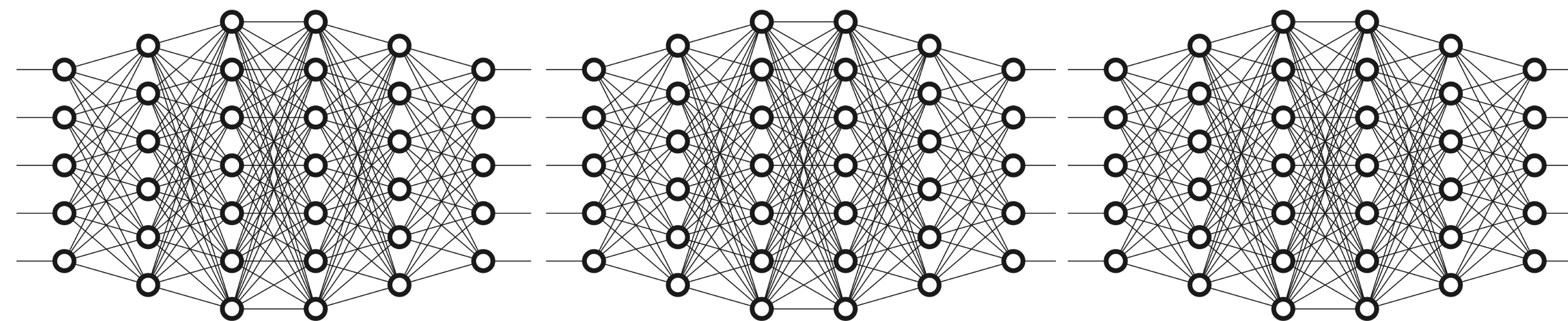




# CPSC 425: Computer Vision



**Lecture 23:** Detection, Segmentation

# Menu for Today

## Topics:

- Classification, Detection, Segmentation
- Attention, Transformers

## Readings:

- **Today's** Lecture: N/A
- **Next** Lecture: N/A

## Reminders:

- **Assignment 6:** Deep Learning is out and due **Thursday**
- **Material** for **Final Prep** will be on Canvas **tonight**
- **Quiz 6** is due **Thursday**

# Computer **Vision Problems**

## Categorization



Multi-**class**: Horse  
Church  
Toothbrush  
**Person**

IMAGENET

Multi-**label**: **Horse**  
Church  
Toothbrush  
**Person**



# Computer **V**ision **P**roblems

## Categorization

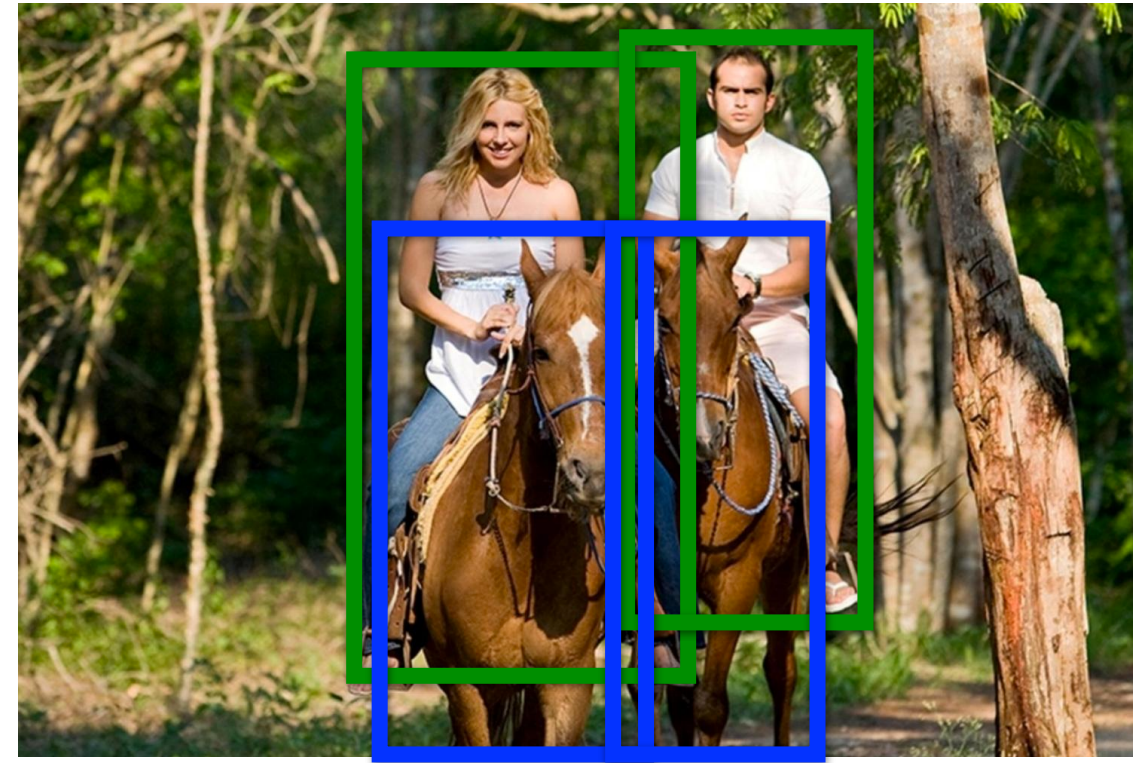


Multi-**class**:  
Horse  
Church  
Toothbrush  
**Person**

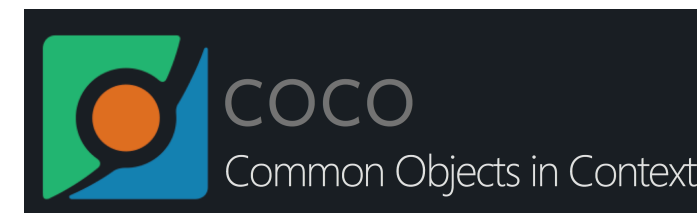
IMAGENET

Multi-**label**:  
**Horse**  
Church  
Toothbrush  
**Person**

## Detection



Horse (x, y, w, h)  
Horse (x, y, w, h)  
Person (x, y, w, h)  
Person (x, y, w, h)





# Computer **V**ision **P**roblems

## Categorization

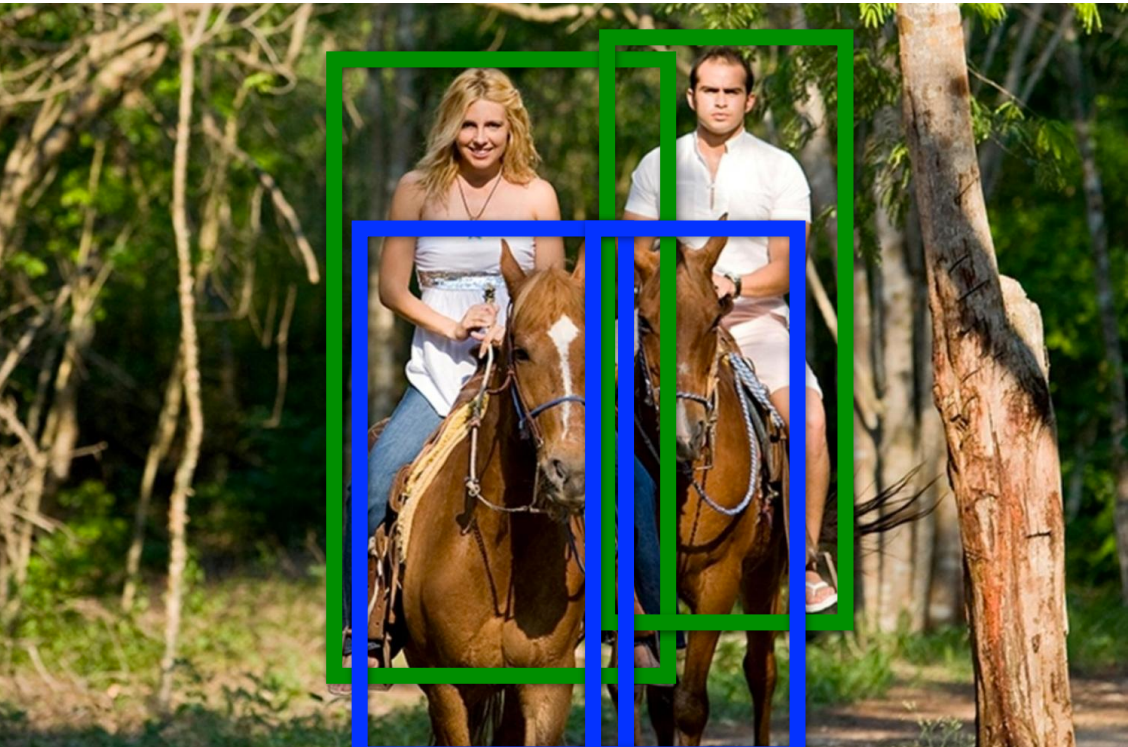


Multi-**class**: Horse  
Church  
Toothbrush  
**Person**



Multi-**label**: **Horse**  
Church  
Toothbrush  
**Person**

## Detection



Horse (x, y, w, h)  
Horse (x, y, w, h)  
Person (x, y, w, h)  
Person (x, y, w, h)



## Segmentation



Horse  
Person





# Computer **V**ision **P**roblems

## Categorization

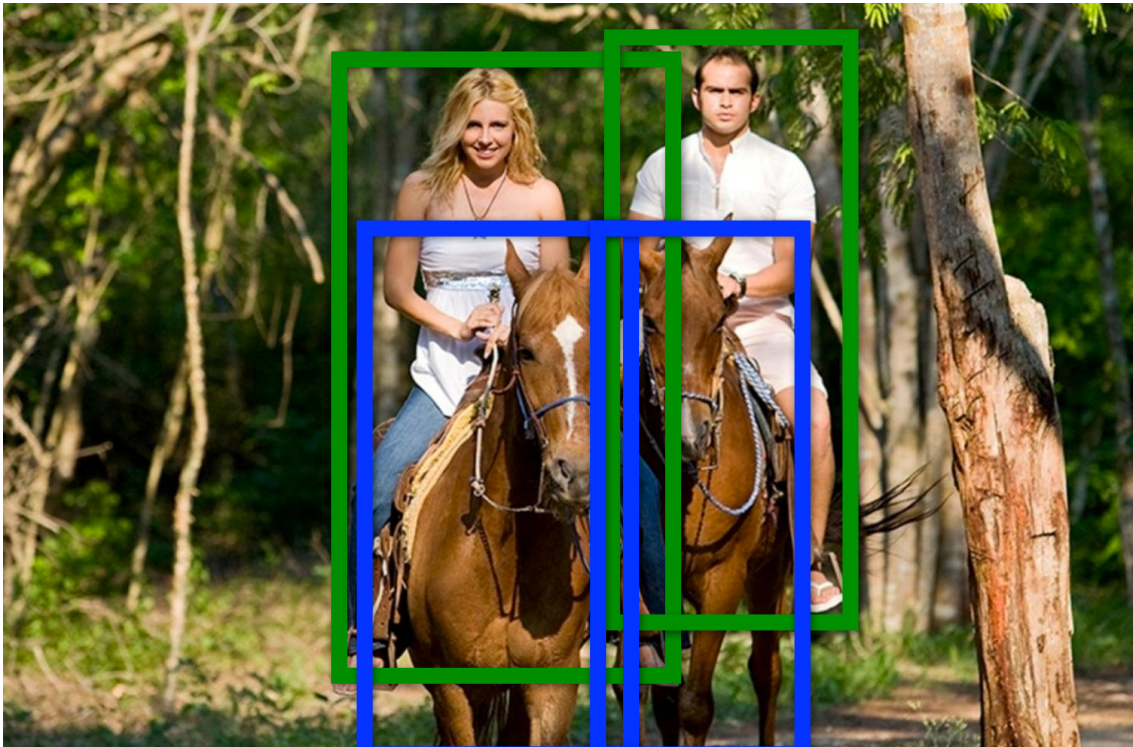


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## Detection



Horse (x, y, w, h)  
Horse (x, y, w, h)  
Person (x, y, w, h)  
Person (x, y, w, h)



## Segmentation



Horse  
Person



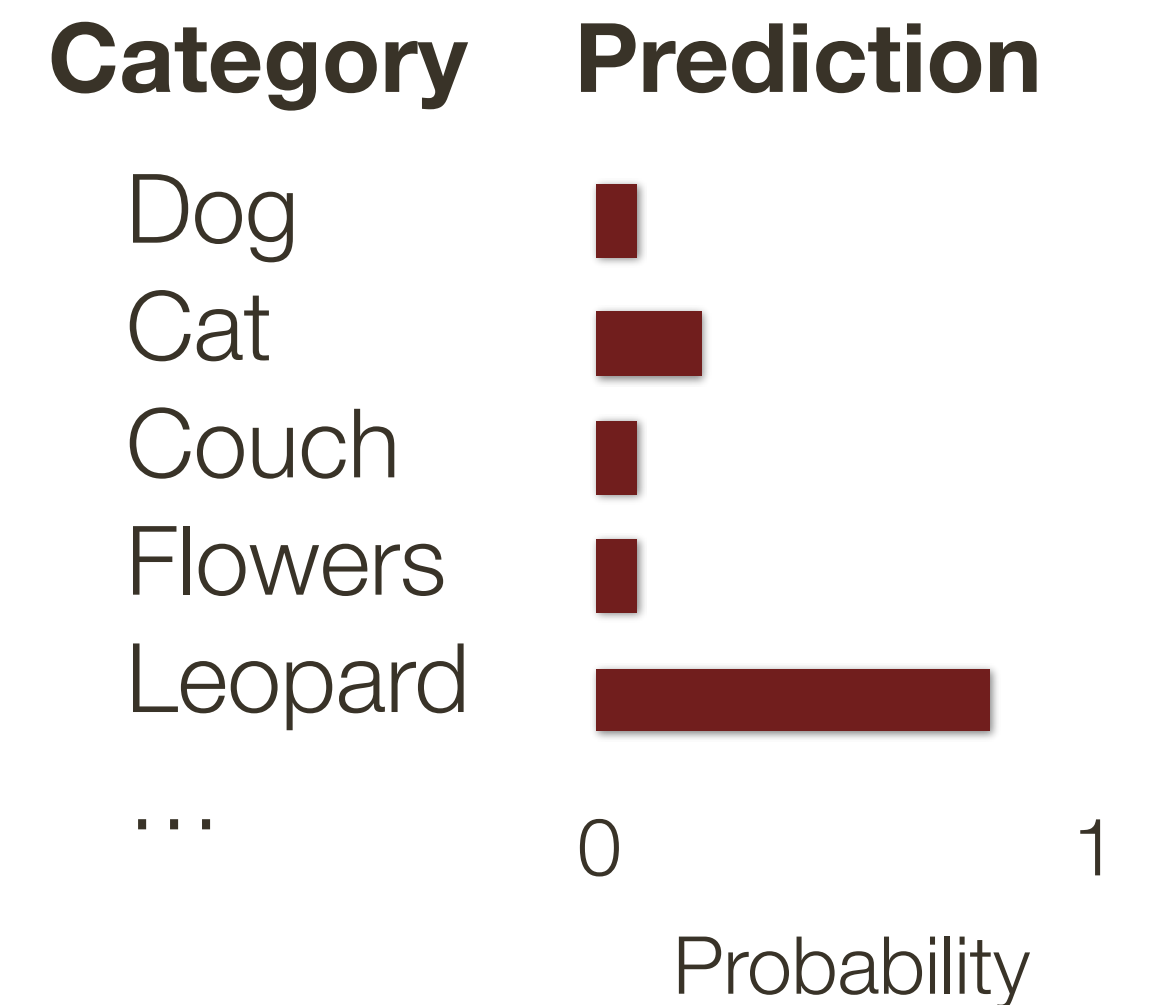
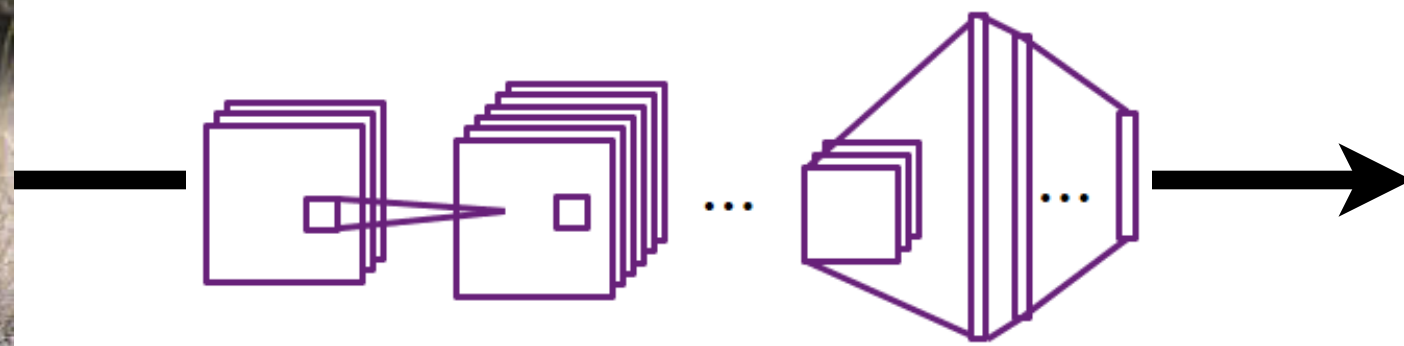
## Instance Segmentation



Horse1  
Horse2  
Person1  
Person2



# Object Classification



**Problem:** For each image predict which category it belongs to out of a fixed set

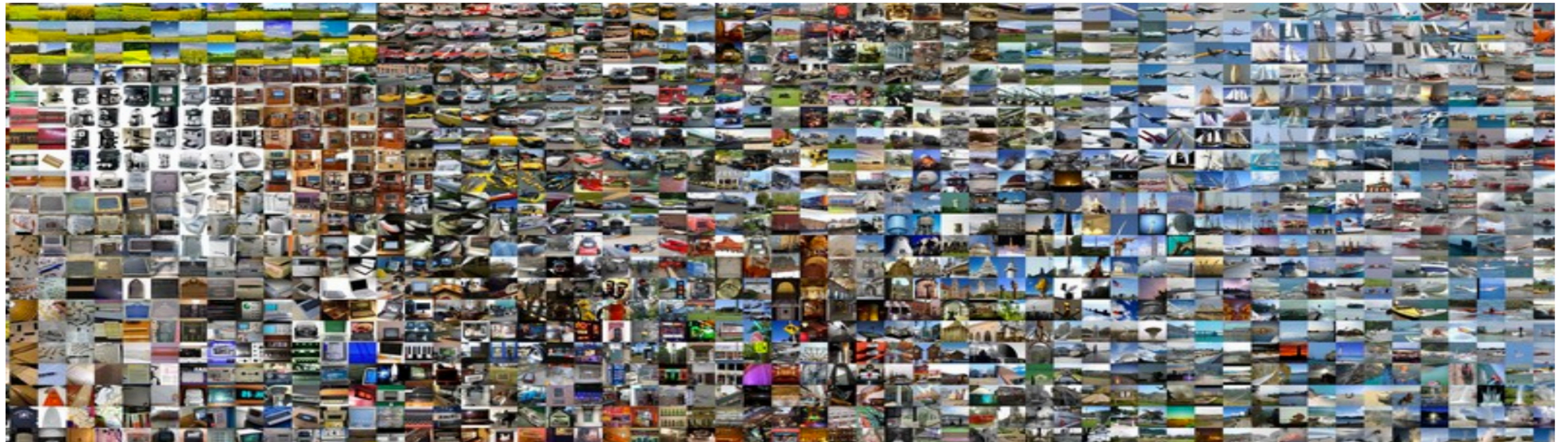


# ImageNet **Competition** (ILSVRC)

Annual competition of image classification at scale

Focuses on a subset of **1K synset** categories

**Scoring:** need to predict true label within top K (K=5)

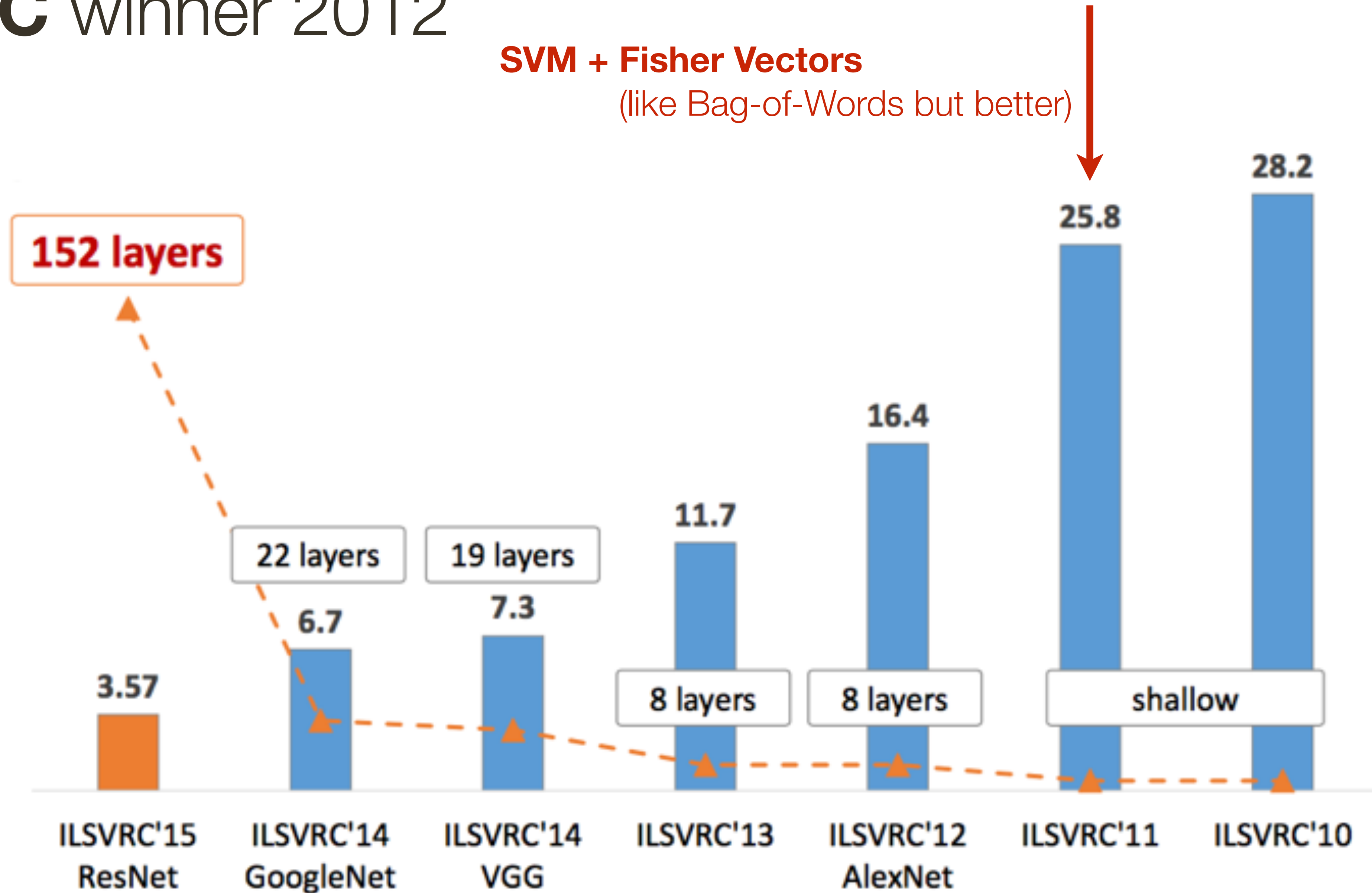




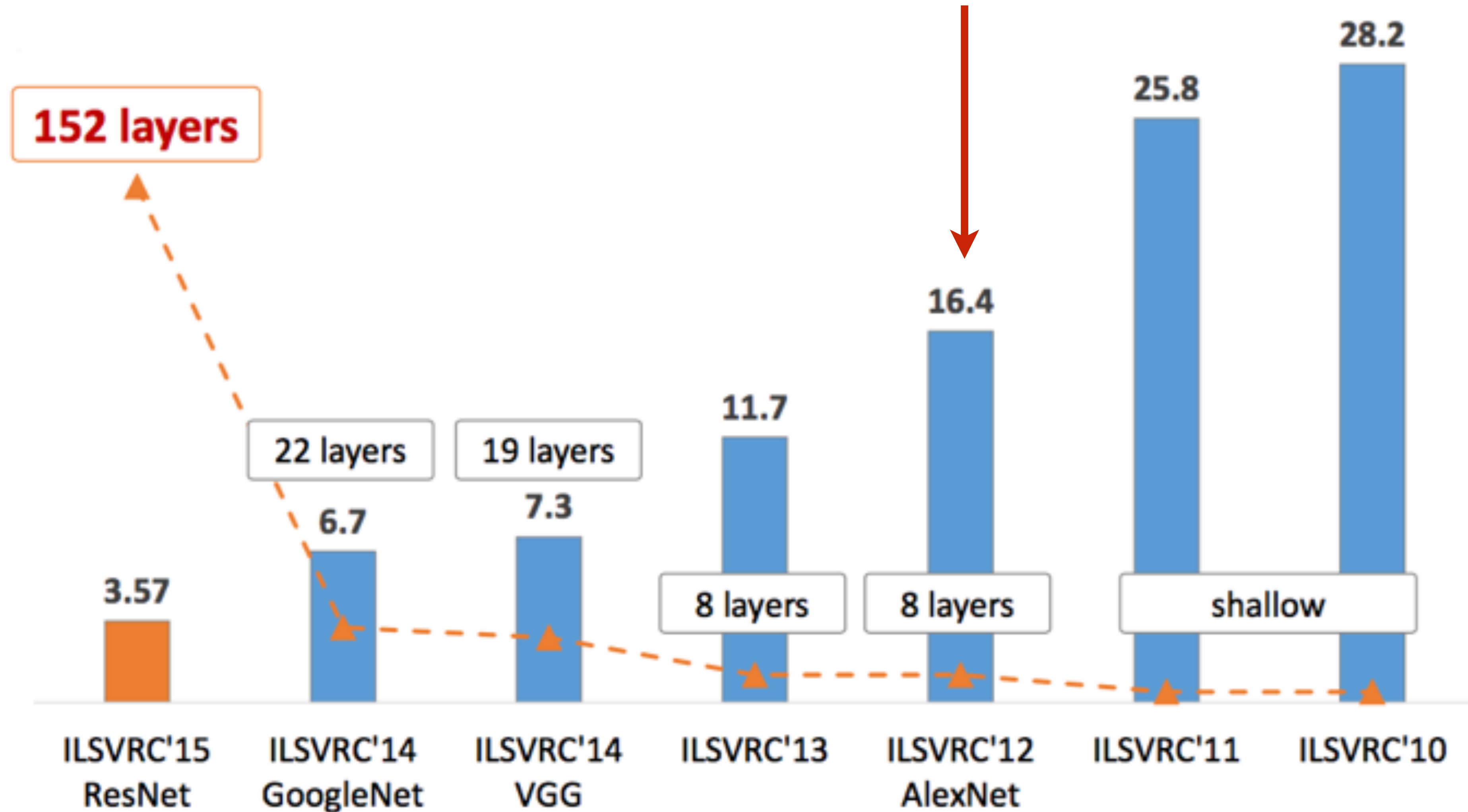
# ILSVRC winner 2012

**SVM + Fisher Vectors**

(like Bag-of-Words but better)

