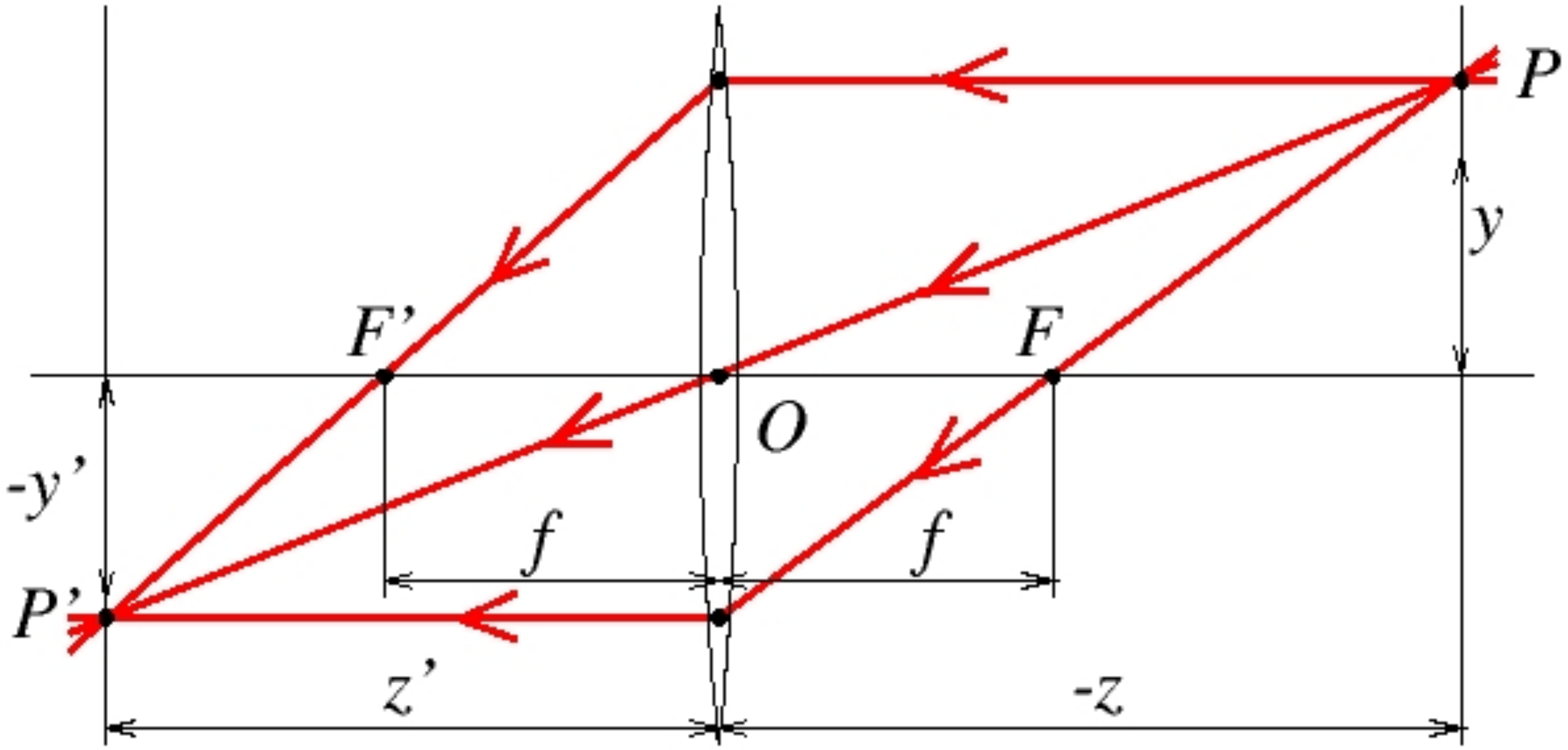
# **Possible Uses** of Thin Lens Abstraction




Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

# **Possible Uses** of Thin Lens Abstraction



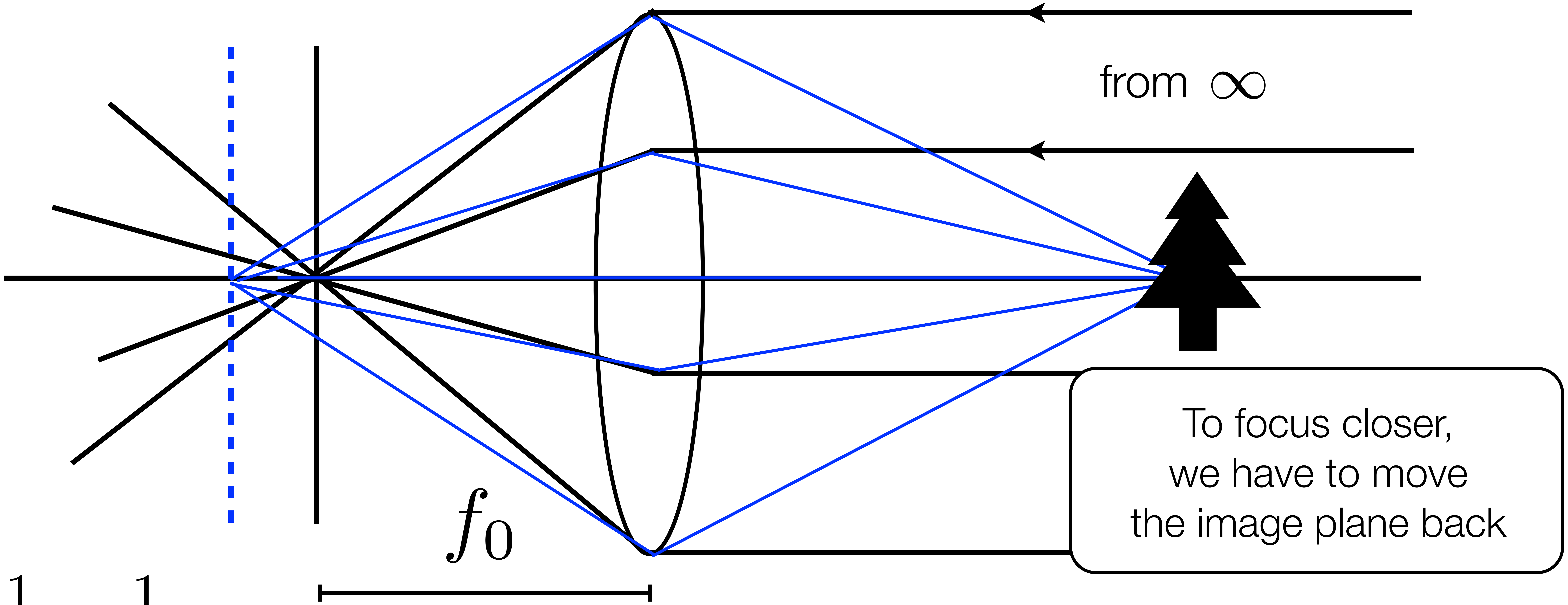

Forsyth & Ponce (1st ed.) Figure 1.9

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

# **Lens** Basics

A lens focuses parallel rays (from points at infinity) at focal length of the lens