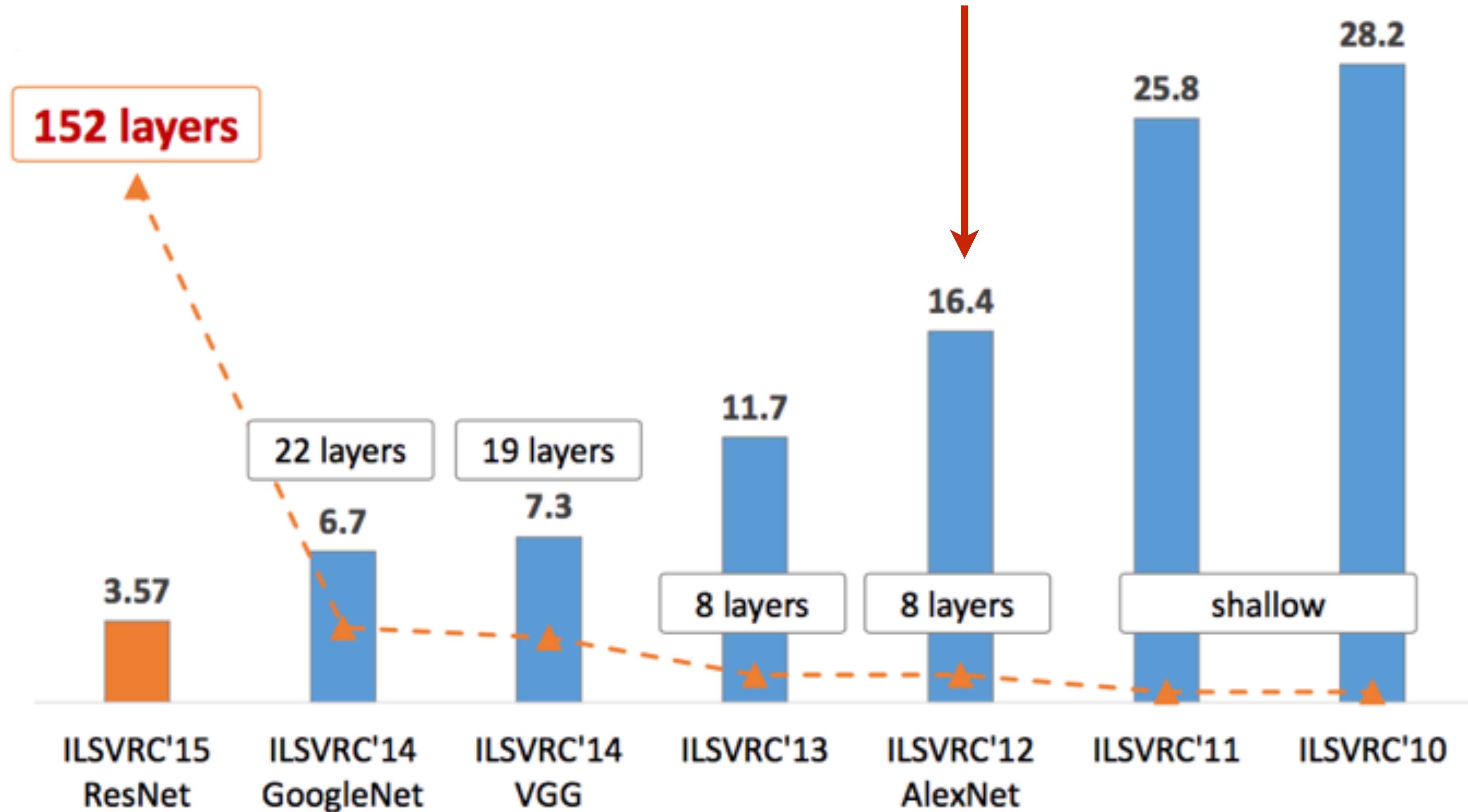


# ILSVRC winner 2012

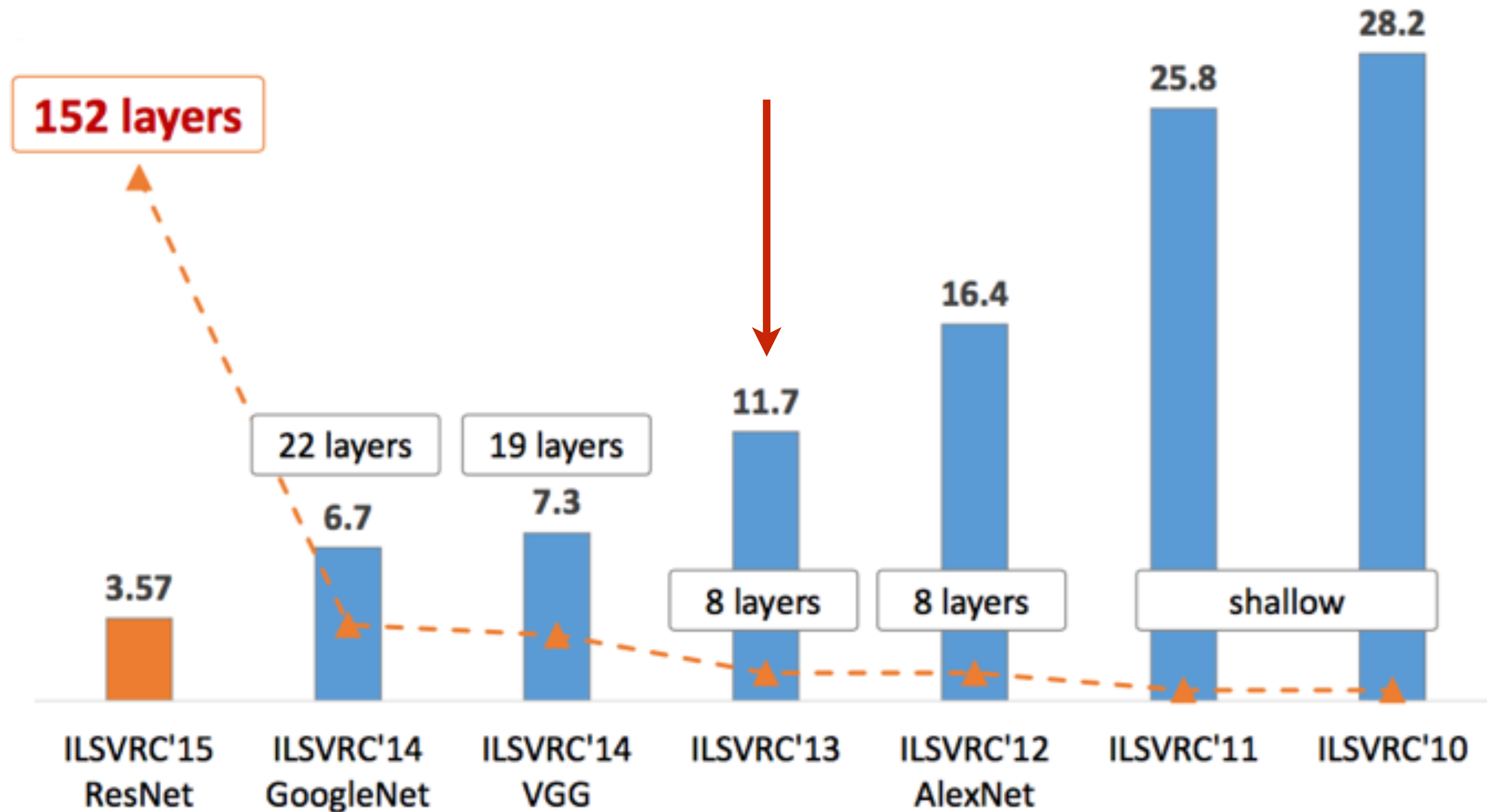




# ILSVRC winner 2012

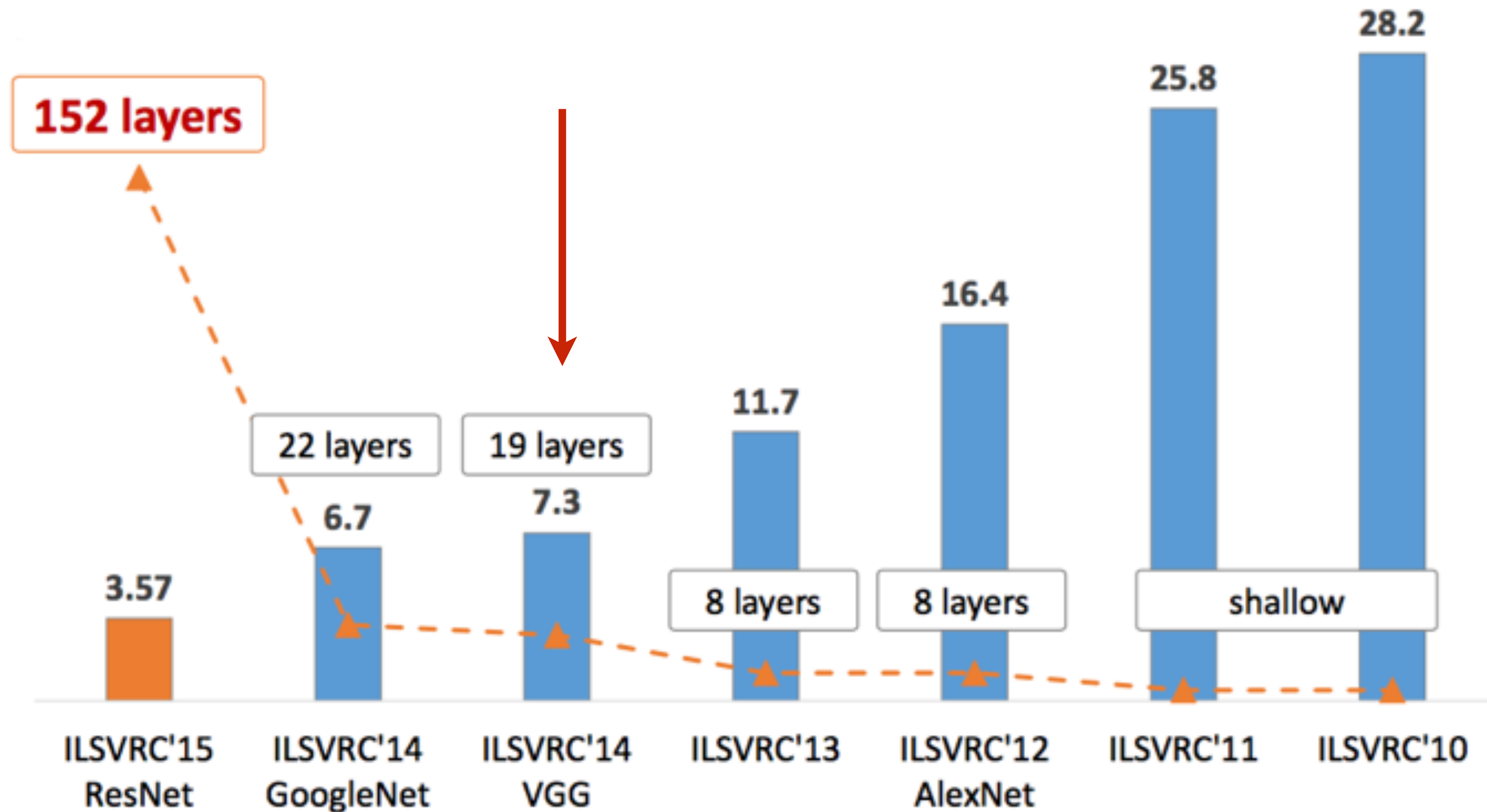


# ILSVRC winner 2012

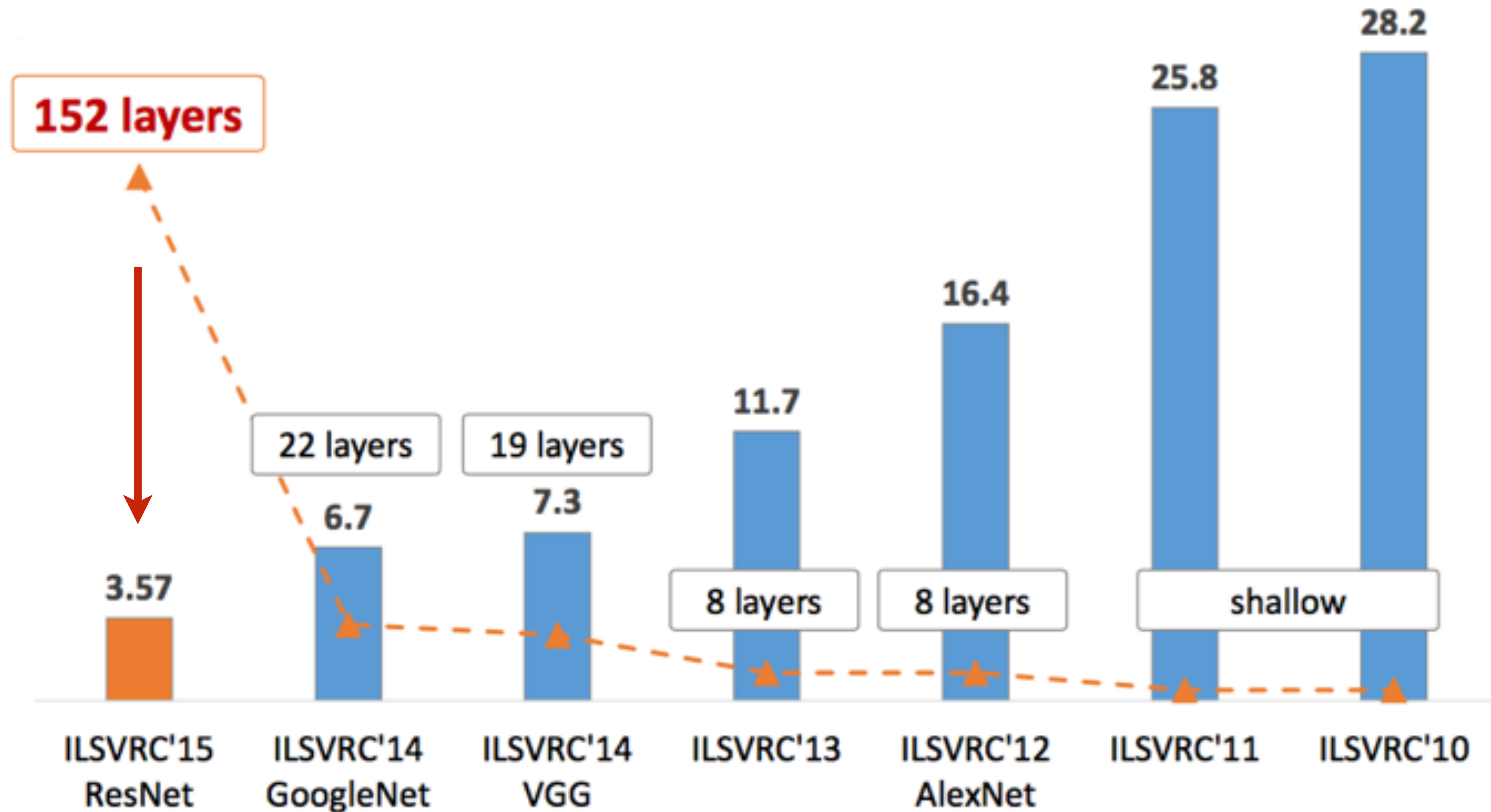




# ILSVRC winner 2012



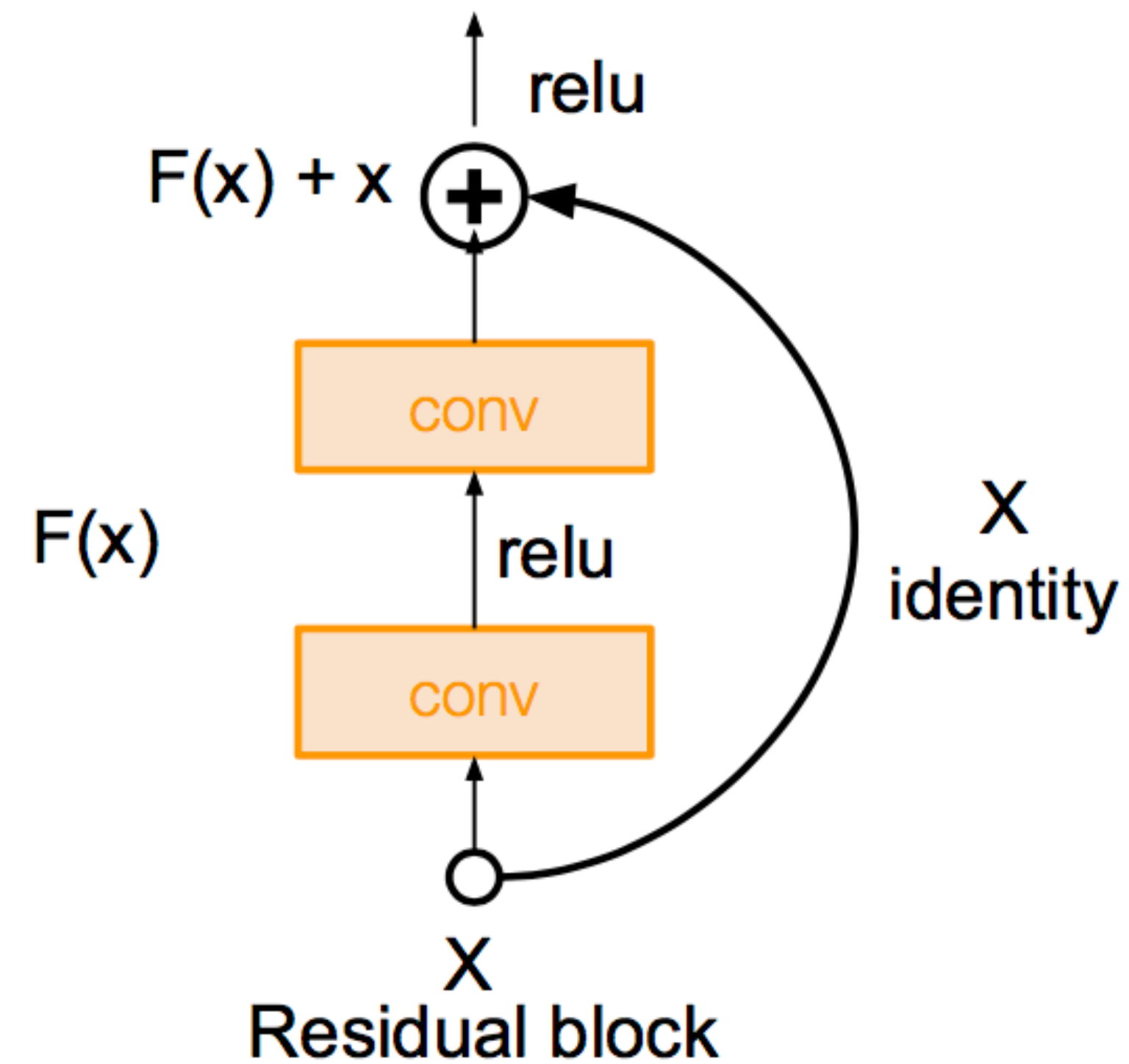
# ILSVRC winner 2012



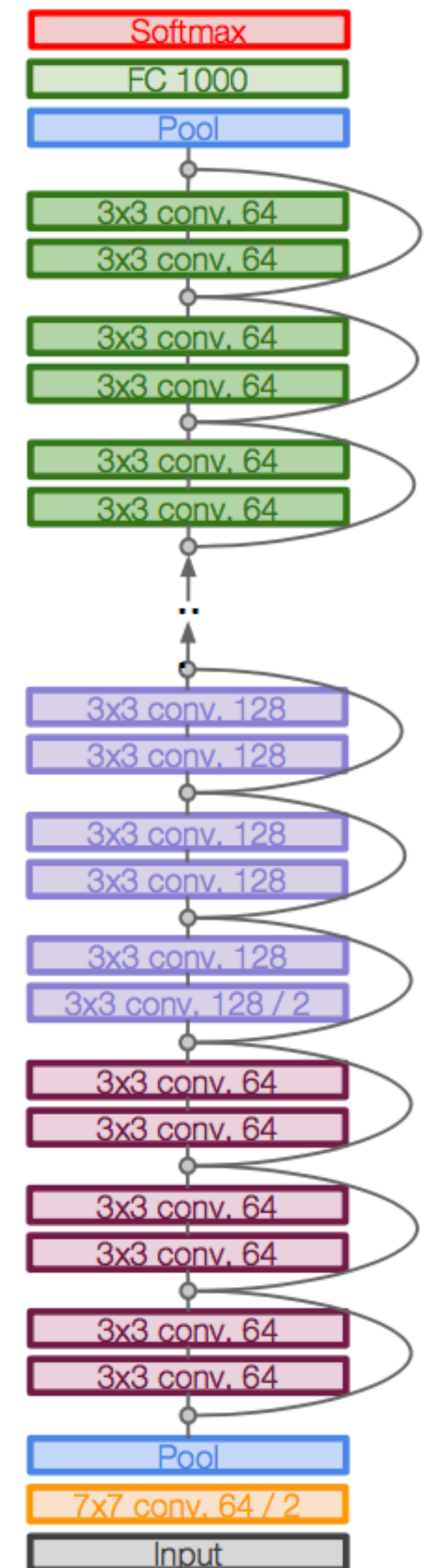
# ResNet

even deeper — **152 layers!**

using residual connections



[ He et al., 2015 ]

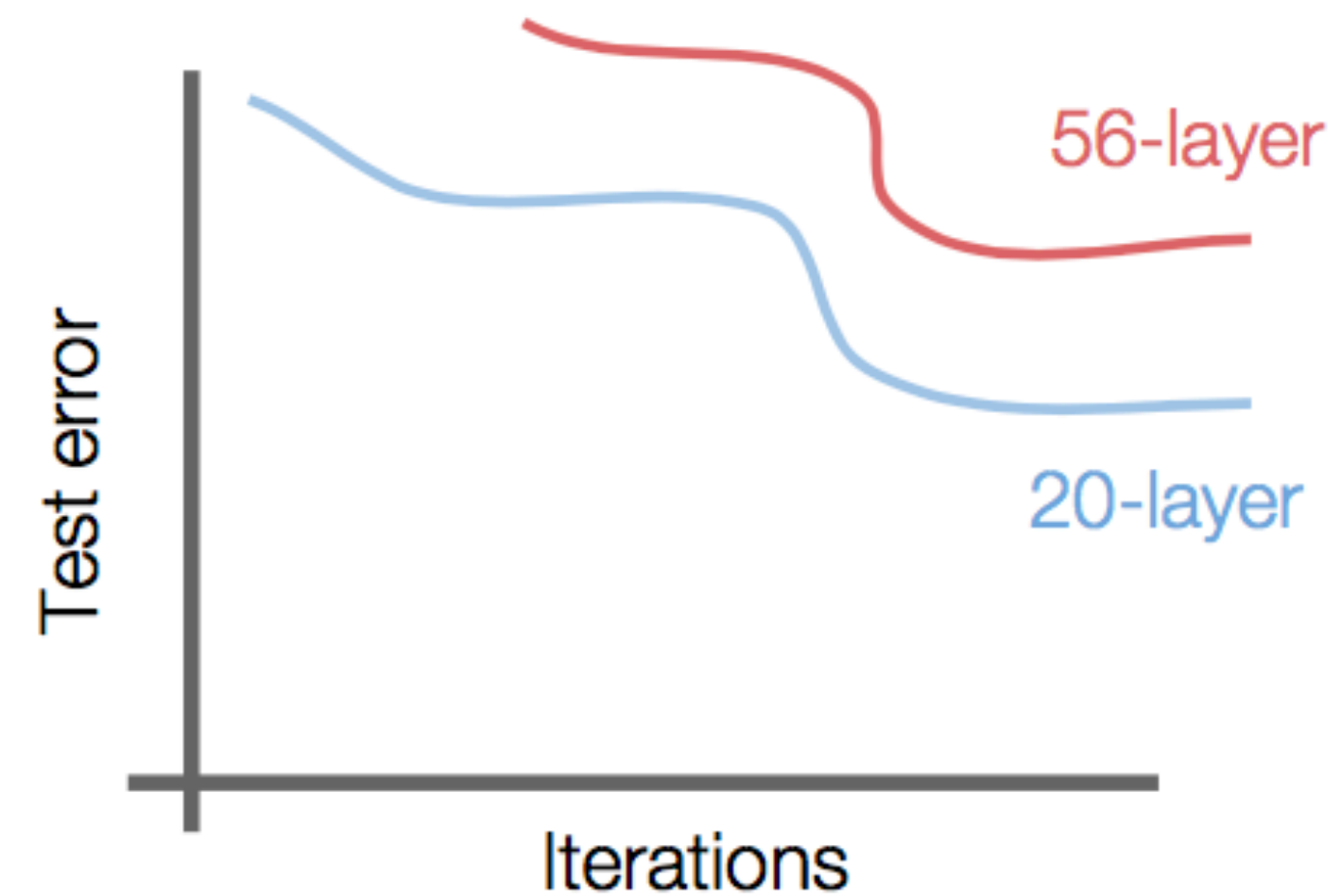




# ResNet: Motivation

[ He et al., 2015 ]

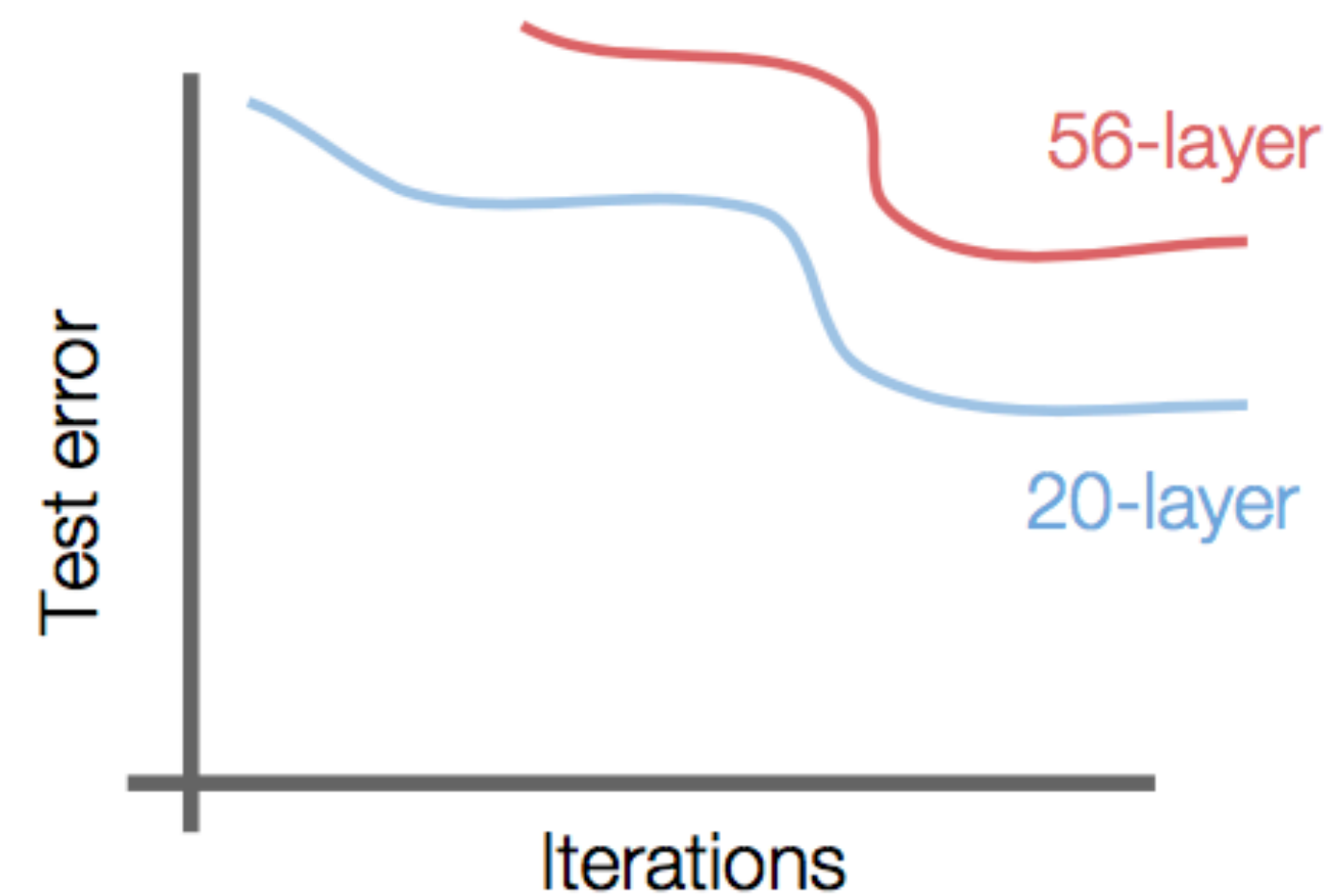
What happens when we continue to stacking deeper layers on a “plain” CNN



# ResNet: Motivation

[ He et al., 2015 ]

What happens when we continue to stacking deeper layers on a “plain” CNN

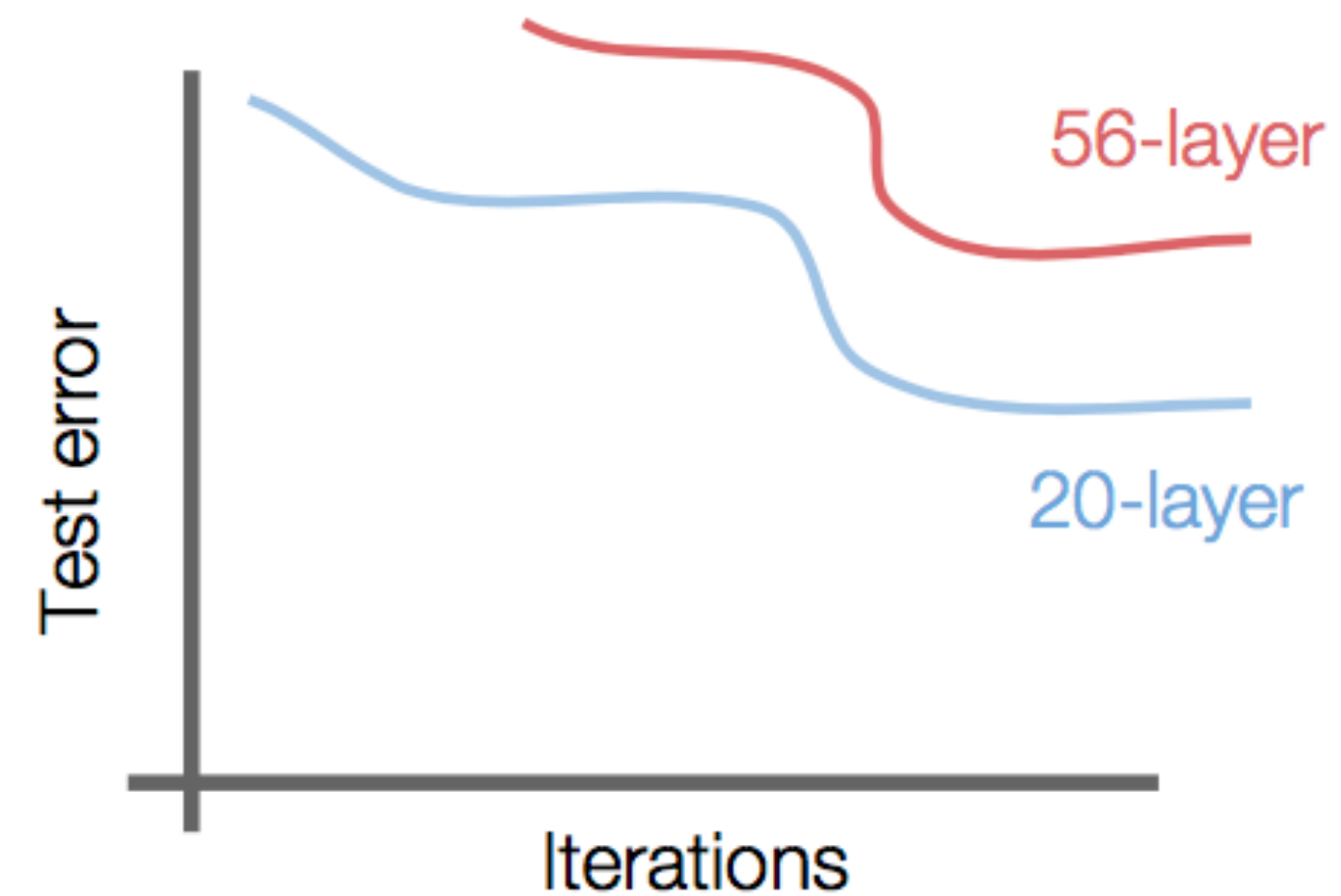
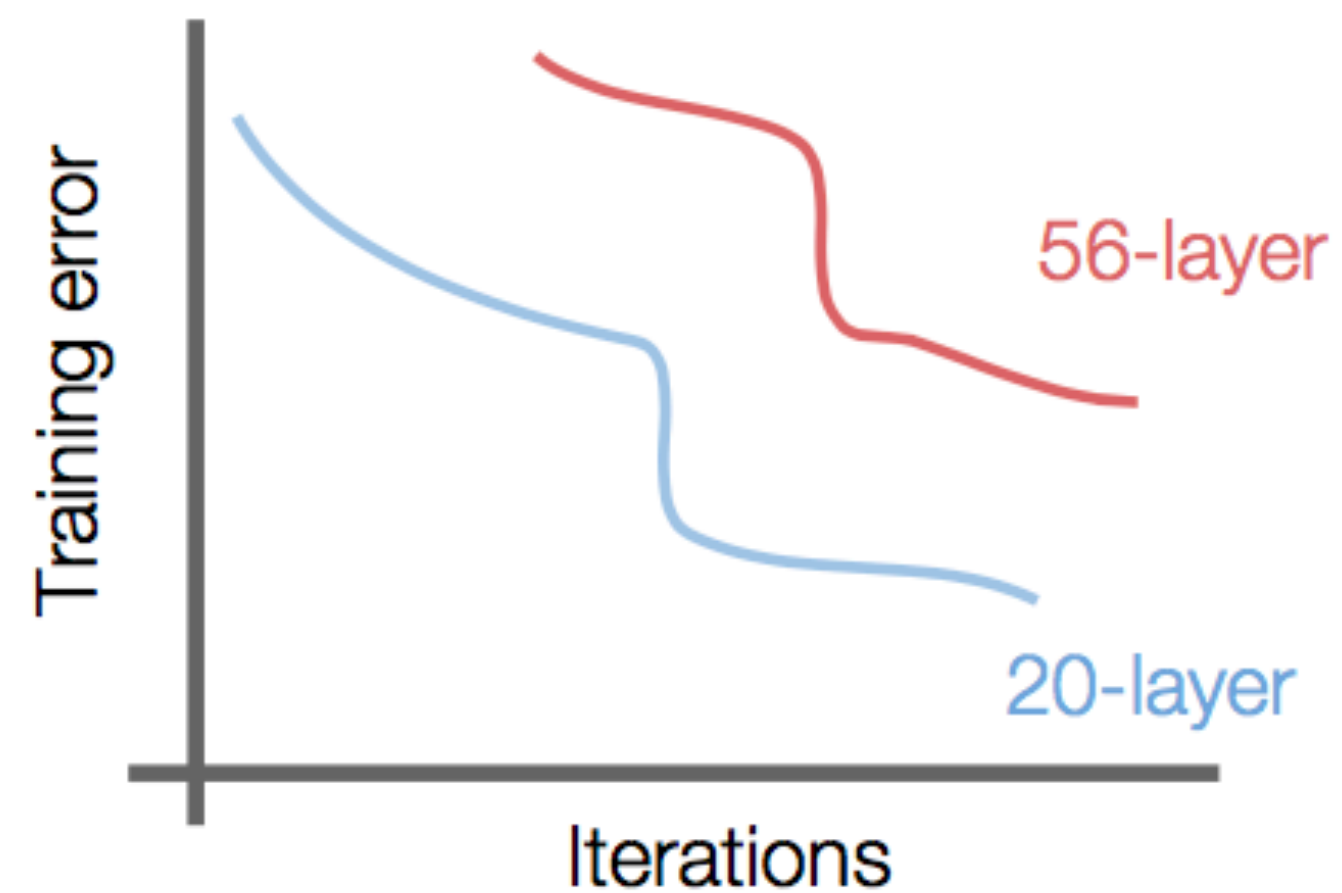


Whats the **problem**?

# ResNet: Motivation

[ He et al., 2015 ]

What happens when we continue to stacking deeper layers on a “plain” CNN

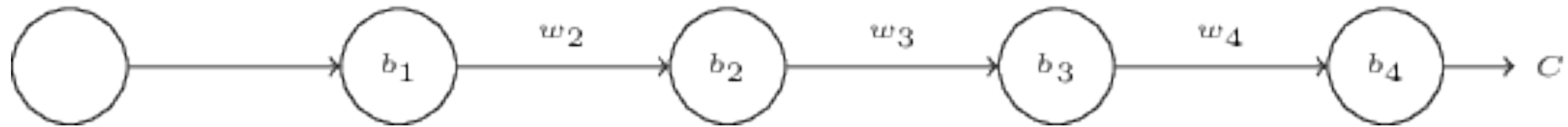


Whats the **problem**?



# Optimizing **Deep** Neural Networks

$$\frac{\partial C}{\partial b_1} = \sigma'(z_1) \times w_2 \times \sigma'(z_2) \times w_3 \times \sigma'(z_3) \times w_4 \times \sigma'(z_4) \times \frac{\partial C}{\partial a_4}$$



$$\frac{\partial C}{\partial b_1} = \sigma'(z_1) \underbrace{w_2 \sigma'(z_2)}_{< \frac{1}{4}} \underbrace{w_3 \sigma'(z_3)}_{< \frac{1}{4}} \underbrace{w_4 \sigma'(z_4)}_{\text{common terms}} \frac{\partial C}{\partial a_4}$$

This is called **vanishing gradient** problem

- makes deep networks hard to train
- later layers learn faster than earlier ones

$$\frac{\partial C}{\partial b_3} = \sigma'(z_3) \underbrace{w_4 \sigma'(z_4)}_{\text{common terms}} \frac{\partial C}{\partial a_4}$$

# ResNet: Motivation

[ He et al., 2015 ]

**Hypothesis:** deeper models are harder to optimize (optimization problem)



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[ He et al., 2015 ]

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**Observation:** the deeper model should (conceptually) perform just as well (e.g., take shallower model and use identity for all remaining layers)



# ResNet: Motivation

[ He et al., 2015 ]

**Hypothesis:** deeper models are harder to optimize (optimization problem)

**Observation:** the deeper model should (conceptually) perform just as well (e.g., take shallower model and use identity for all remaining layers)

How do we implement this idea in practice



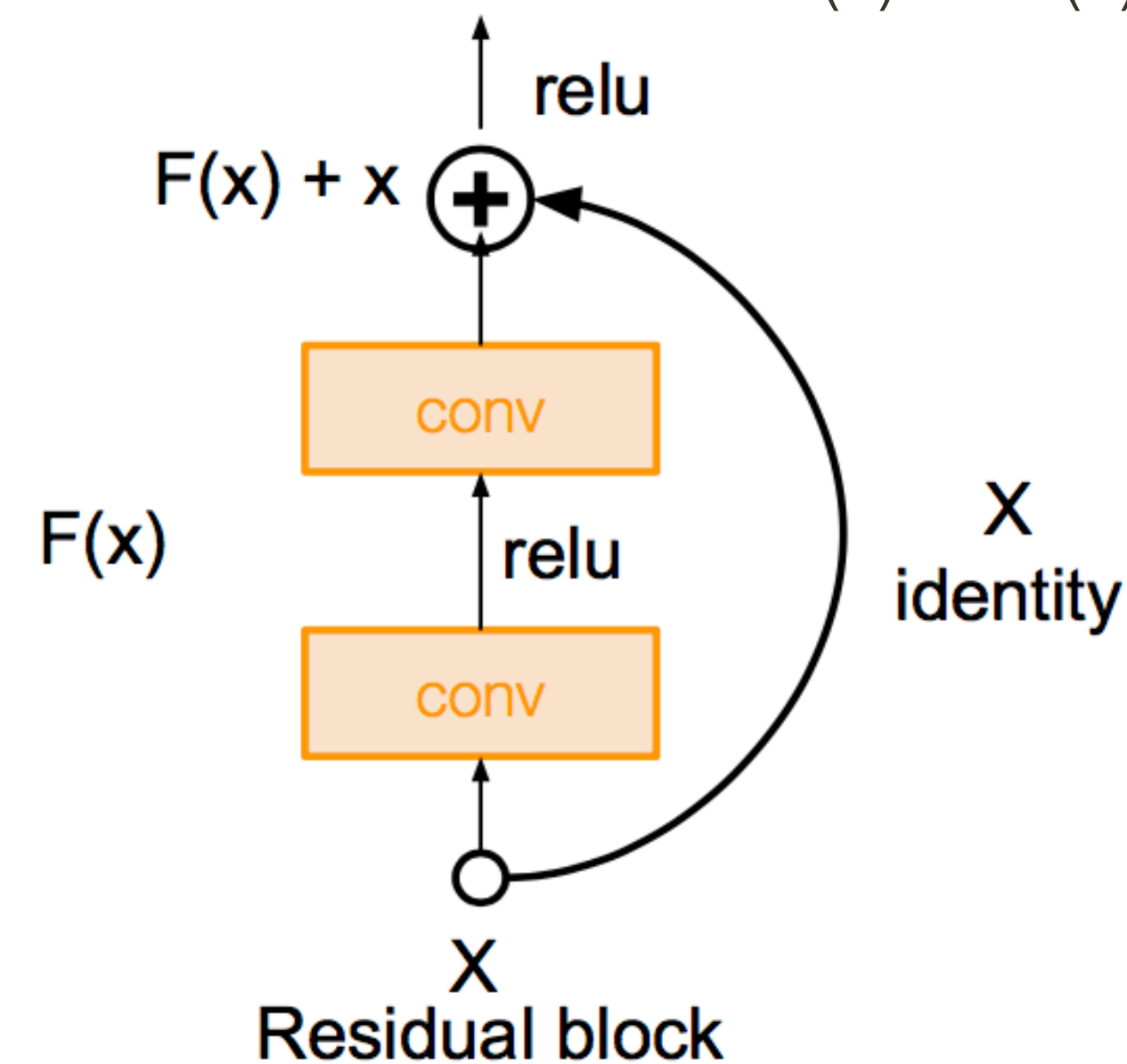
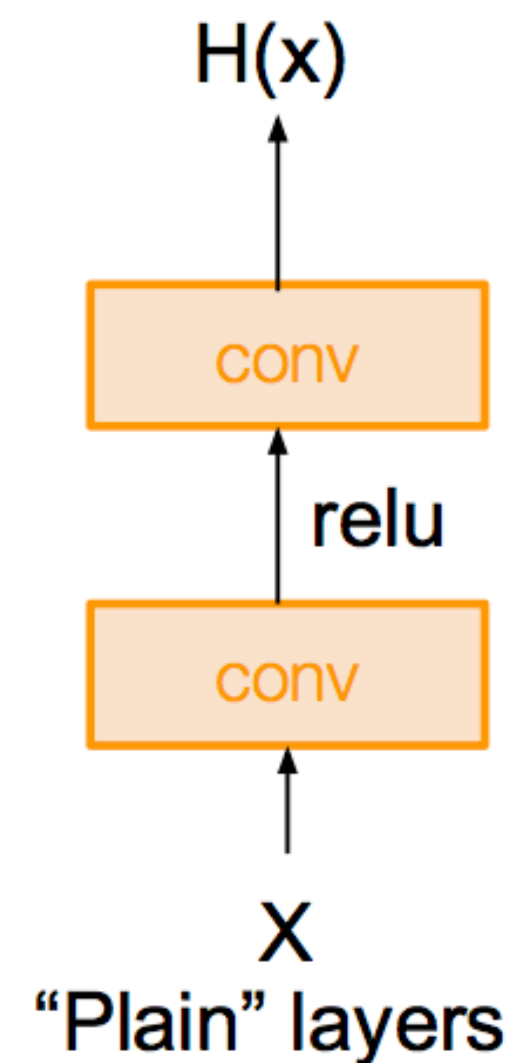
# ResNet

[ He et al., 2015 ]

**Solution:** use network to fit residual mapping instead of directly trying to fit a desired underlying mapping

$$H(x) = F(x) + X$$

Use layers to fit **residual**  
 $F(x) = H(x) - X$  instead of  $H(x)$  directly

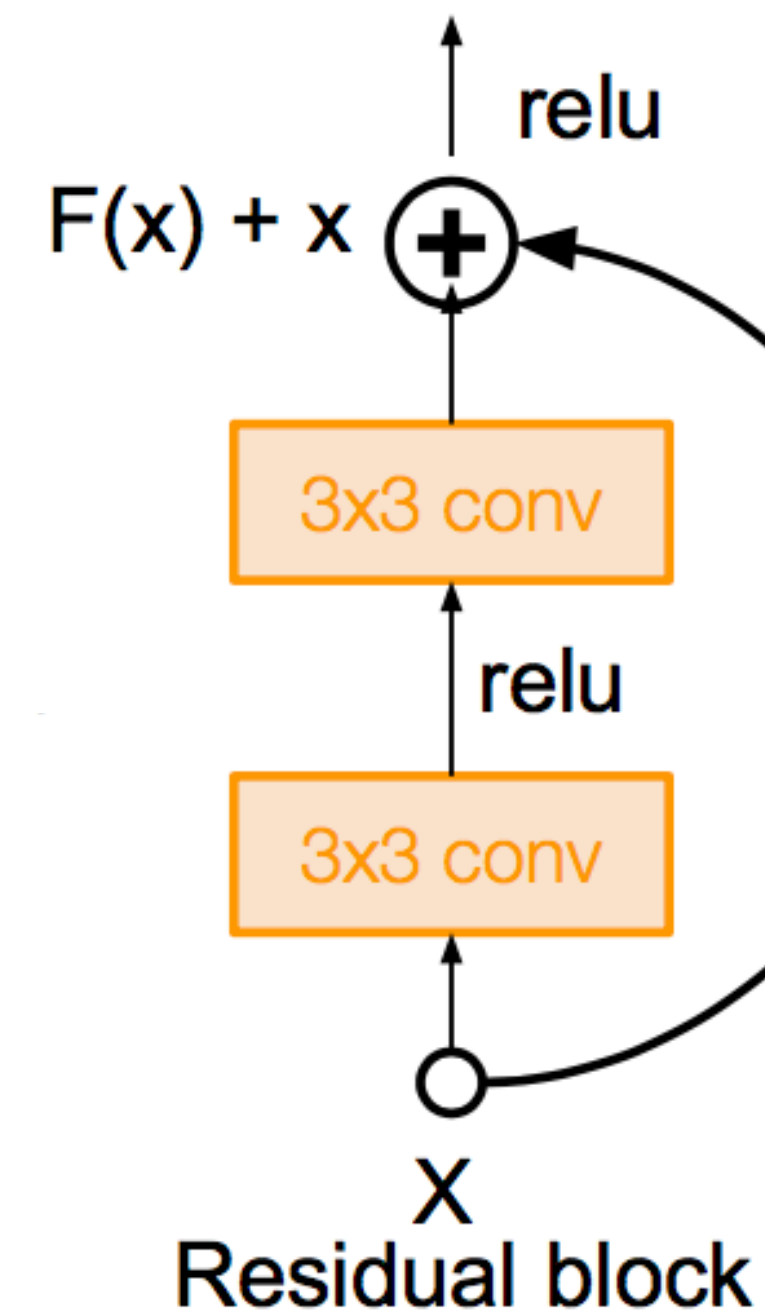


# ResNet

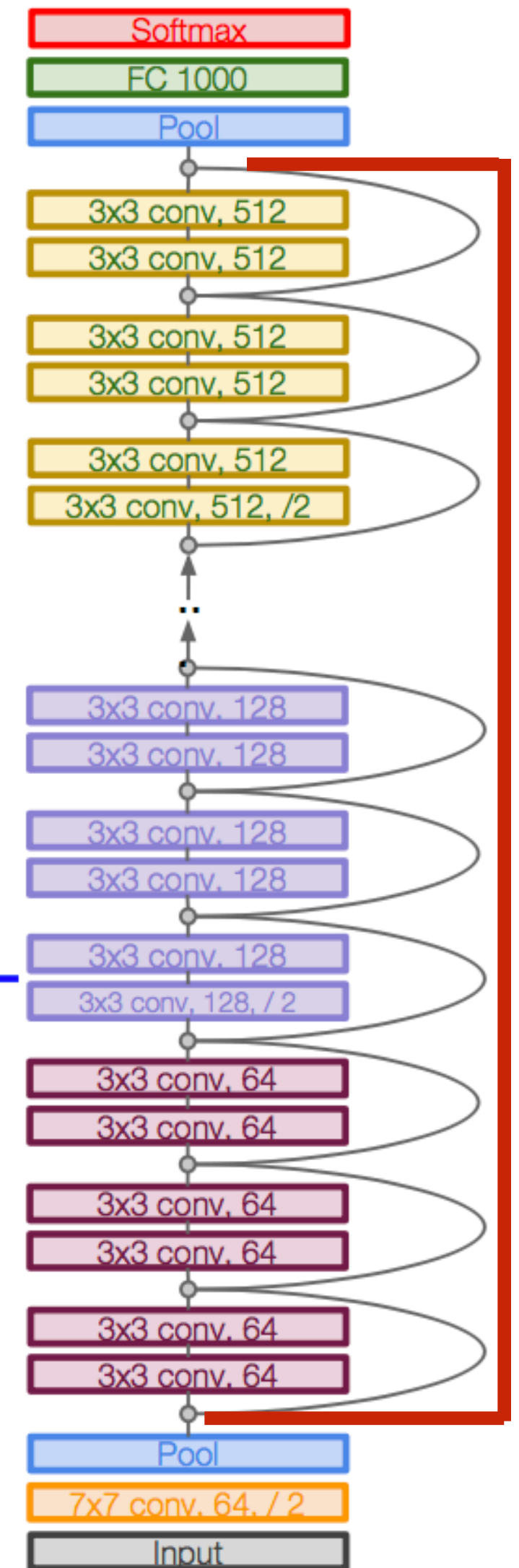
[ He et al., 2015 ]

## Full details

- Stacked **residual blocks**
- Every residual block consists of **two 3x3 filters**
- Periodically double # of filters and downsample spatially using stride of 2
- Additional convolutional layer in the beginning
- **No FC layers** at the end (only FC to output 1000 classes)