Rays passing through the center of the lens are not bent

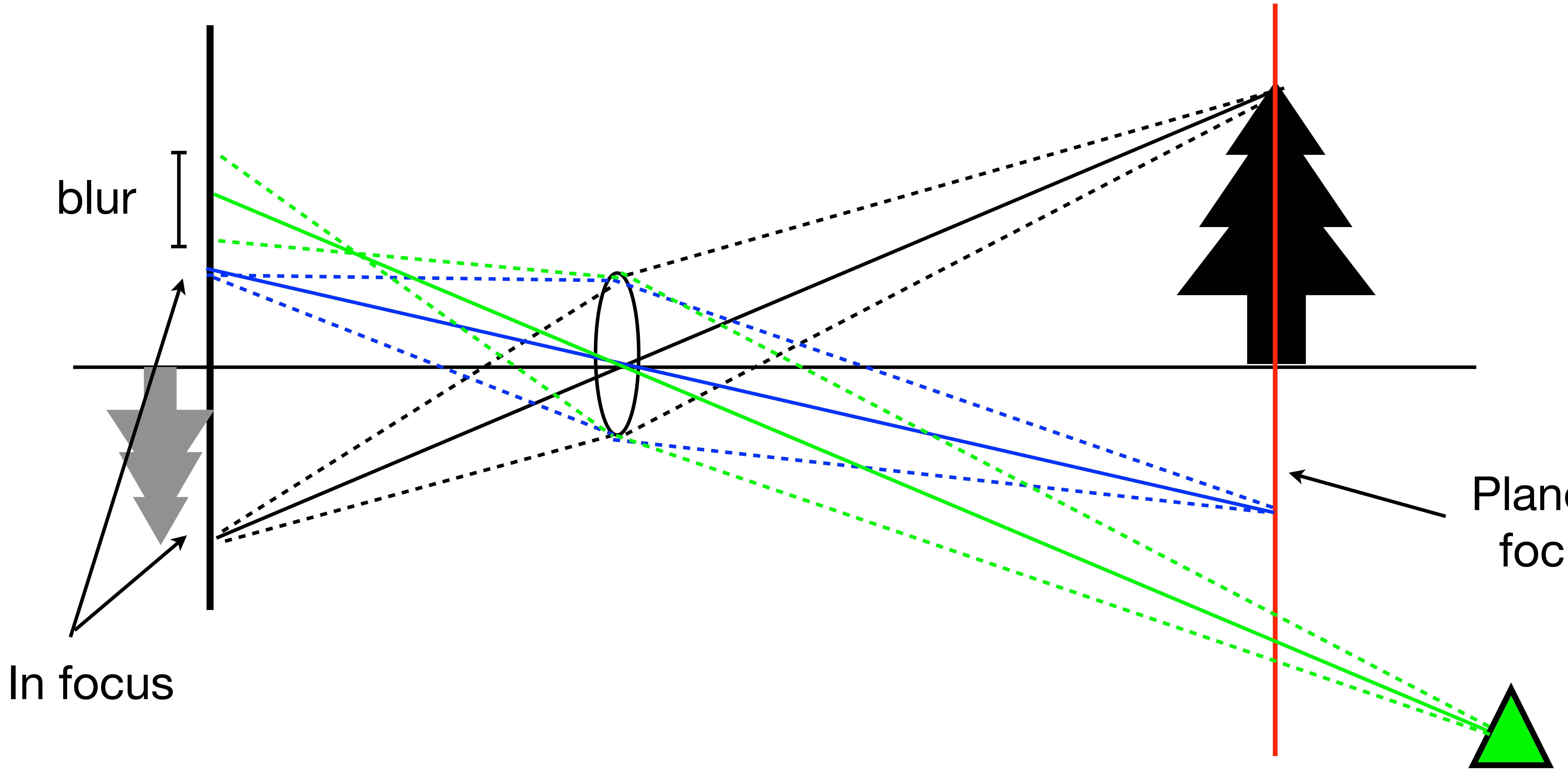



$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

# **Lens** Basics

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$

Lenses focus all rays from a (parallel to lens) plane in the world




Objects off the plane are blurred depending on the distance

# **Pinhole Camera** with a **Lens**

**Perspective Projection**: location in the image where a 3D world point projects

$$x' = f' \frac{x}{z}$$
$$y' = f' \frac{y}{z}$$

**Thin Lens Equation**: depth of the imaging plane itself where this point will be in focus

$$\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}$$