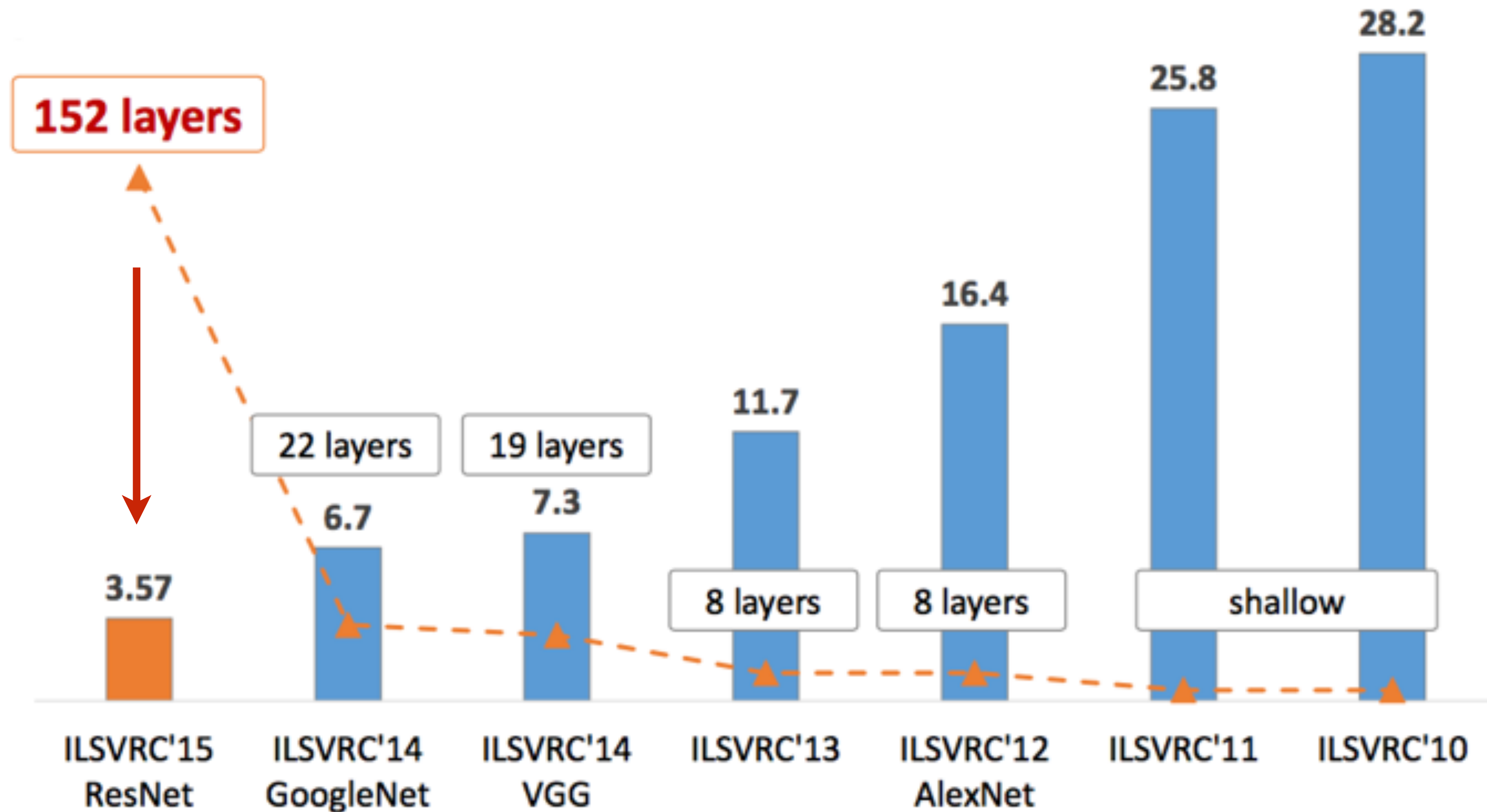


$X$   
identity

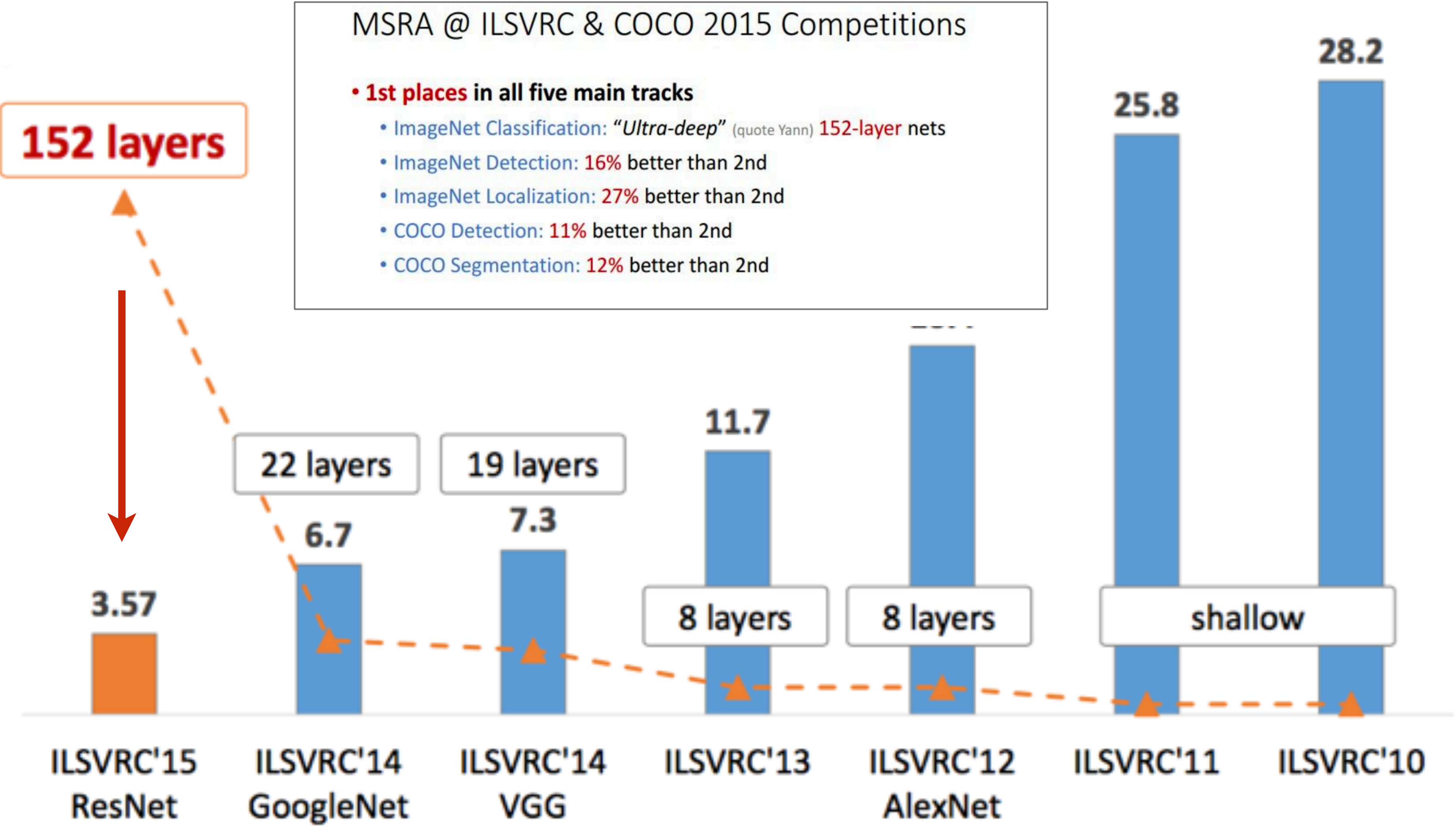




# ILSVRC winner 2012



# ILSVRC winner 2012



\* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**



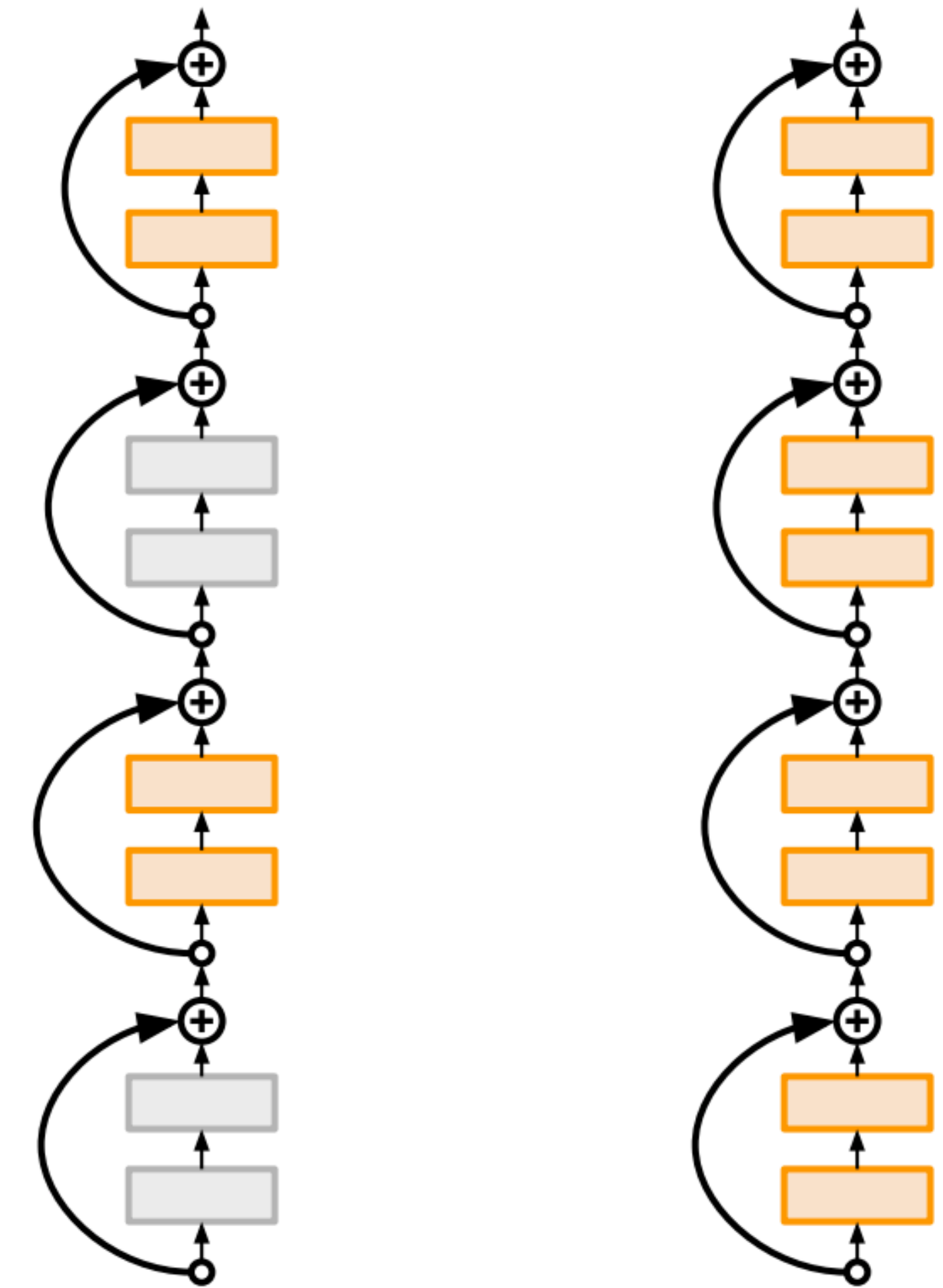
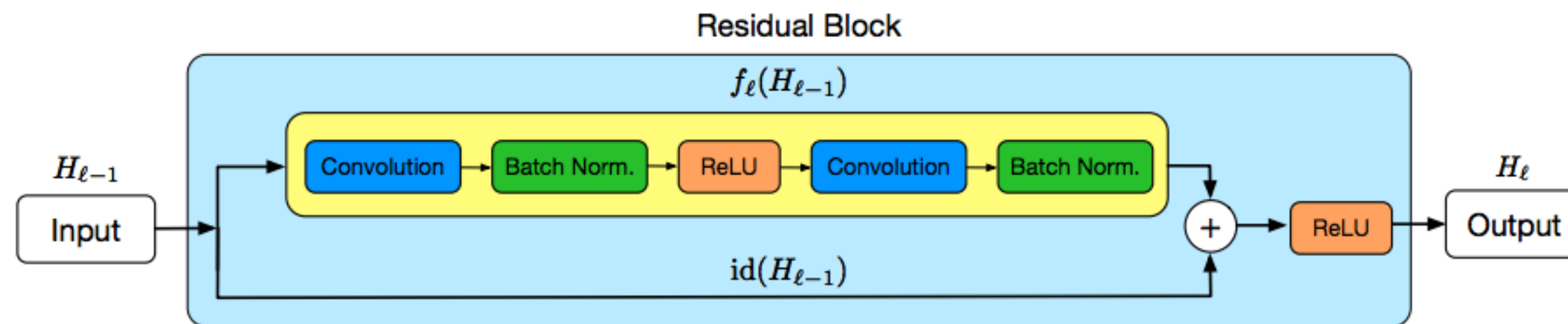
# Regularization: Stochastic Depth

[ Huang et al., ECCV 2016 ]

Effectively “dropout” but for layers

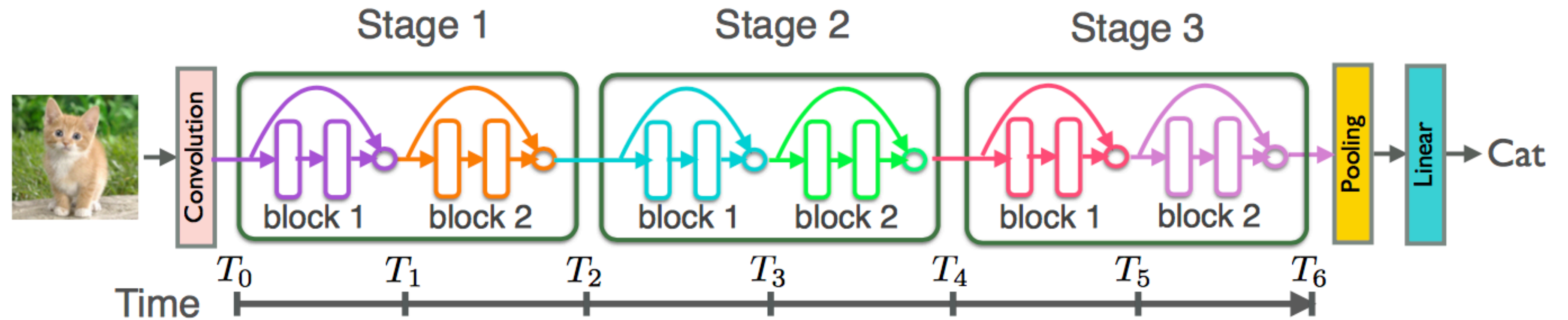
Stochastically with some probability **turn off some layer** (for each batch)

Effectively trains a collection of neural networks



# ResNet: A little theory

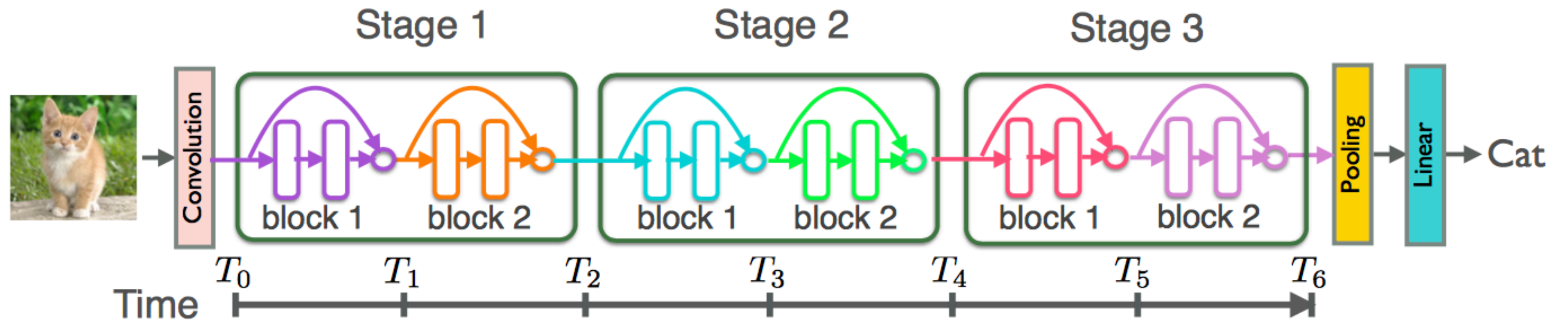
One can view a sequence of outputs from residual layers as a **Dynamical System**



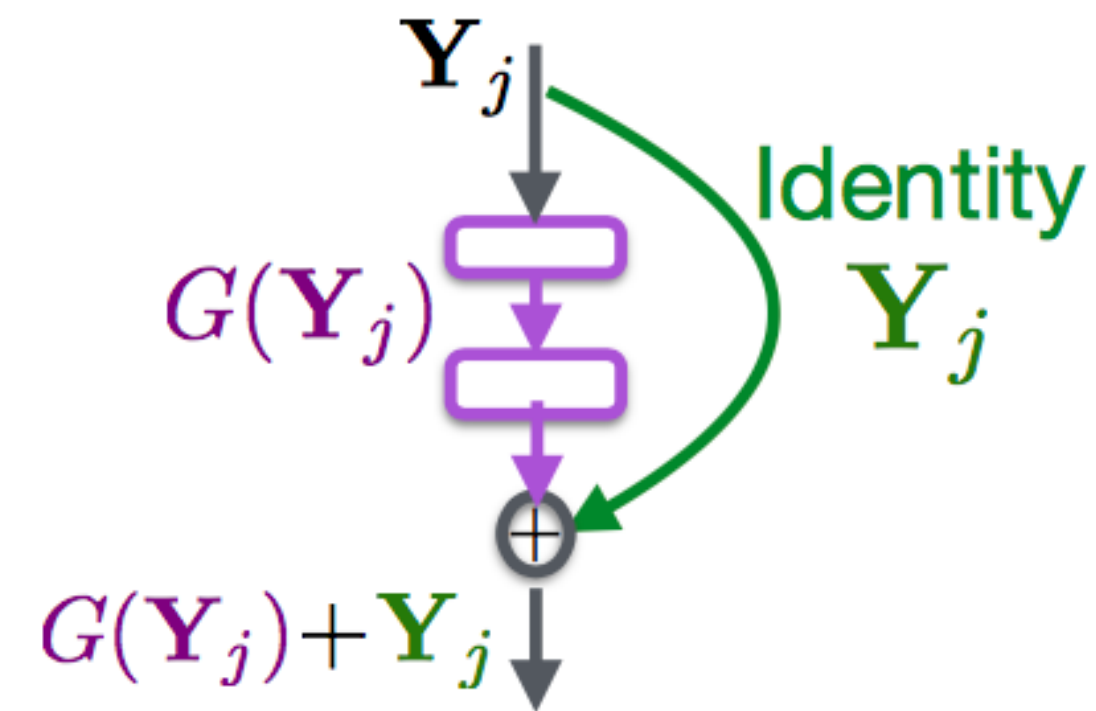


# ResNet: A little theory

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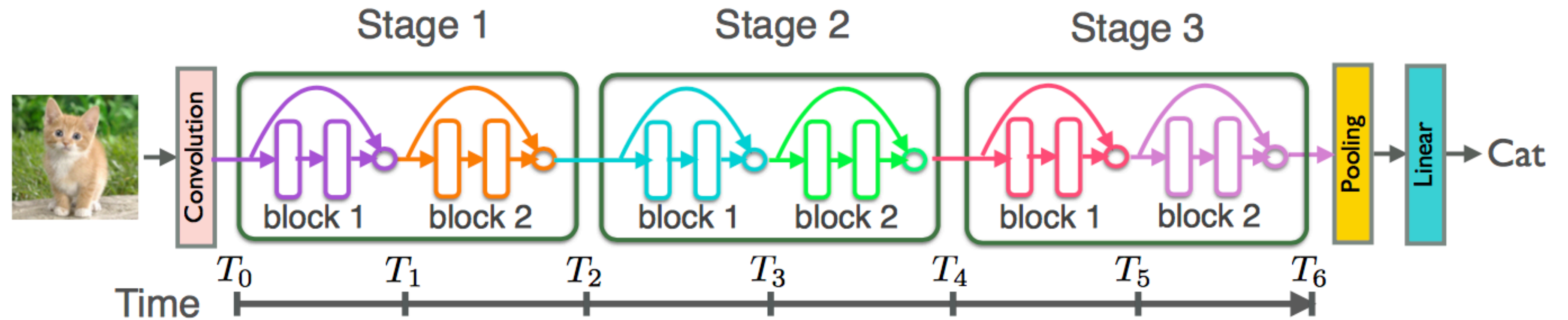


$$\mathbf{Y}_{j+1} = \mathbf{Y}_j + \mathbf{G}(\mathbf{Y}_j, \theta_j)$$



# ResNet: A little theory

One can view a sequence of outputs from residual layers as a **Dynamical System**

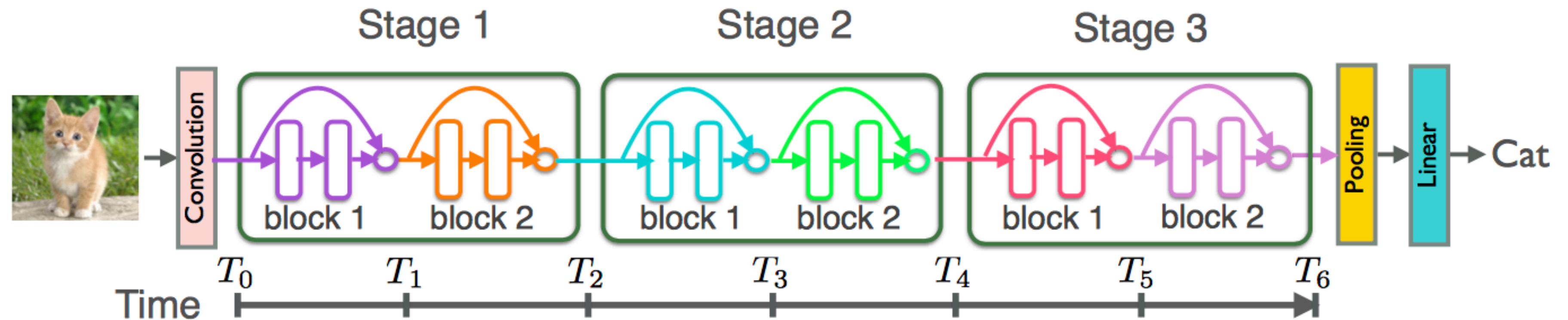


What happens if you take more layers and take smaller steps?



# ResNet: A little theory

One can view a sequence of outputs from residual layers as a **Dynamical System**



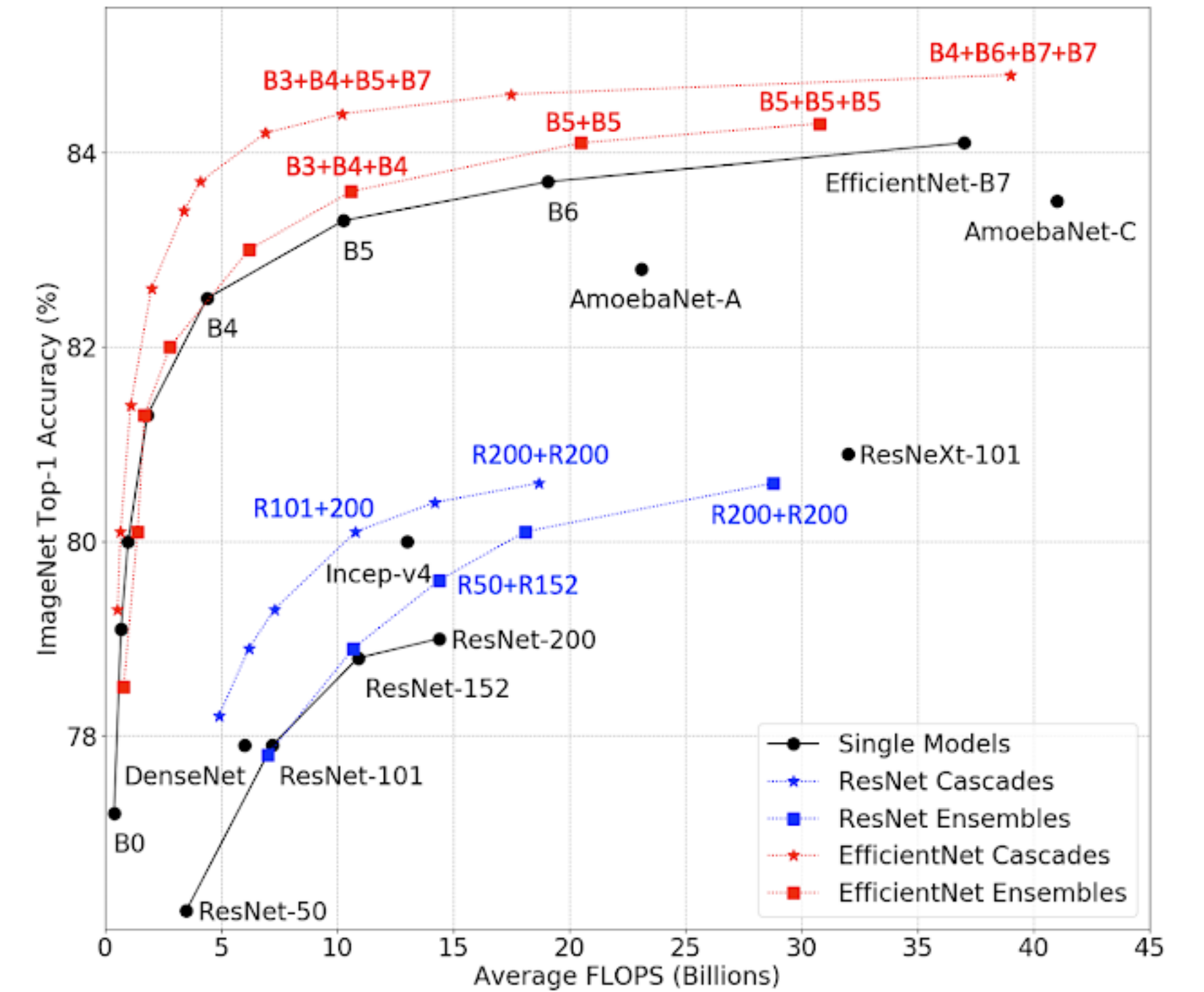
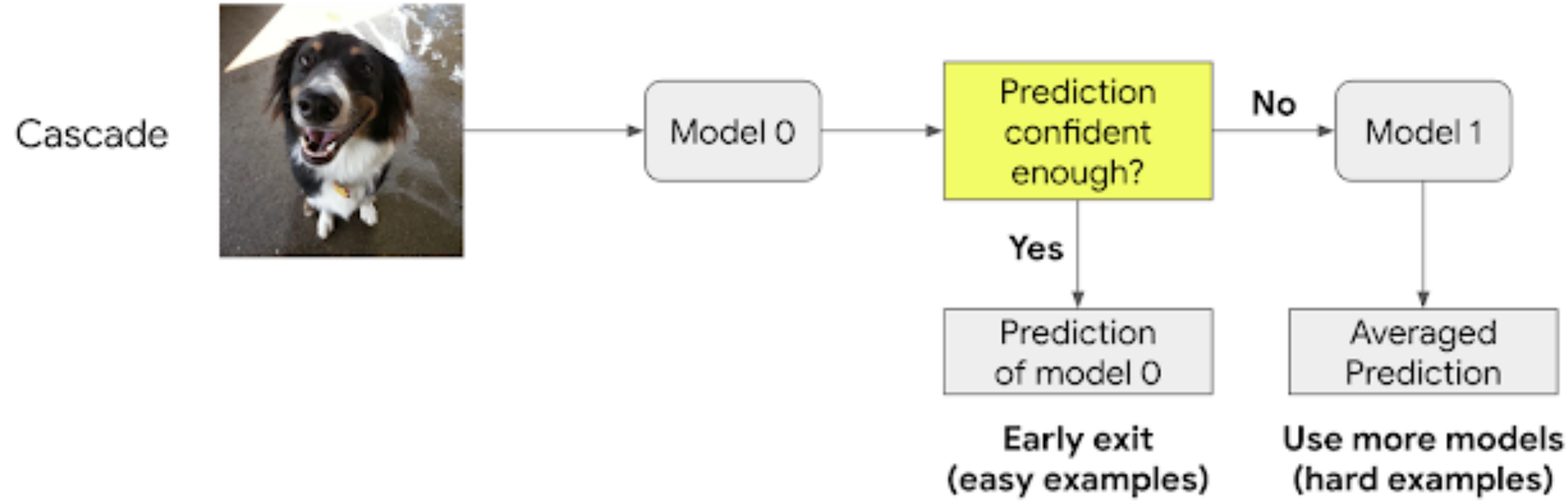
What happens if you take more layers and take smaller steps?

You can actually treat a neural network as an **ODE**: 
$$\frac{d\mathbf{h}(t)}{dt} = f(\mathbf{h}(t), t, \theta)$$

[ Chen et al., NIPS 2018 **best paper** ]

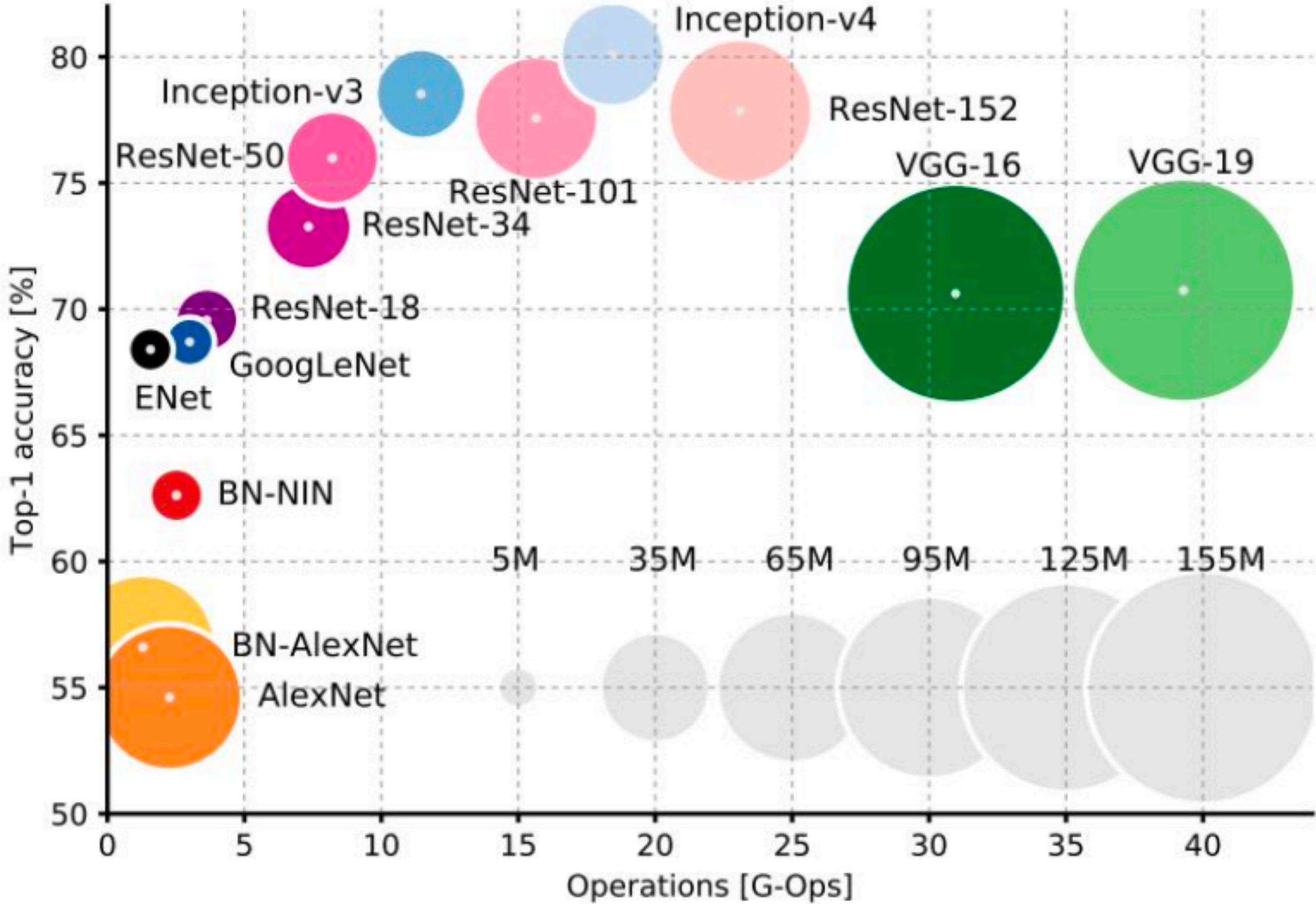
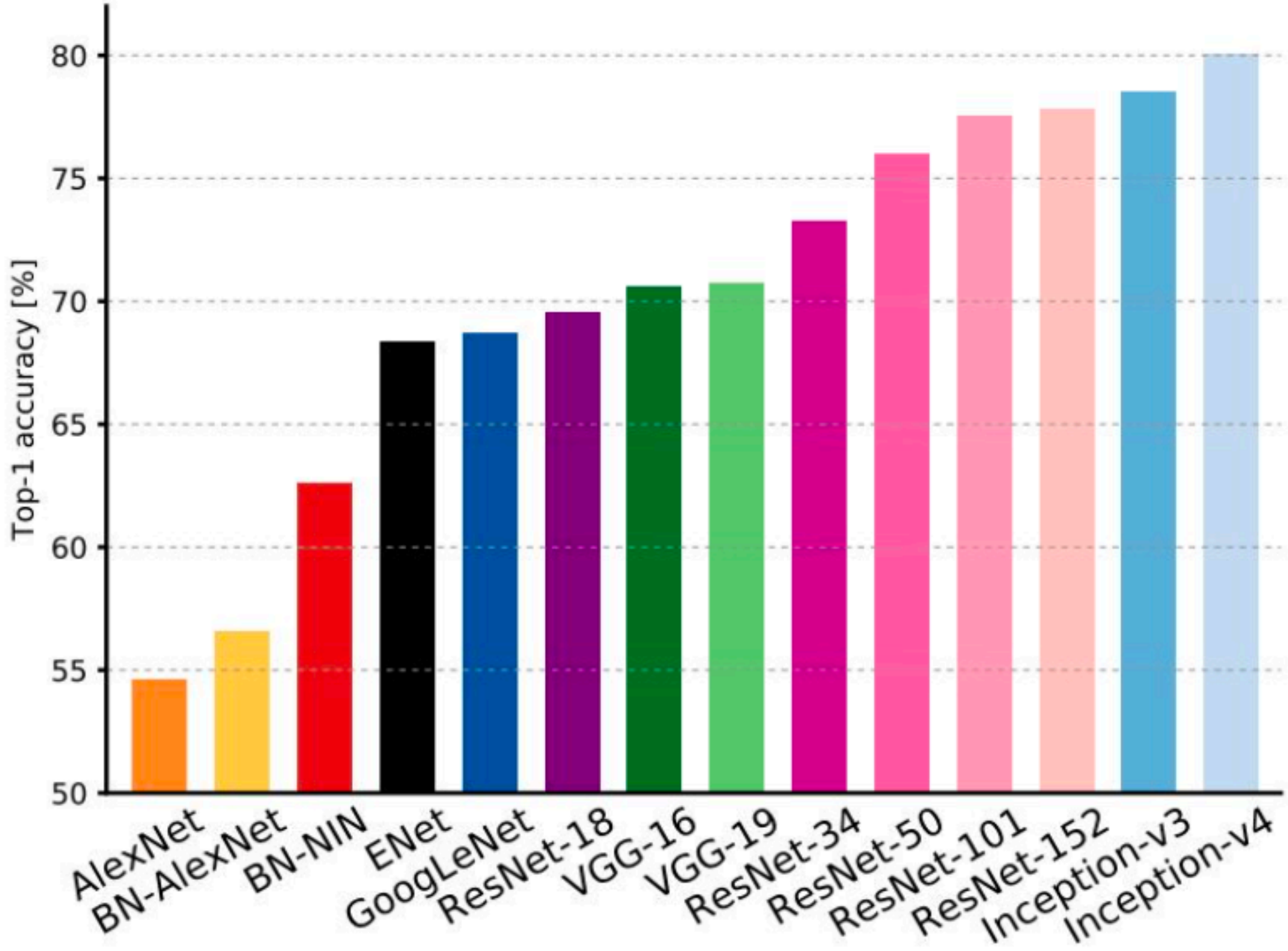
# An Aside: Neural Network Cascades

[ Wang et al., ICLR 2022 ]





# Comparing Complexity



An Analysis of Deep Neural Network Models for Practical Applications, 2017.

\* adopted from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**



# Computer **Vision Problems** (no language for now)

## Categorization

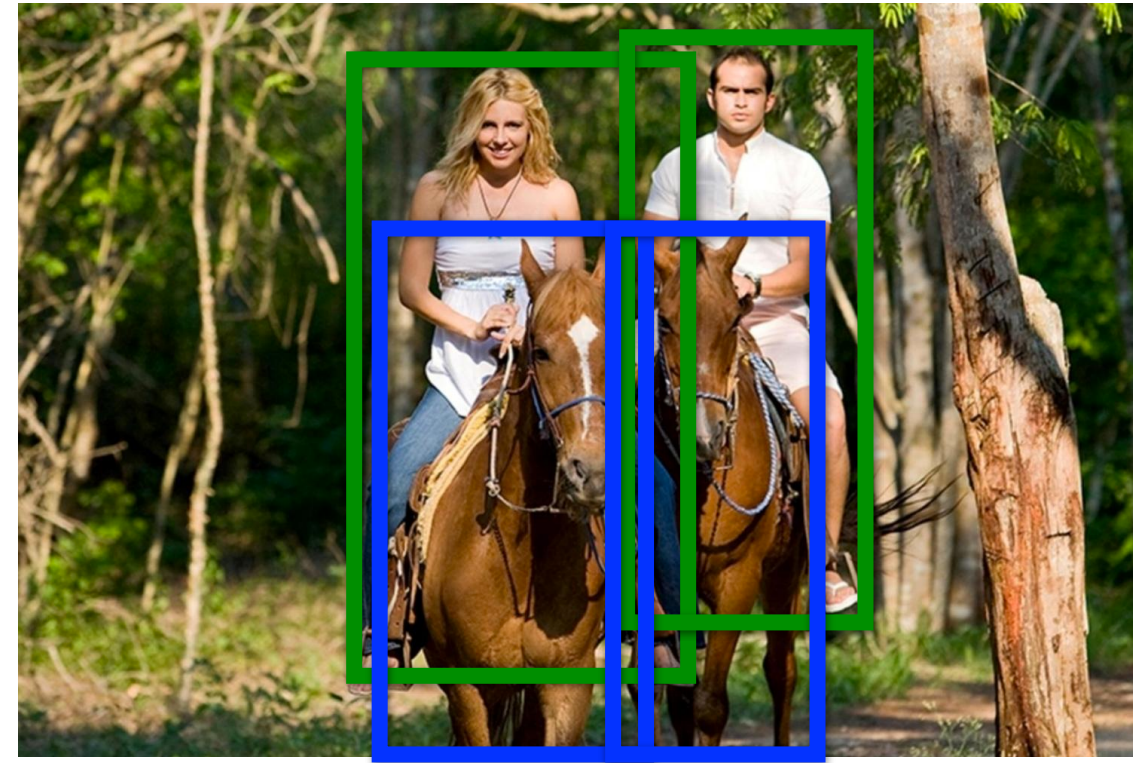


Multi-**class**:  
Horse  
Church  
Toothbrush  
**Person**

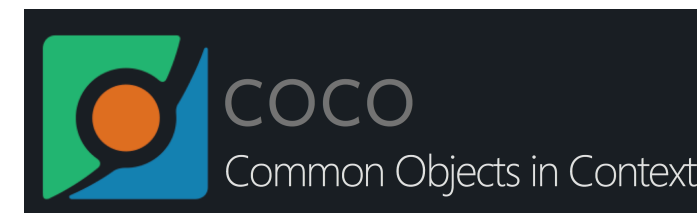
IMAGENET

Multi-**label**:  
**Horse**  
Church  
Toothbrush  
**Person**

## Detection

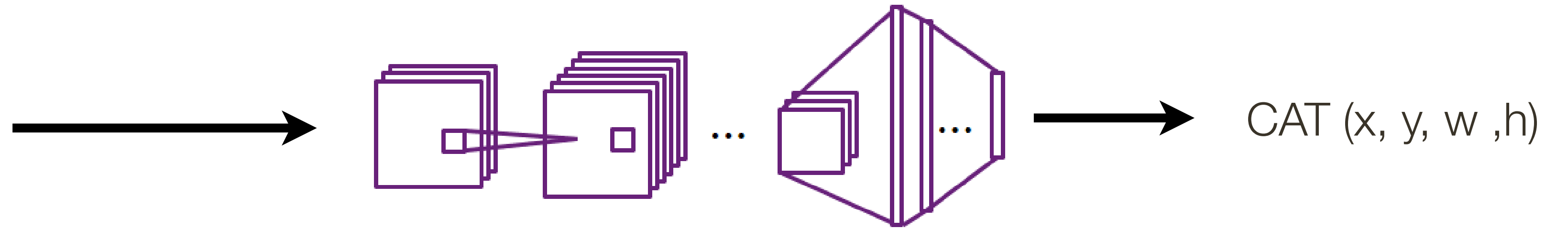


Horse (x, y, w, h)  
Horse (x, y, w, h)  
Person (x, y, w, h)  
Person (x, y, w, h)



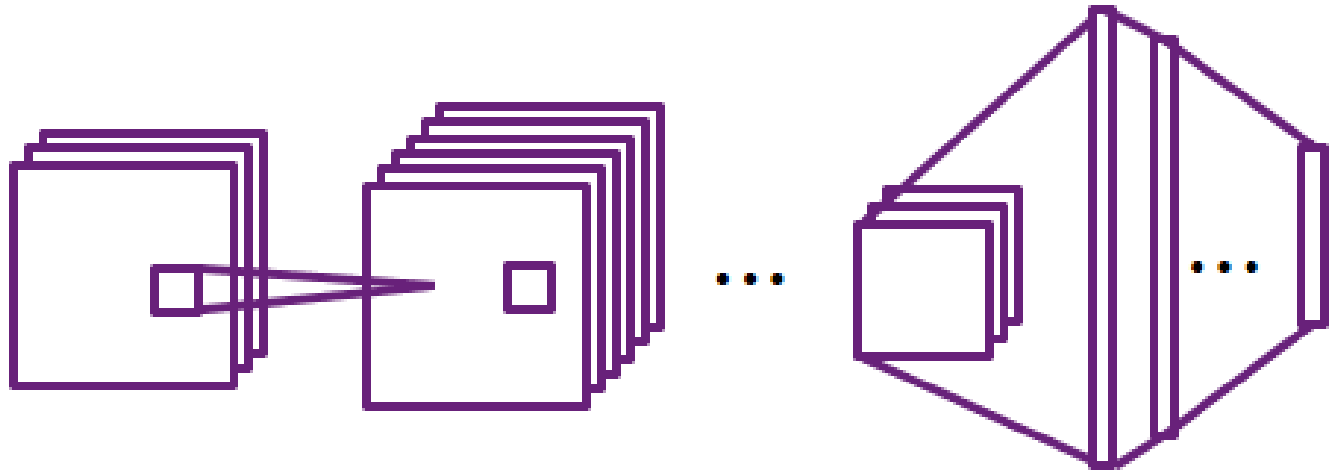


# Object **Detection** as Regression Problem

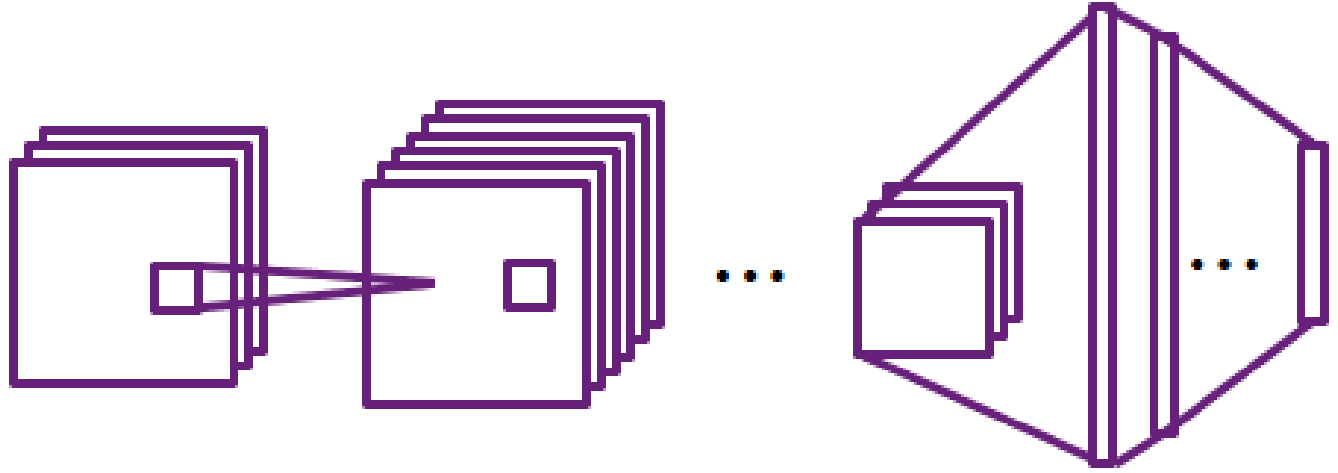




# Object **Detection** as Regression Problem



CAT (x, y, w, h)

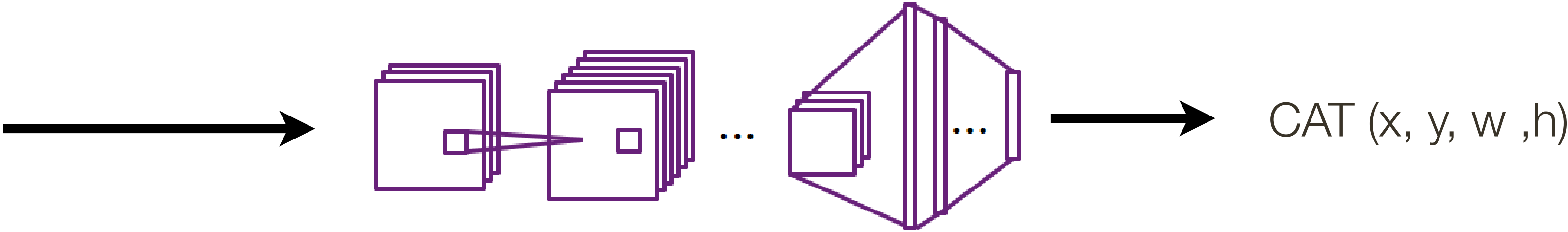


DUCK (x, y, w, h)  
DUCK (x, y, w, h)  
DUCK (x, y, w, h)  
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DUCK (x, y, w, h)  
...

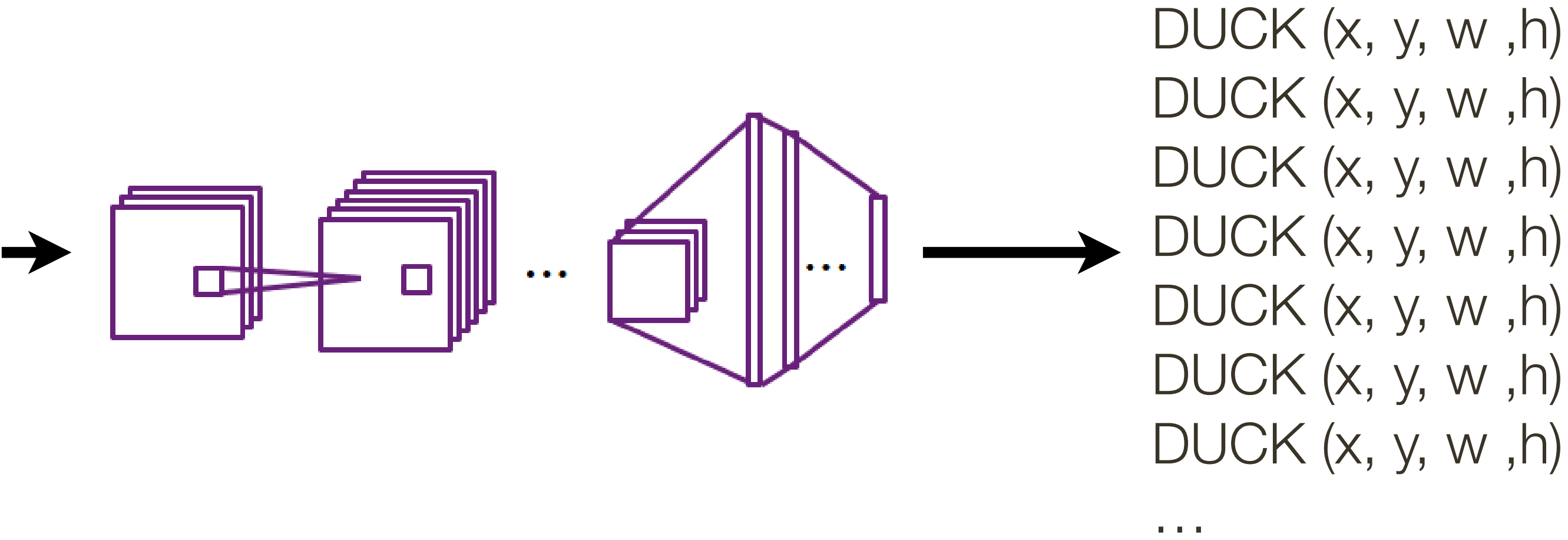
\* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**



# Object **Detection** as Regression Problem



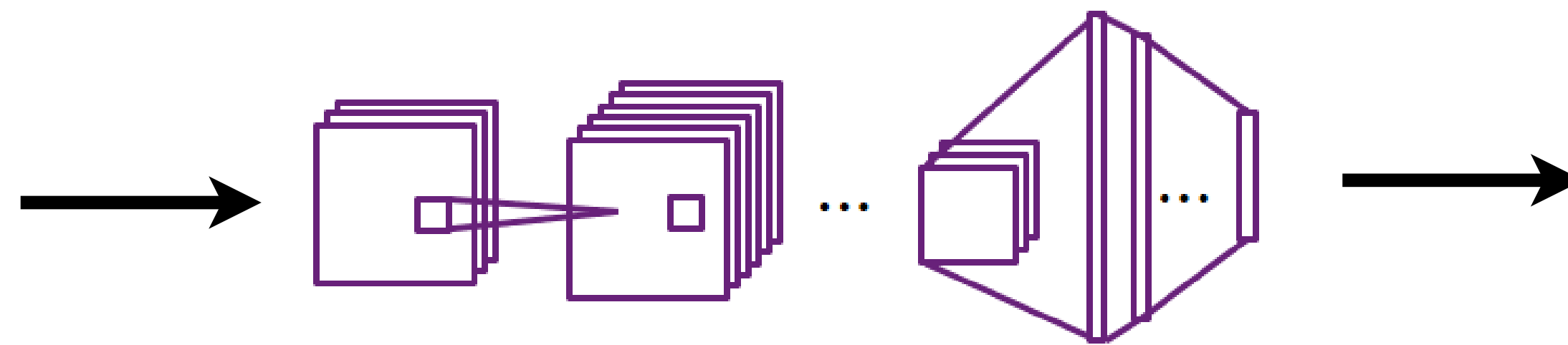
**Problem:** each image needs a different number of outputs



\* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**



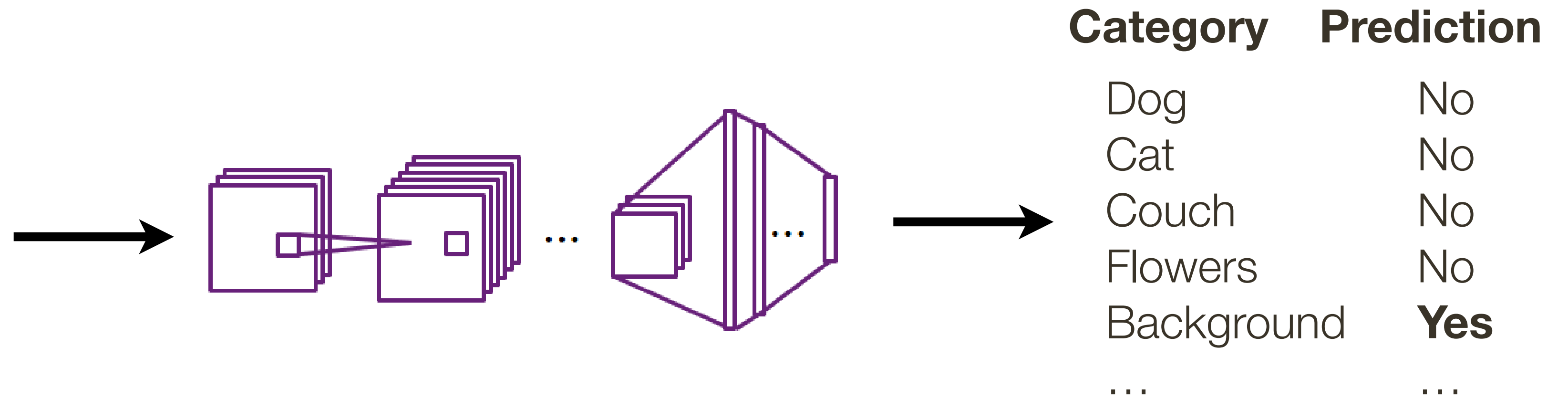
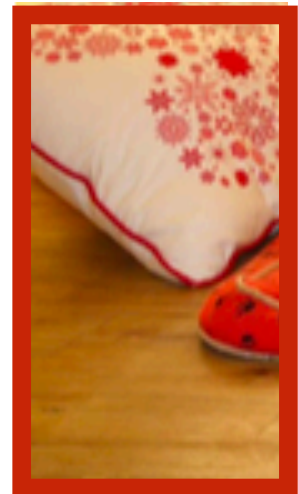
# Object **Detection** as Classification Problem



Category	Prediction
Dog	No
Cat	No
Couch	No
Flowers	No
Background	<b>Yes</b>
...	...

Apply CNN to many different crops in the image and (classification) CNN classifies each patch as object or background

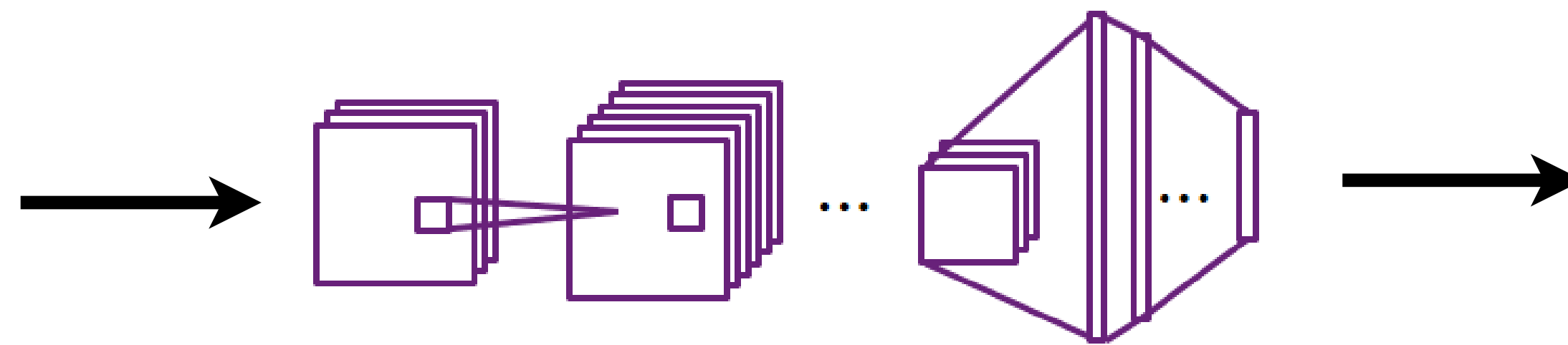
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Apply CNN to many different crops in the image and (classification) CNN classifies each patch as object or background



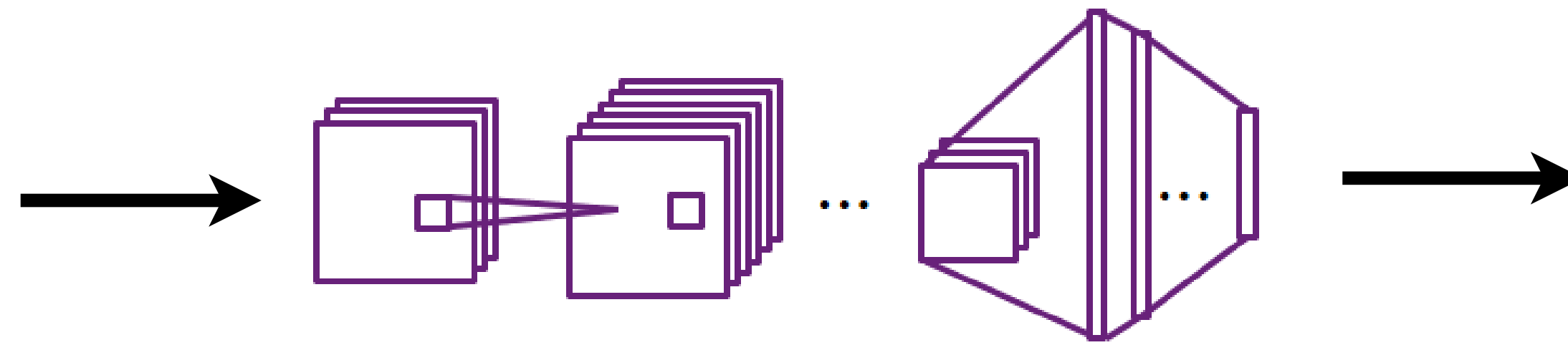
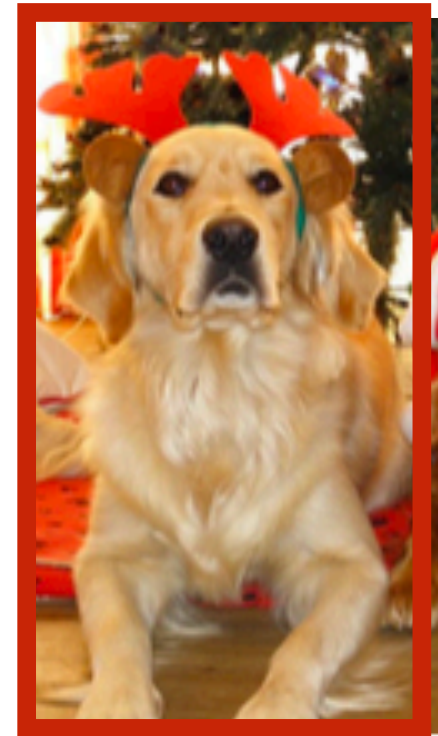
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# Object **Detection** as Classification Problem

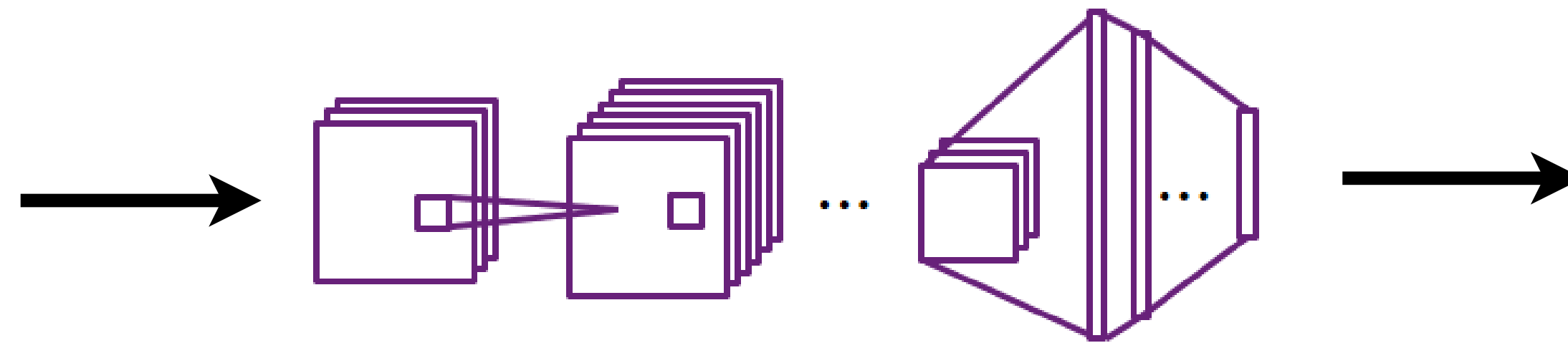


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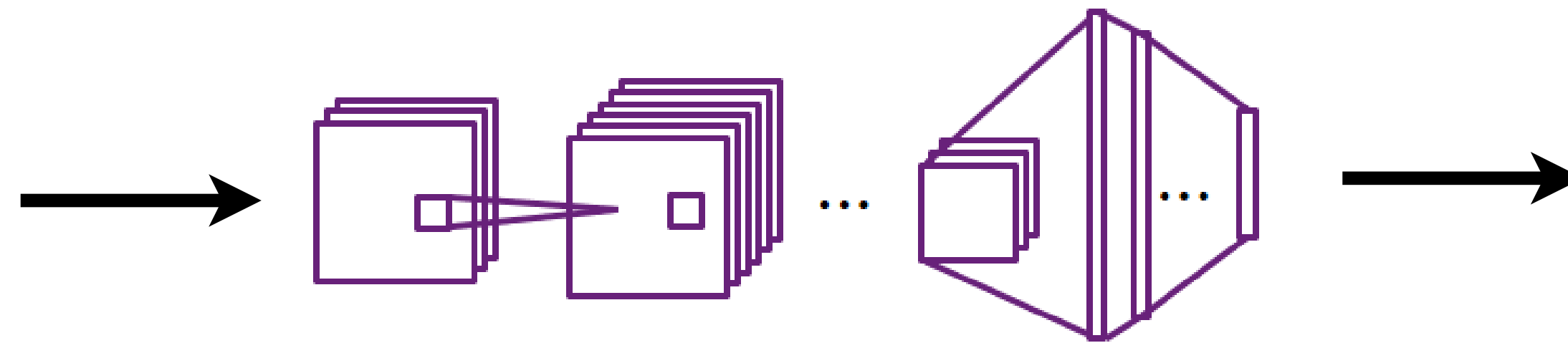
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Apply CNN to many different crops in the image and (classification) CNN classifies each patch as object or background



# Object **Detection** as Classification Problem

**Problem:** Need to apply CNN to **many** patches in each image



Category	Prediction
Dog	No
Cat	<b>Yes</b>
Couch	No
Flowers	No
Background	No
...	...

Apply CNN to many different crops in the image and (classification) CNN classifies each patch as object or background



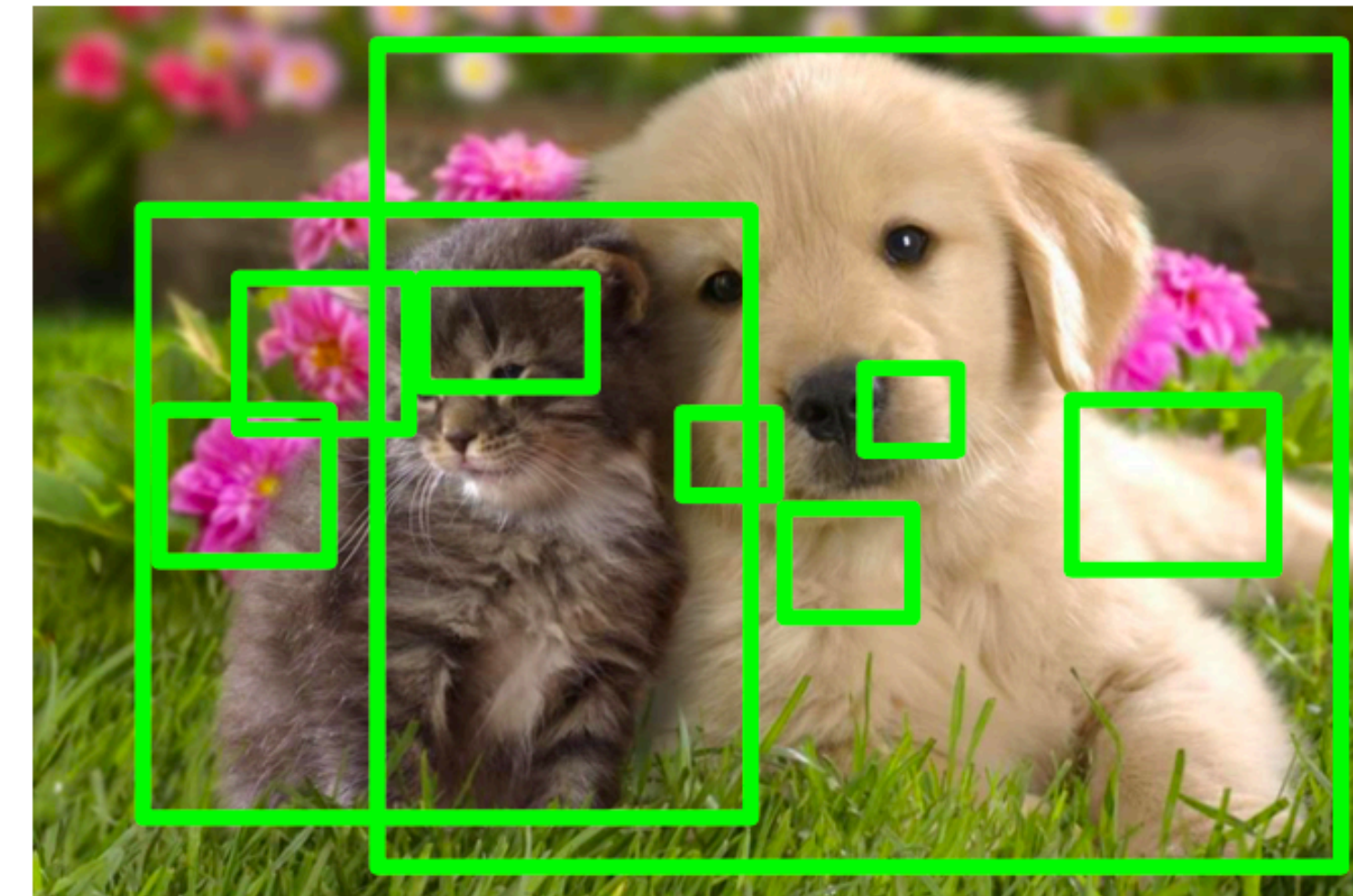
# Region Proposals (older idea in vision)

[ Alexe et al, TPAMI 2012 ]  
[ Ujkings et al, IJCV 2013 ]  
[ Cheng et al, CVPR 2014 ]  
[ Zitnick and Dollar, ECCV 2014 ]

Find image **regions that are likely contain objects** (any object at all)

- typically works by looking at histogram distributions, region aspect ratio, closed contours, coherent color

Relatively **fast to run** (Selective Search gives 1000 region proposals in a few seconds on a CPU)



**Goal:** Get “true” object regions to be in as few top K proposals as possible



# R-CNN

[ Girshick et al, CVPR 2014 ]



Input **Image**

\* image from Ross Girshick

# R-CNN

[ Girshick et al, CVPR 2014 ]

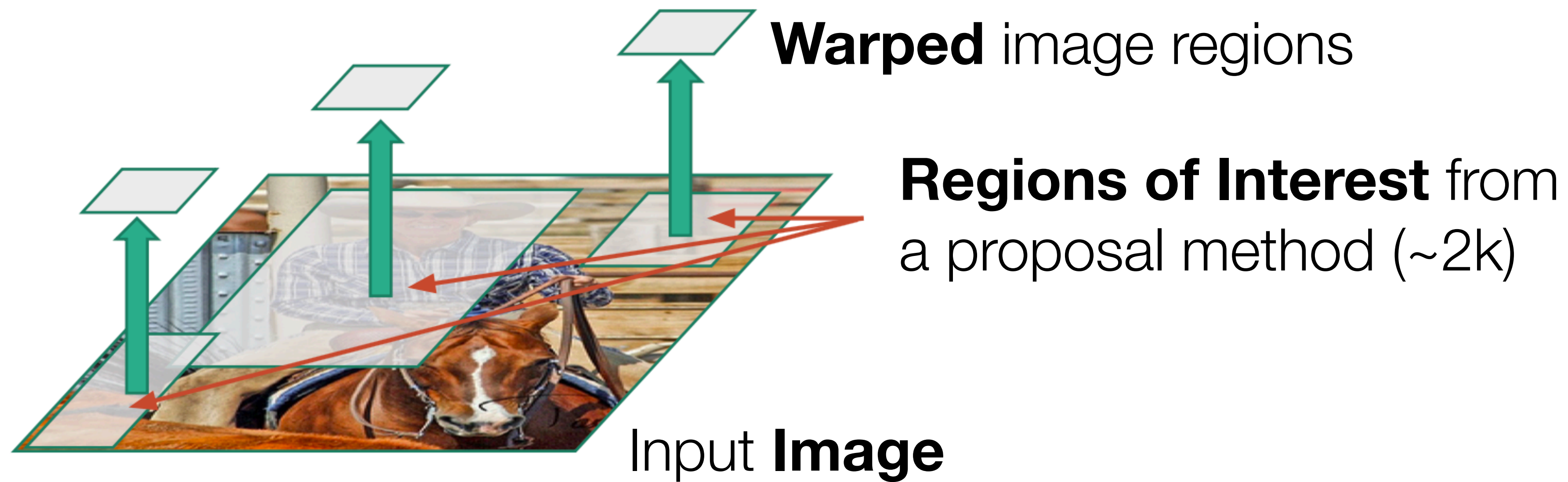


**Regions of Interest** from  
a proposal method (~2k)



# R-CNN

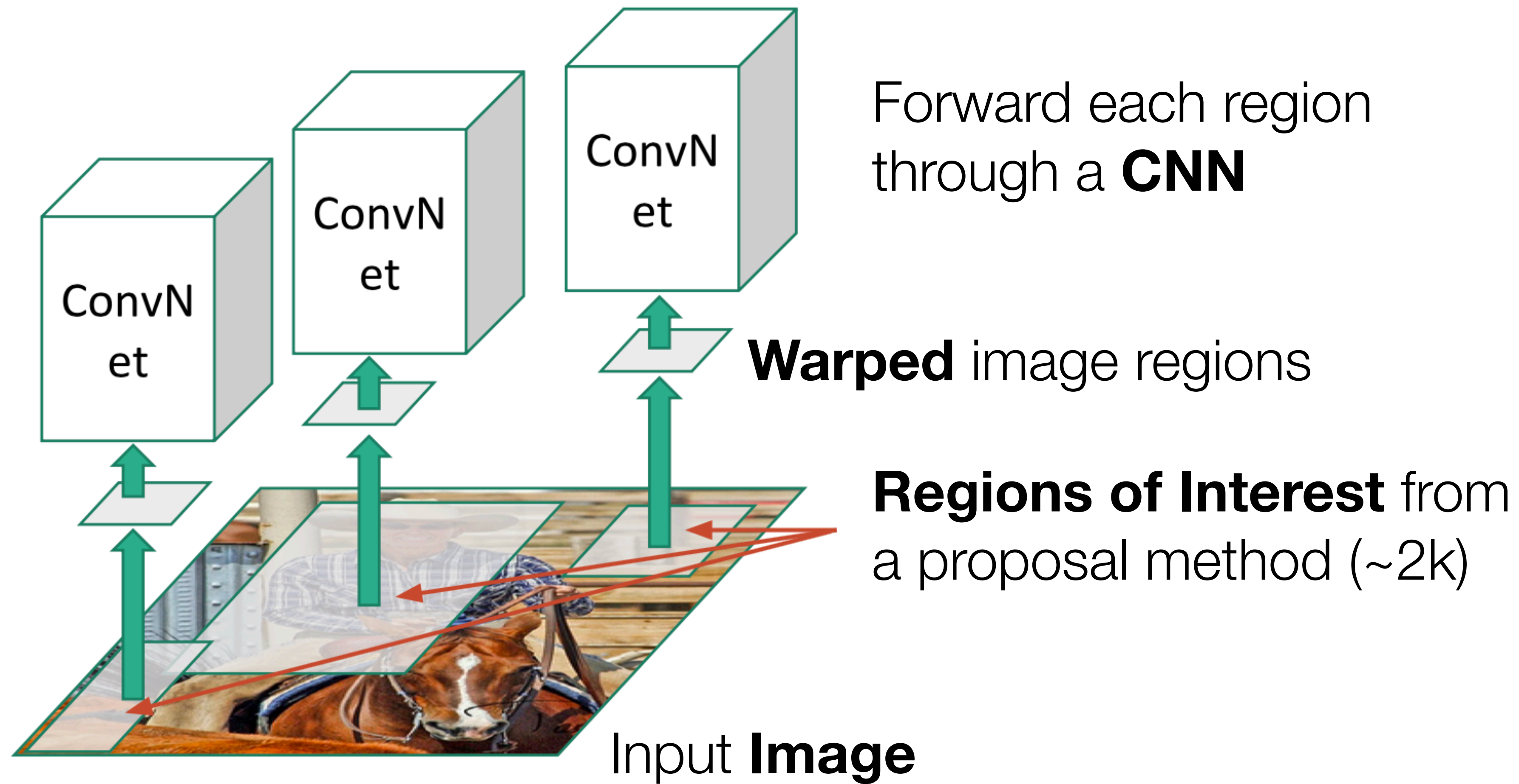
[ Girshick et al, CVPR 2014 ]



\* image from Ross Girshick

# R-CNN

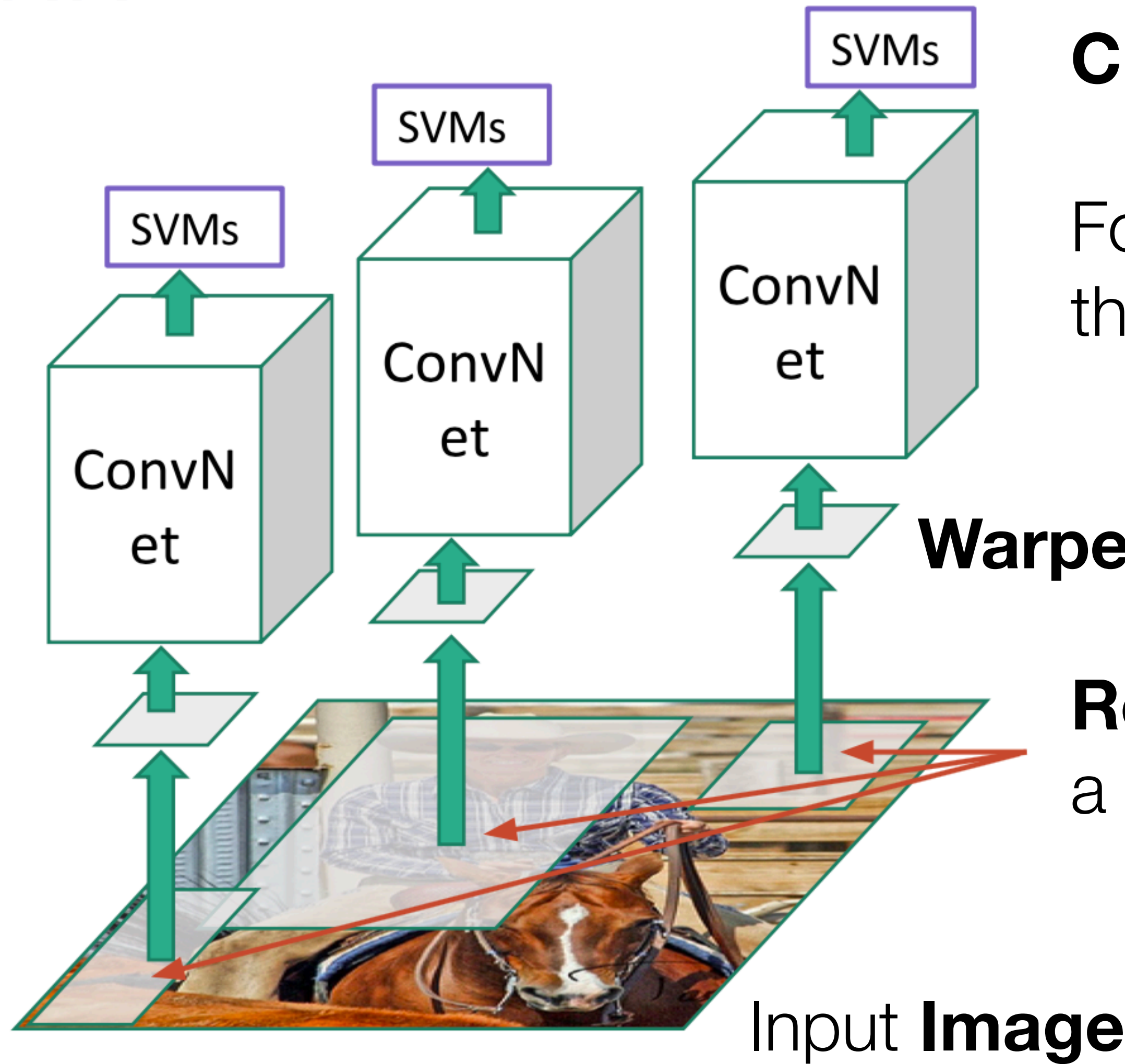
[ Girshick et al, CVPR 2014 ]





# R-CNN

[ Girshick et al, CVPR 2014 ]



**Classify** regions with SVM

Forward each region through a **CNN**

**Warped** image regions

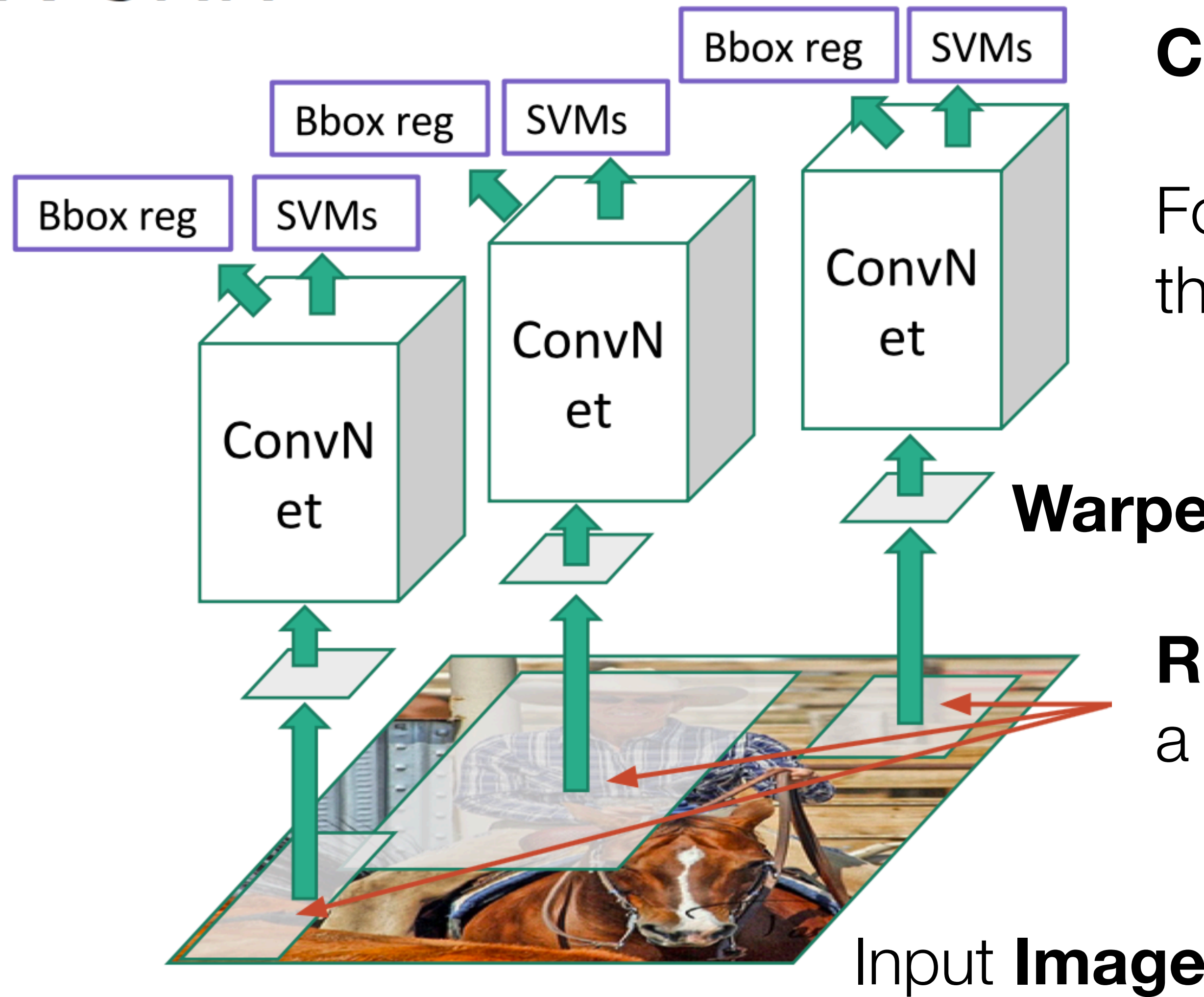
**Regions of Interest** from a proposal method (~2k)

Input **Image**

# R-CNN

**Linear Regression** for bounding box offsets

[ Girshick et al, CVPR 2014 ]



**Classify** regions with SVM

Forward each region through a **CNN**

**Warped** image regions

**Regions of Interest** from a proposal method (~2k)



# R-CNN

R-CNN (Regions with CNN features) algorithm:

- Extract promising candidate regions using an object proposals algorithm
- Resize each proposal window to the size of the input layer of a trained convolutional neural network
- Input each resized image patch to the convolutional neural network

**Implementation detail:** Instead of using the classification scores of the network directly, the output of the final fully-connected layer can be used as an input feature to a trained support vector machine (SVM)

# Fast R-CNN

[ Girshick et al, ICCV 2015 ]



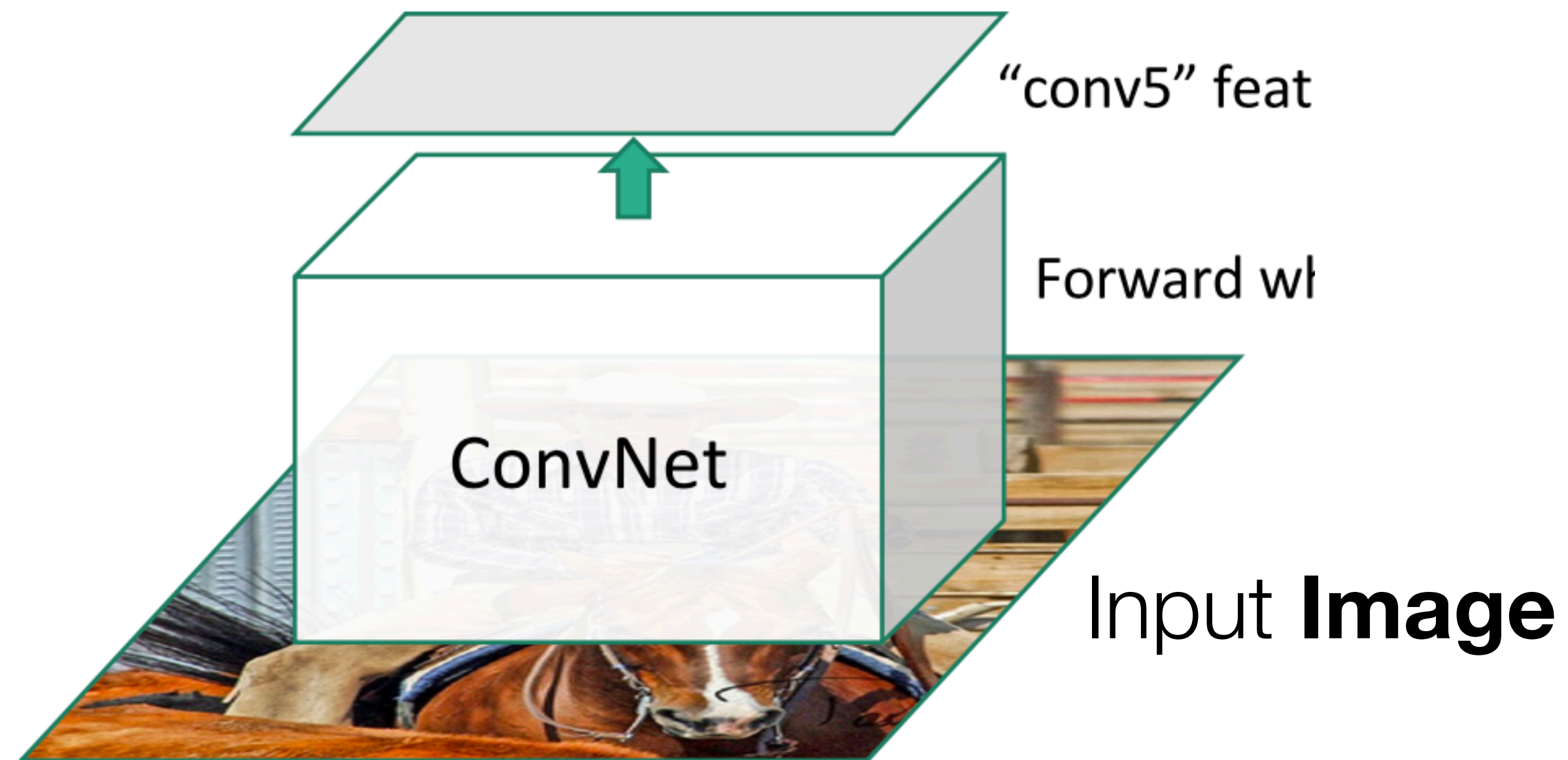
Input **Image**

\* image from Ross Girshick



# Fast R-CNN

[ Girshick et al, ICCV 2015 ]



\* image from Ross Girshick

# Fast R-CNN

[ Girshick et al, ICCV 2015 ]



\* image from Ross Girshick



# Fast R-CNN

[ Girshick et al, ICCV 2015 ]

**Regions of Interest**  
from the  
proposal  
method

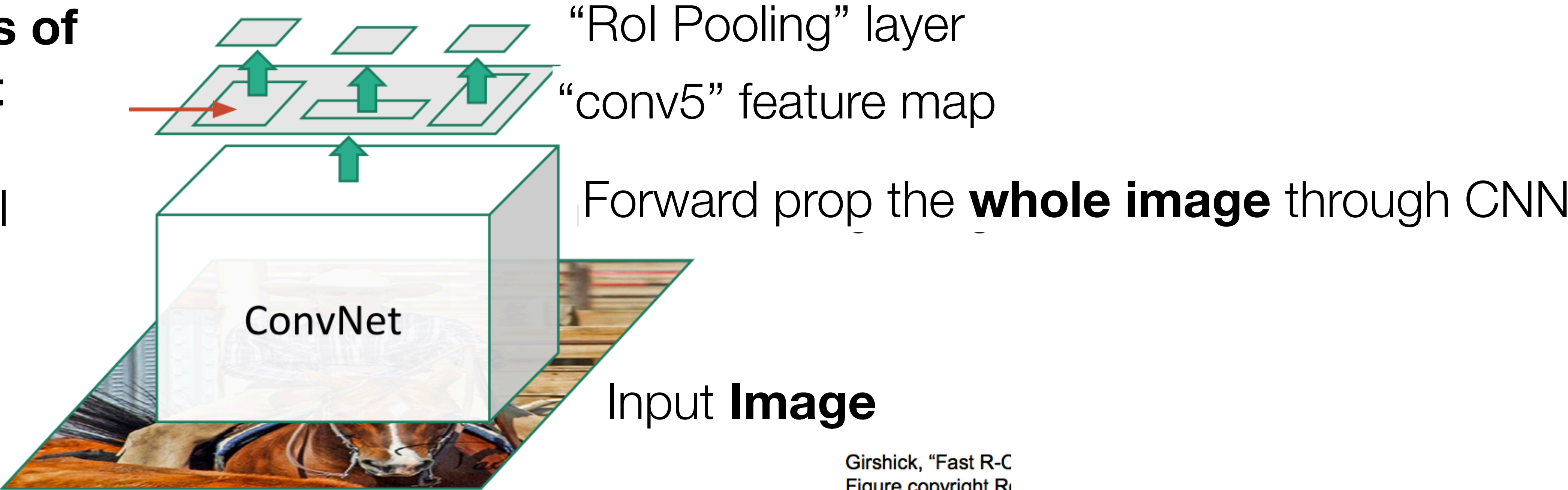


\* image from Ross Girshick

# Fast R-CNN

[ Girshick et al, ICCV 2015 ]

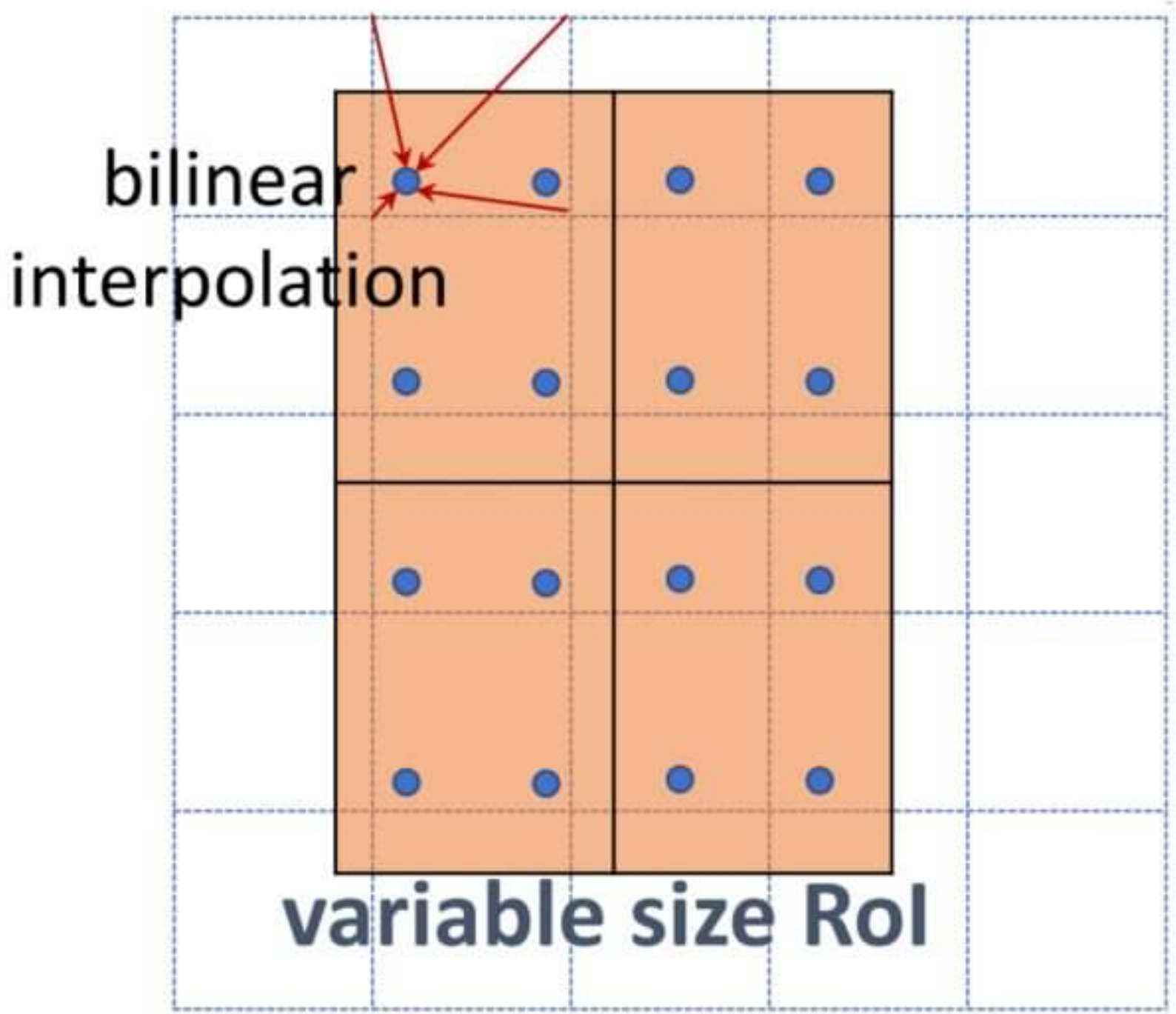
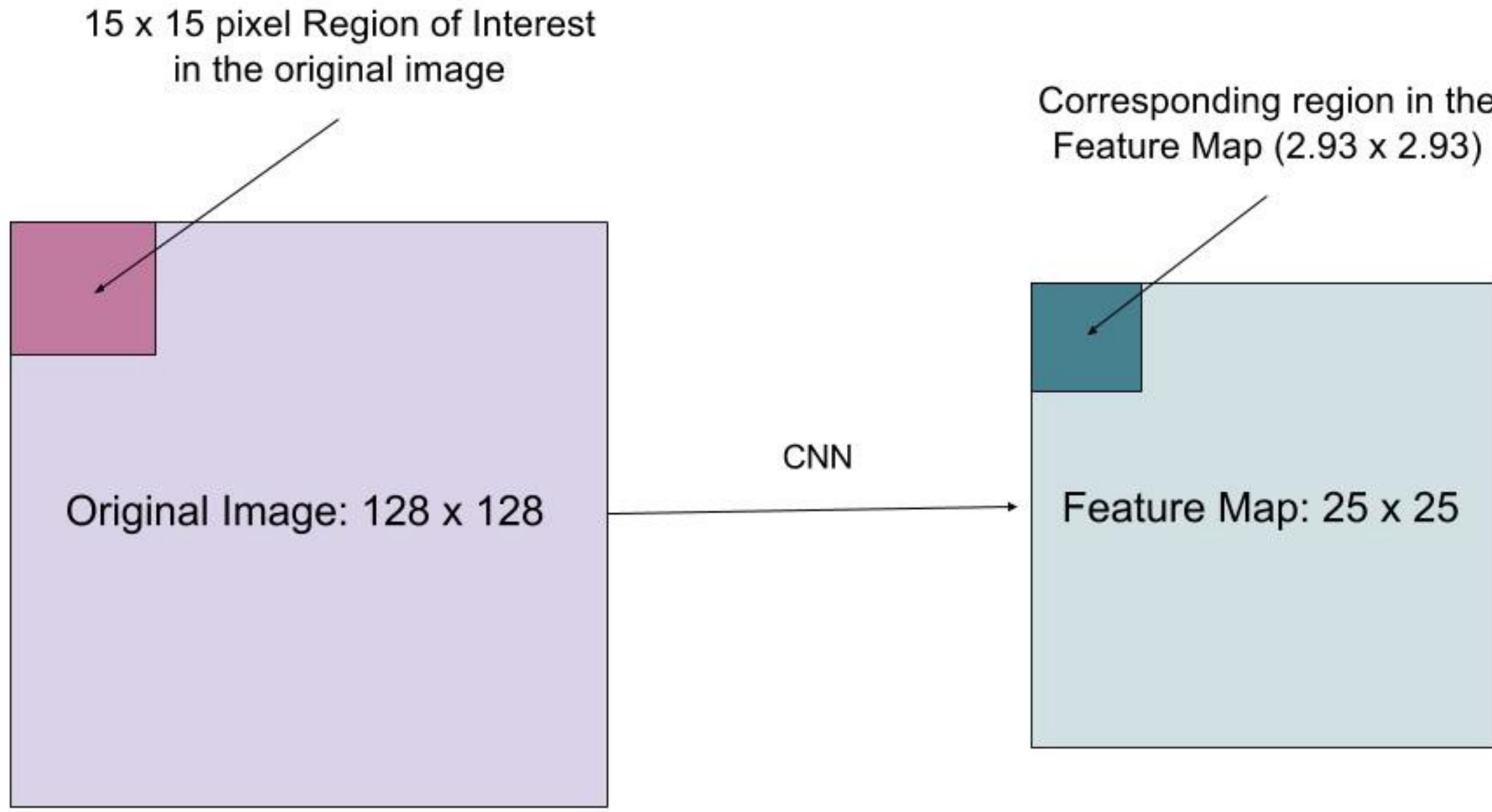
**Regions of Interest** from the proposal method



\* image from Ross Girshick

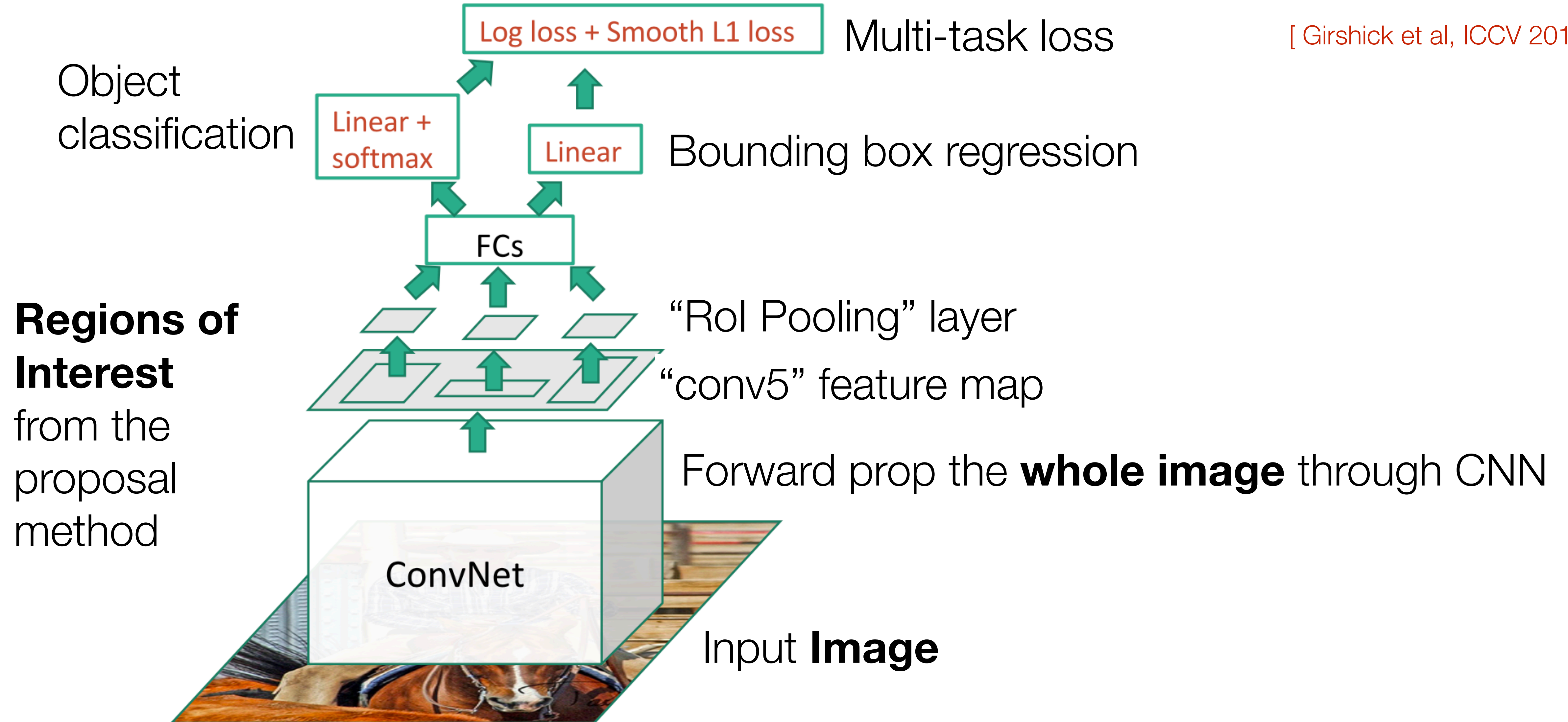


# RoI Align



# Fast R-CNN

[ Girshick et al, ICCV 2015 ]

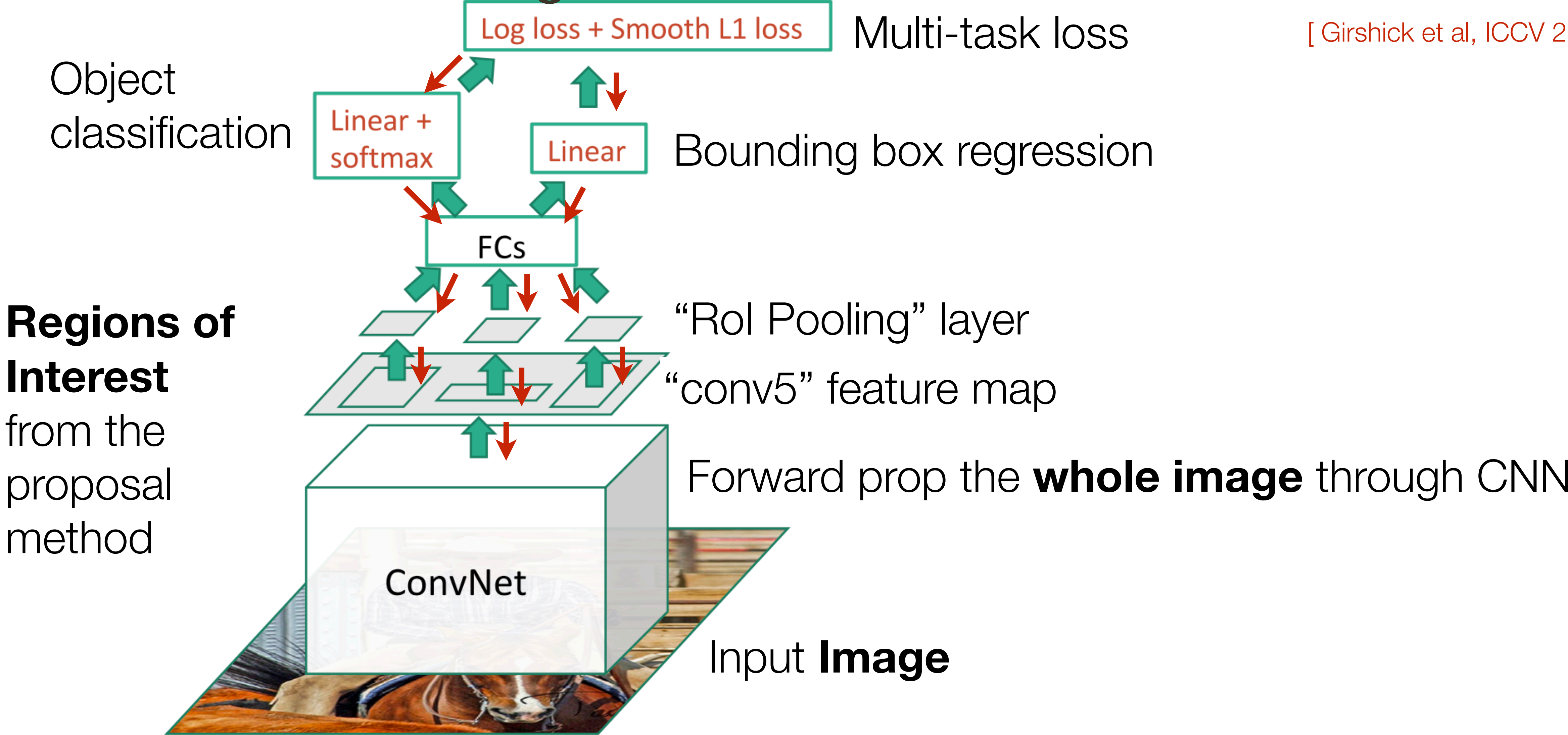


\* image from Ross Girshick



# Fast R-CNN: Training

[ Girshick et al, ICCV 2015 ]



\* image from Ross Girshick

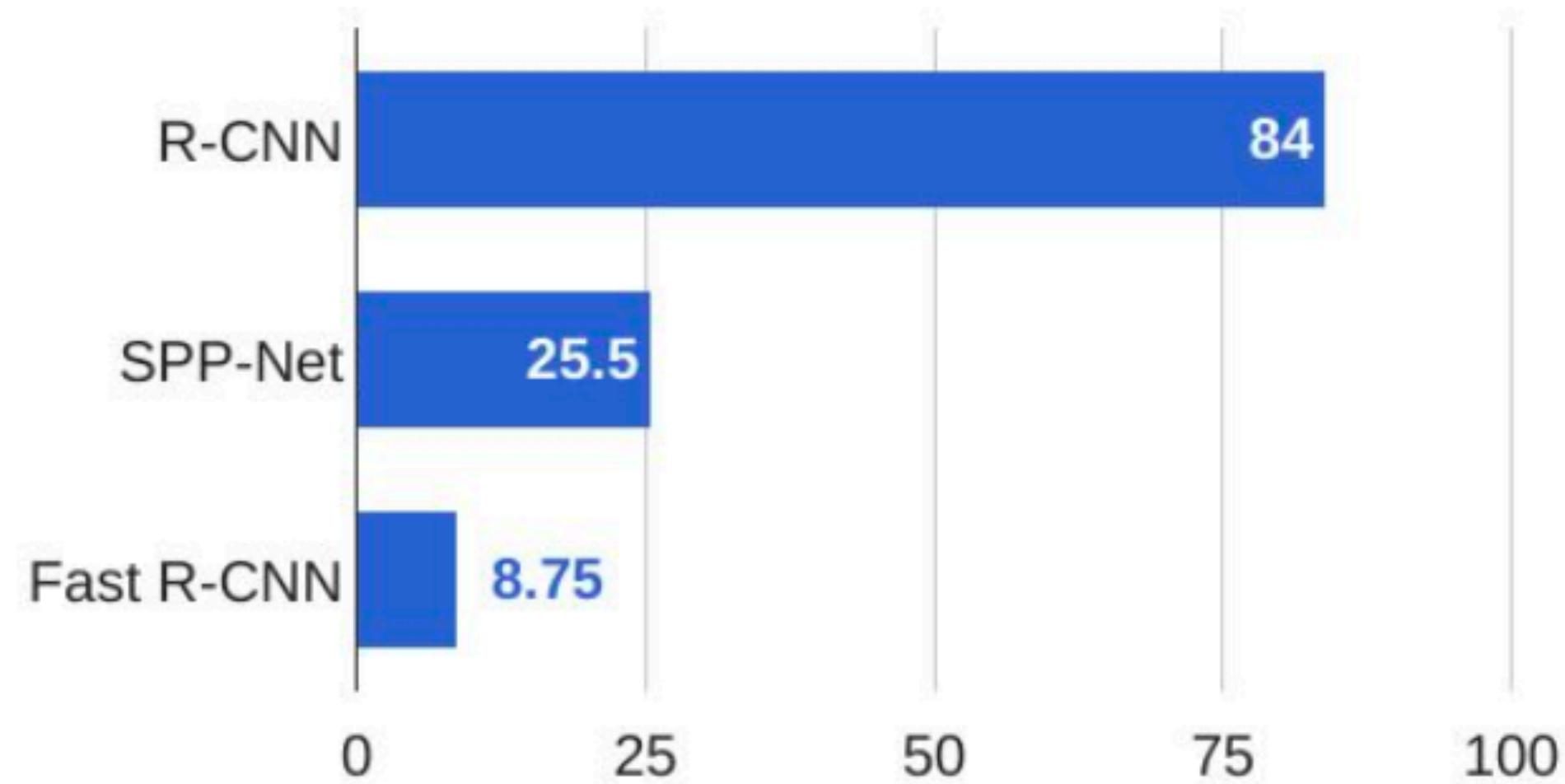
# R-CNN vs. SPP vs. Fast R-CNN

[ Girshick et al, CVPR 2014 ]

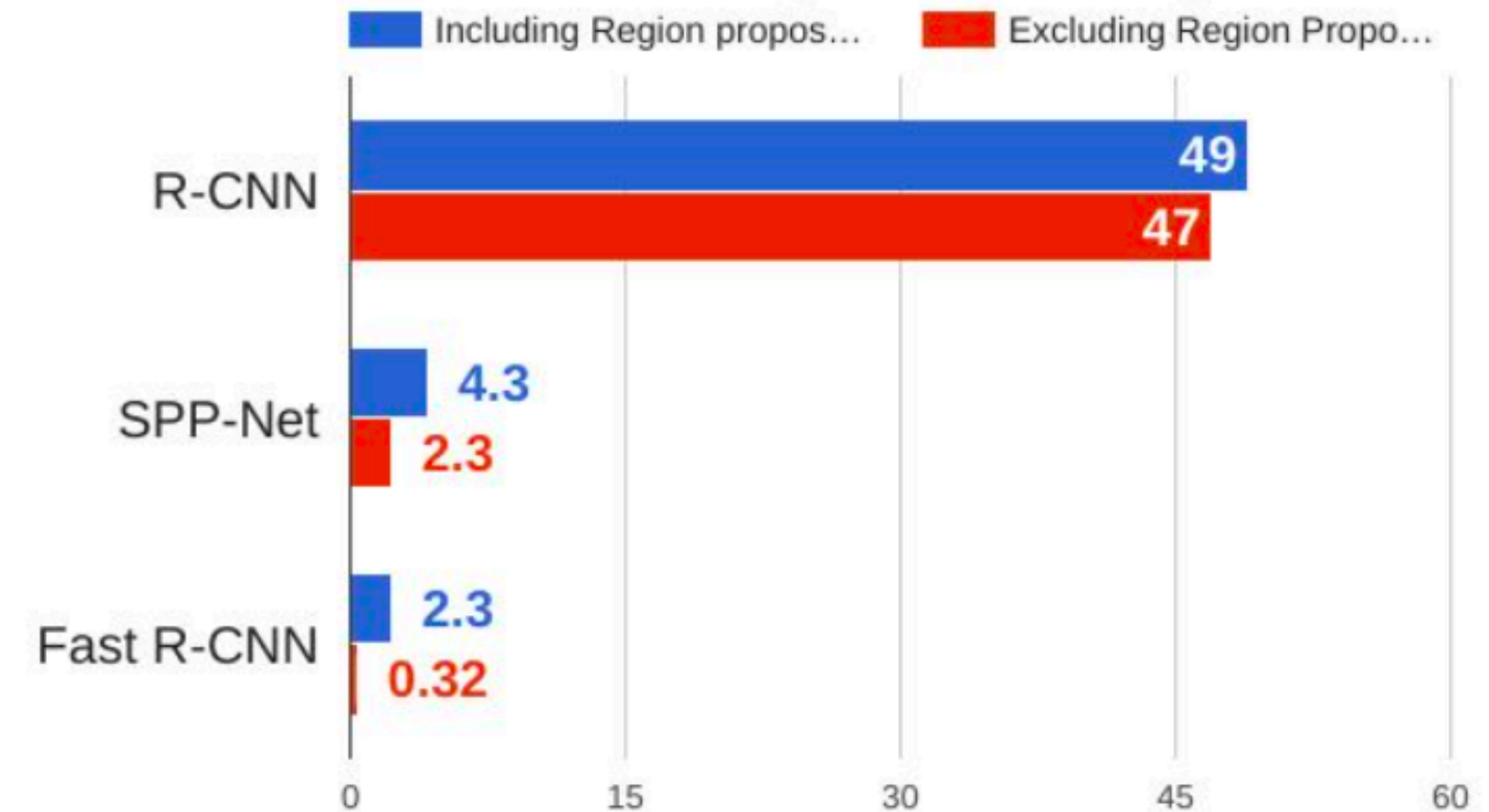
[ Girshick et al, ICCV 2015 ]

[ He et al, ECCV 2014 ]

## Training time (Hours)



## Test time (seconds)



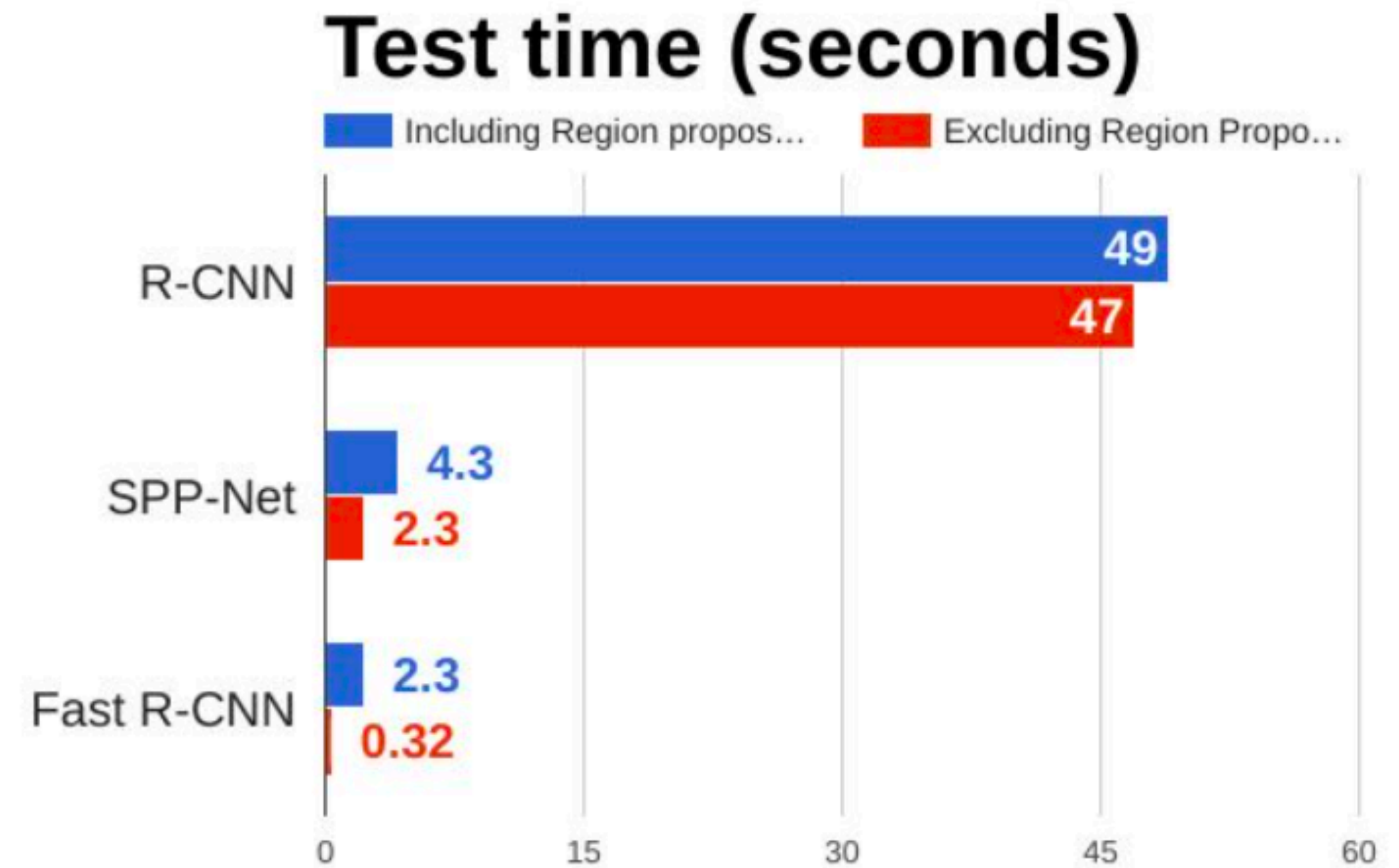
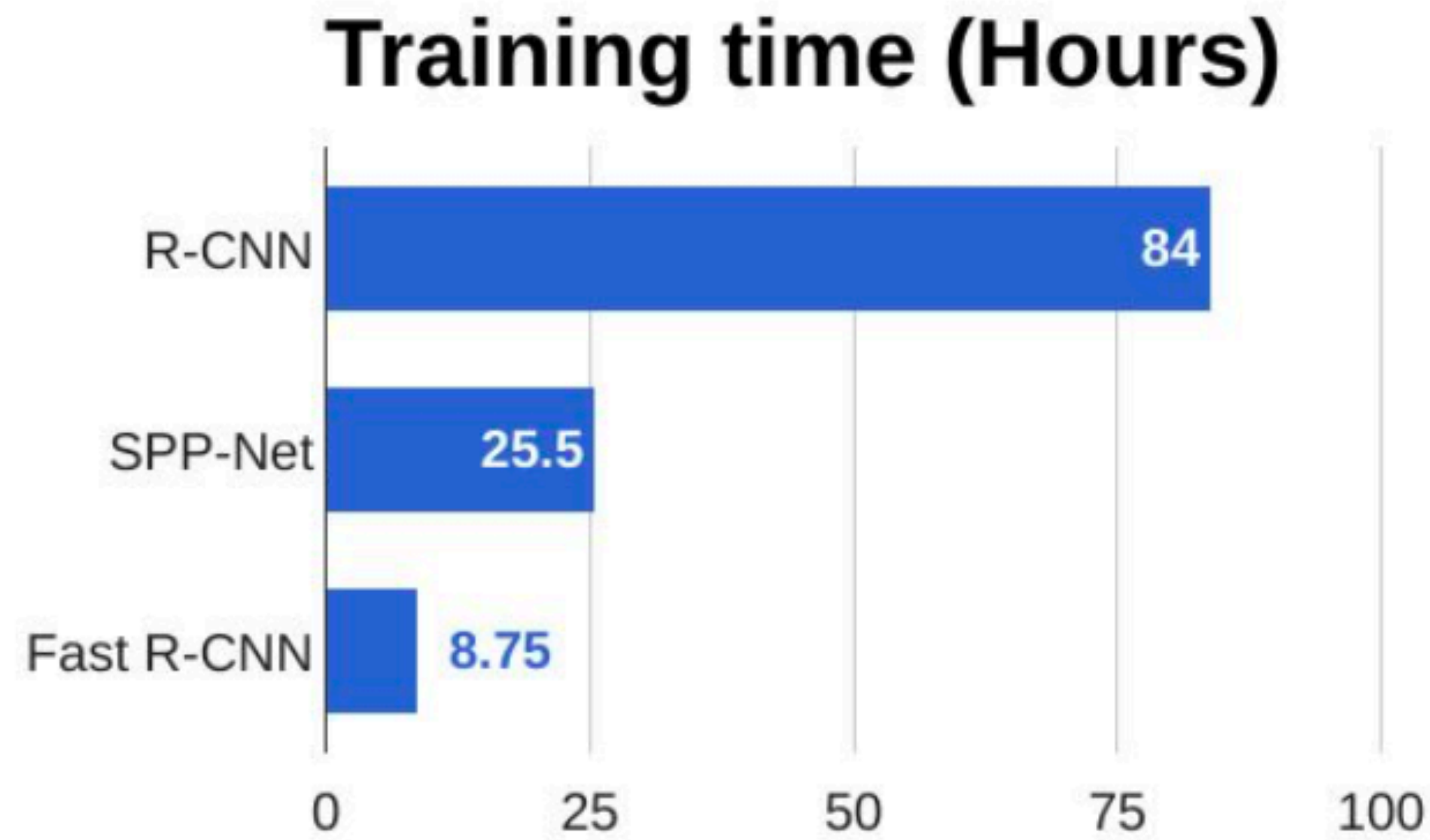


# R-CNN vs. SPP vs. Fast R-CNN

[ Girshick et al, CVPR 2014 ]

[ Girshick et al, ICCV 2015 ]

[ He et al, ECCV 2014 ]



**Observation:** Performance dominated by the region proposals at this point!

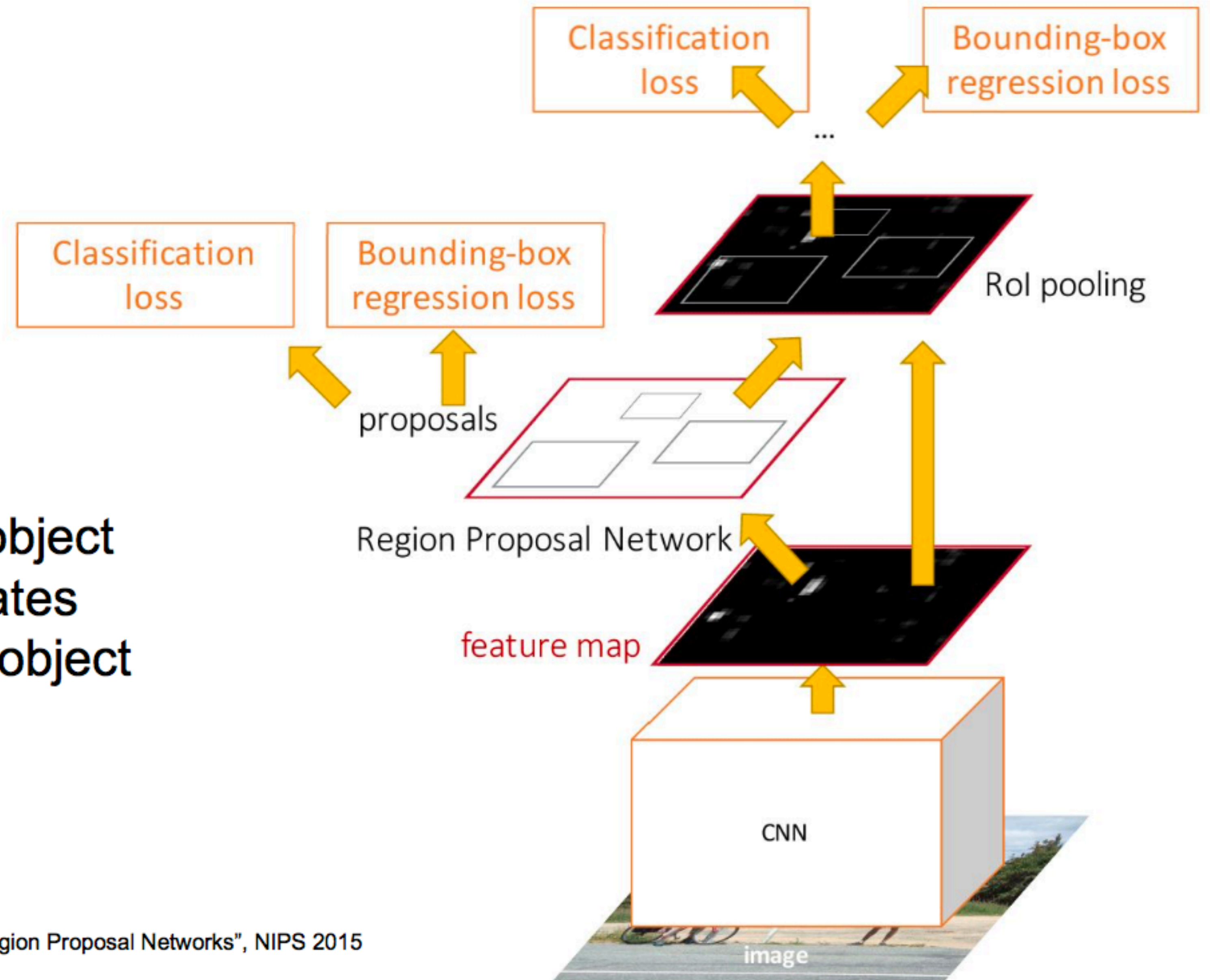
# Faster R-CNN

Make CNN do proposals!

Insert **Region Proposal Network (RPN)** to predict proposals from features

Jointly train with 4 losses:

1. RPN classify object / not object
2. RPN regress box coordinates
3. Final classification score (object classes)
4. Final box coordinates



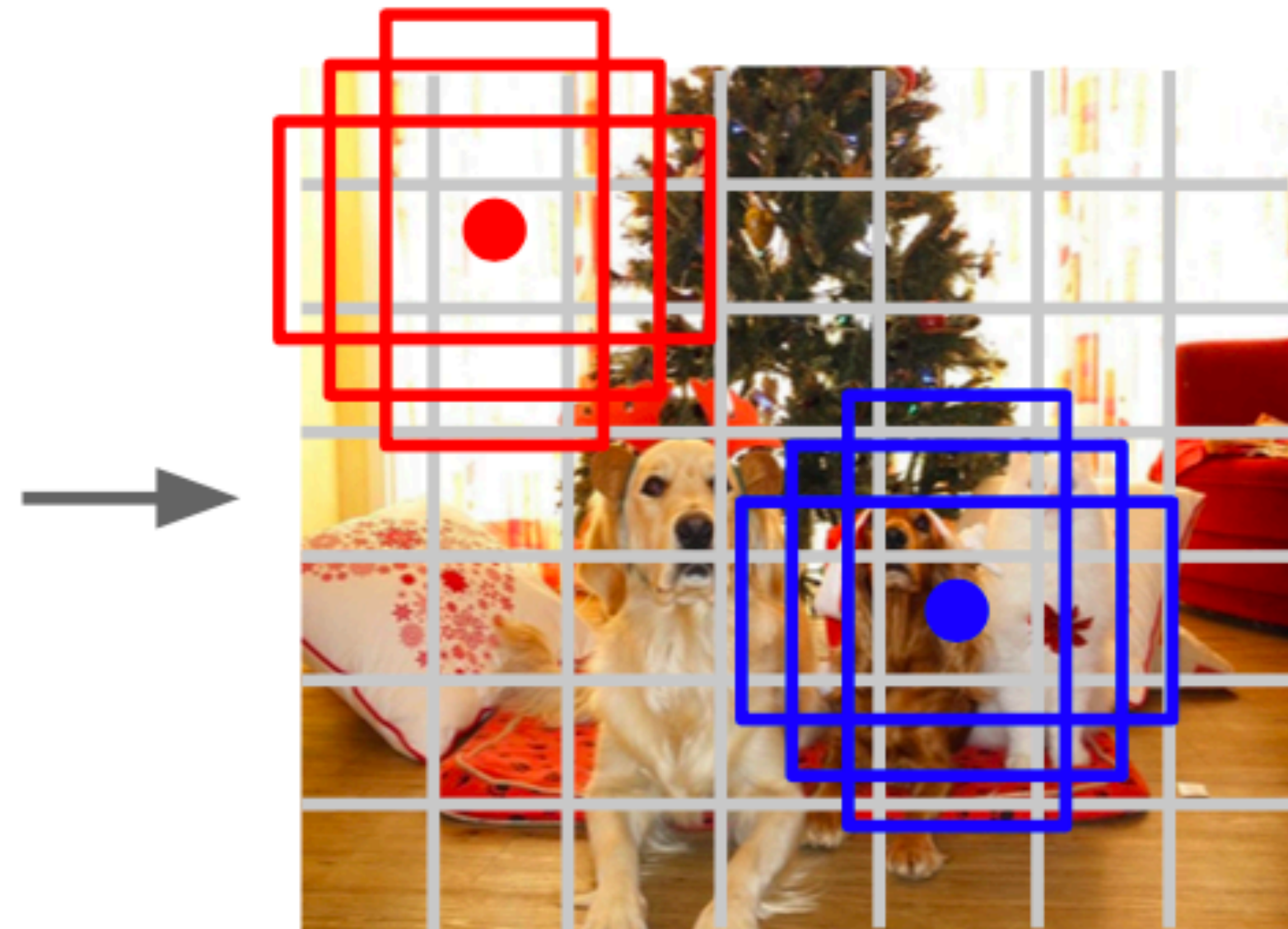


# YOLO: You Only Look Once

[ Redmon et al, CVPR 2016 ]



Input image  
 $3 \times H \times W$



Divide image into grid  
 $7 \times 7$

Image a set of **base boxes**  
centered at each grid cell  
Here  $B = 3$

Within each grid cell:

- Regress from each of the  $B$  base boxes to a final box with 5 numbers:  
(dx, dy, dh, dw, confidence)
- Predict scores for each of  $C$  classes (including background as a class)

Output:  
 $7 \times 7 \times (5 * B + C)$

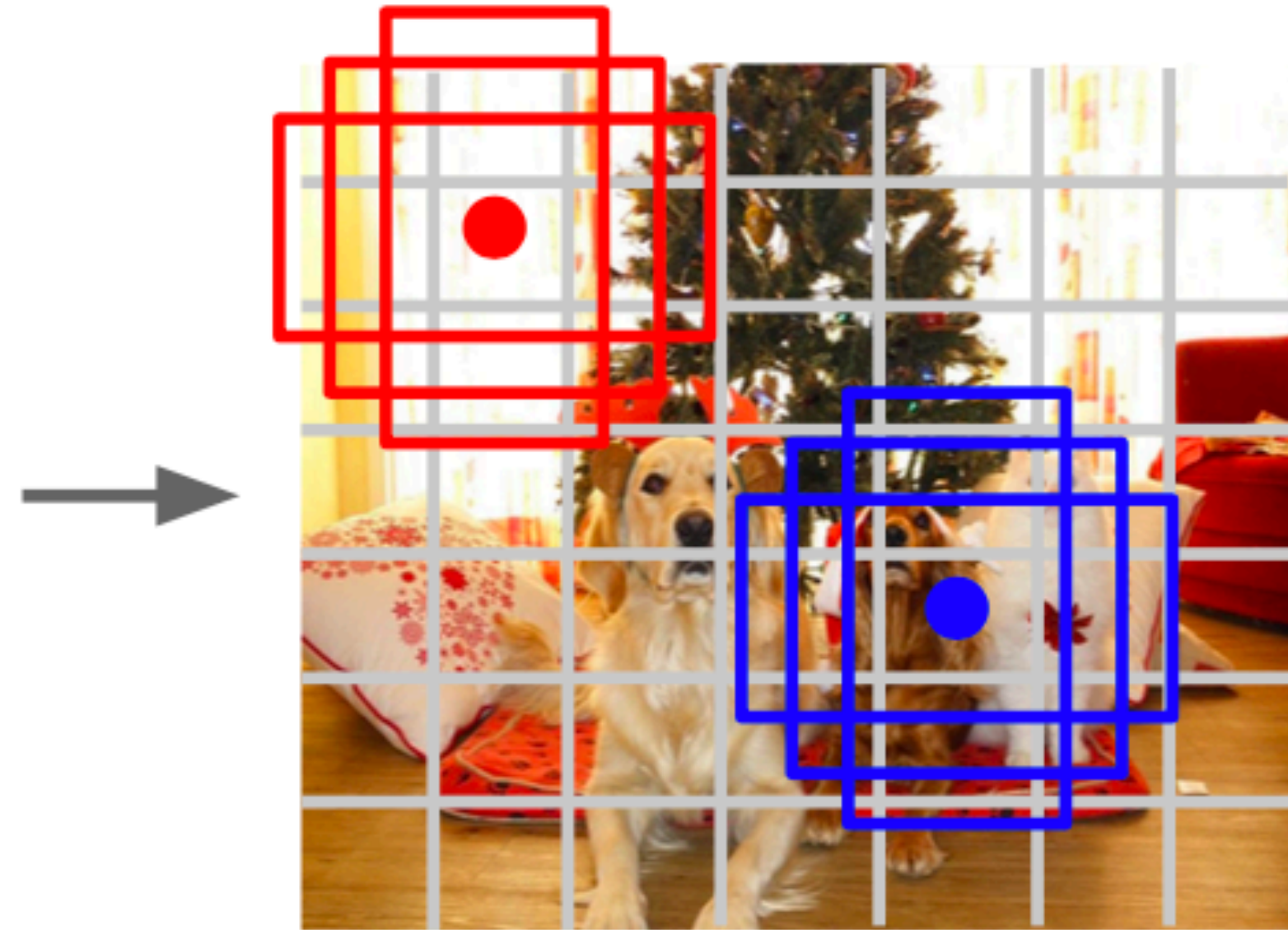


# YOLO: You Only Look Once

[ Redmon et al, CVPR 2016 ]

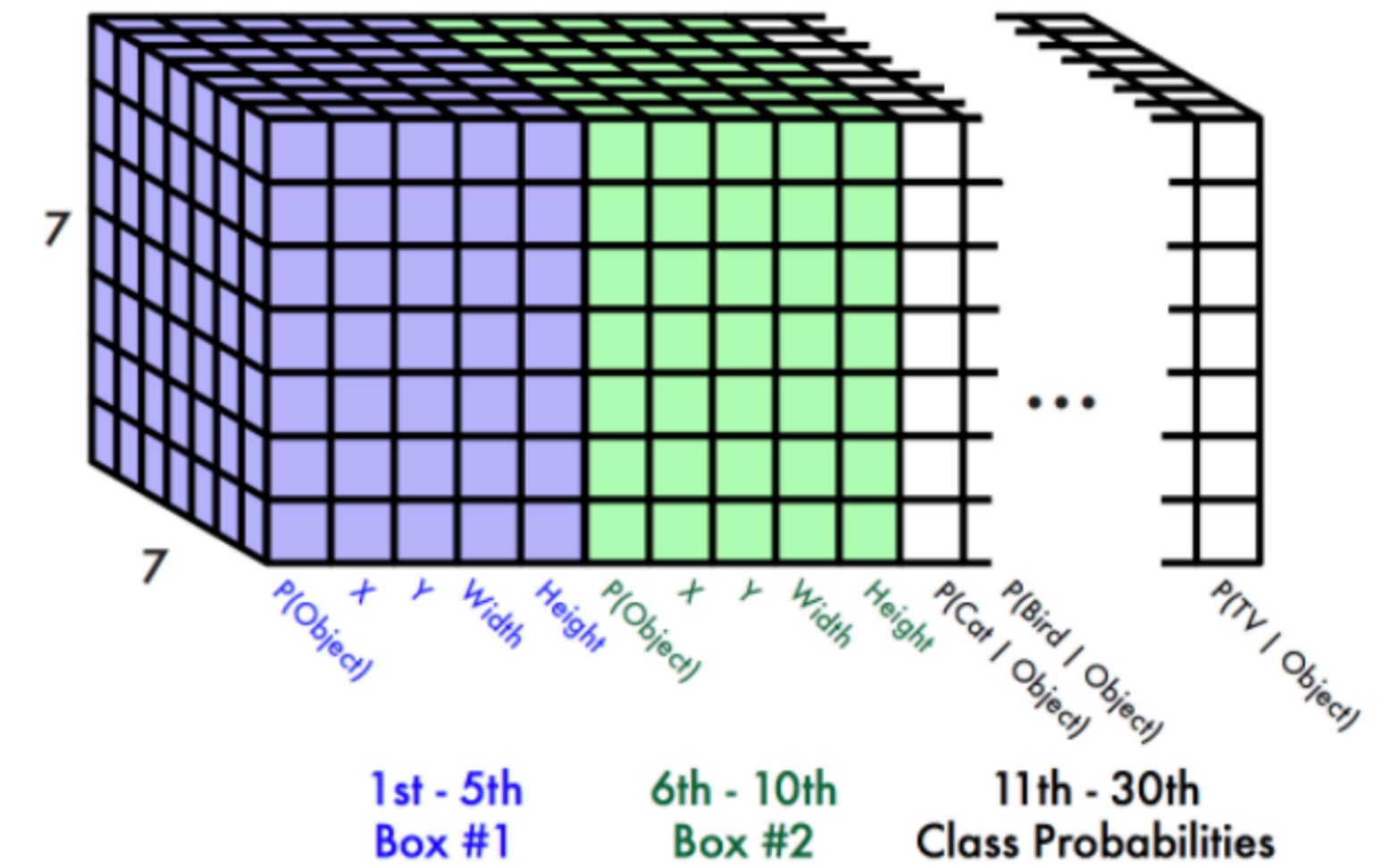


Input image  
 $3 \times H \times W$



Divide image into grid  
 $7 \times 7$

Image a set of **base boxes**  
centered at each grid cell  
Here  $B = 3$







# YOLO v2

<http://pureddie.com/yolo>





# YOLO v2

<http://pureddie.com/yolo>

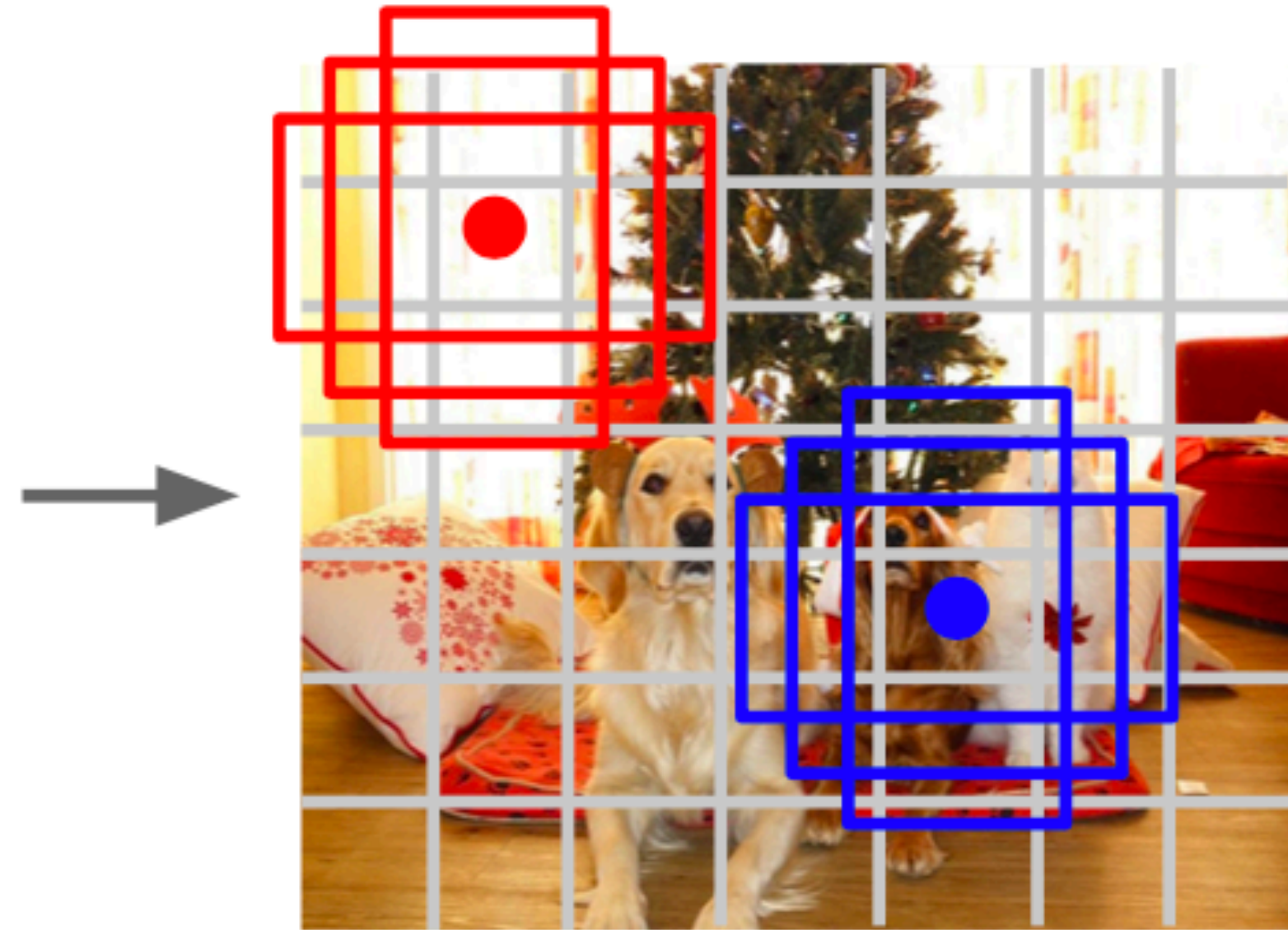


# YOLO: You Only Look Once

[ Redmon et al, CVPR 2016 ]

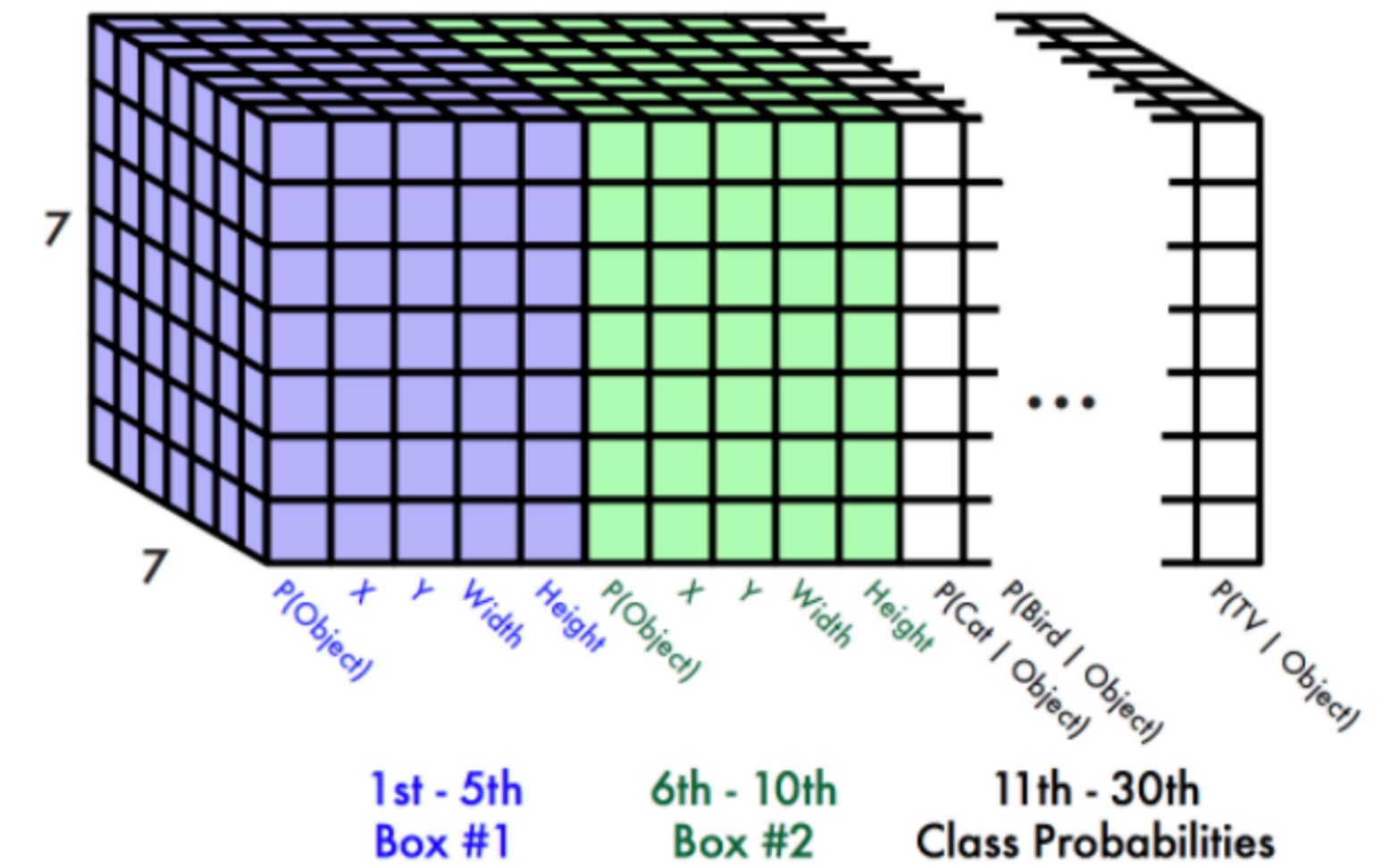


Input image  
 $3 \times H \times W$



Divide image into grid  
 $7 \times 7$

Image a set of **base boxes**  
centered at each grid cell  
Here  $B = 3$



1st - 5th Box #1      6th - 10th Box #2      11th - 30th Class Probabilities

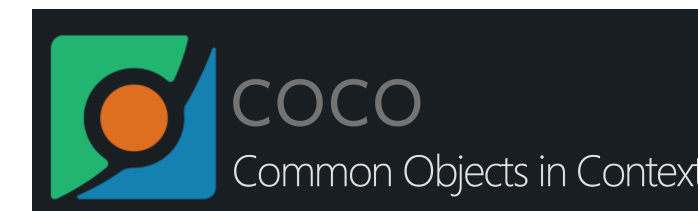


# Computer **Vision Problems** (no language for now)

## Segmentation



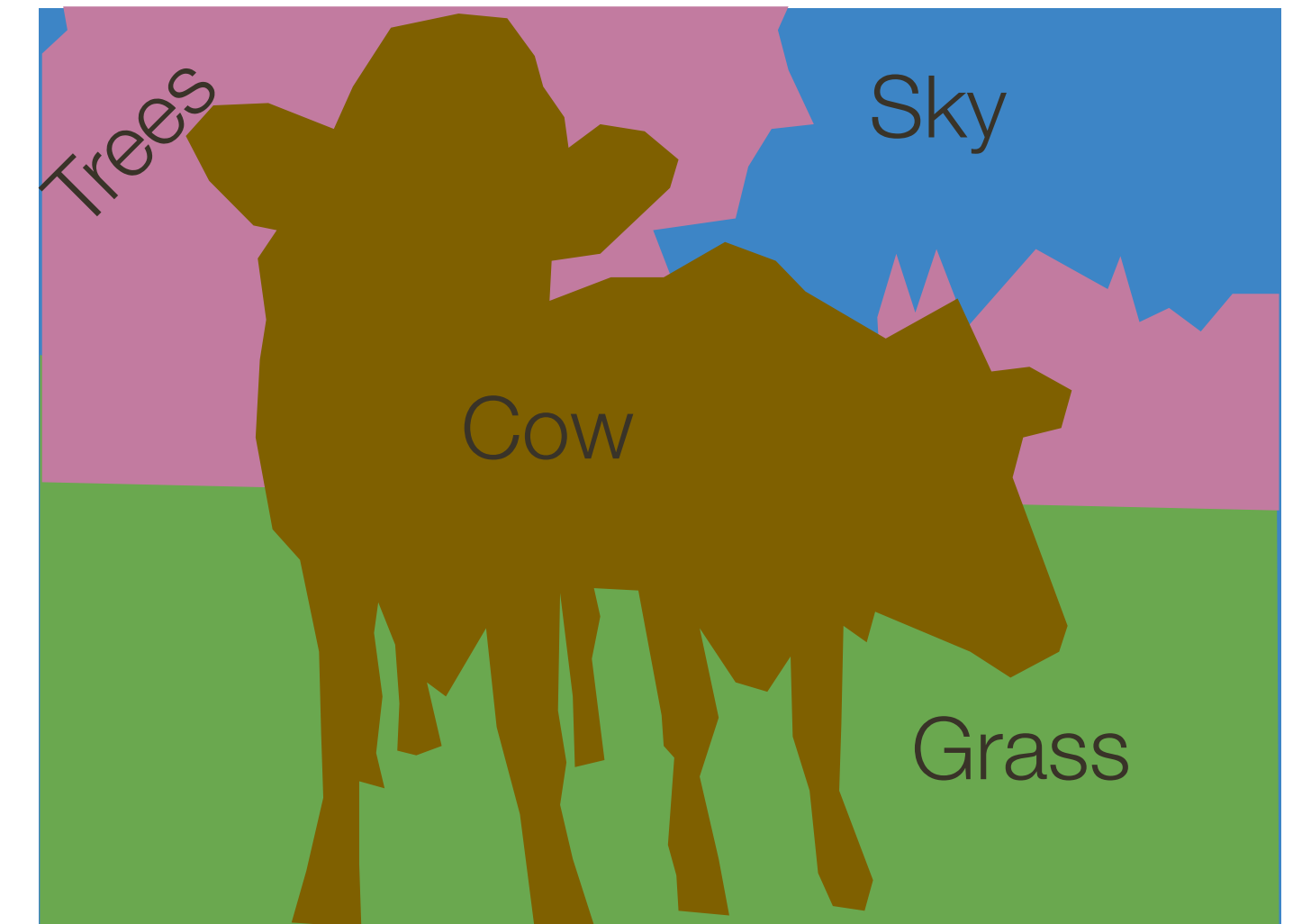
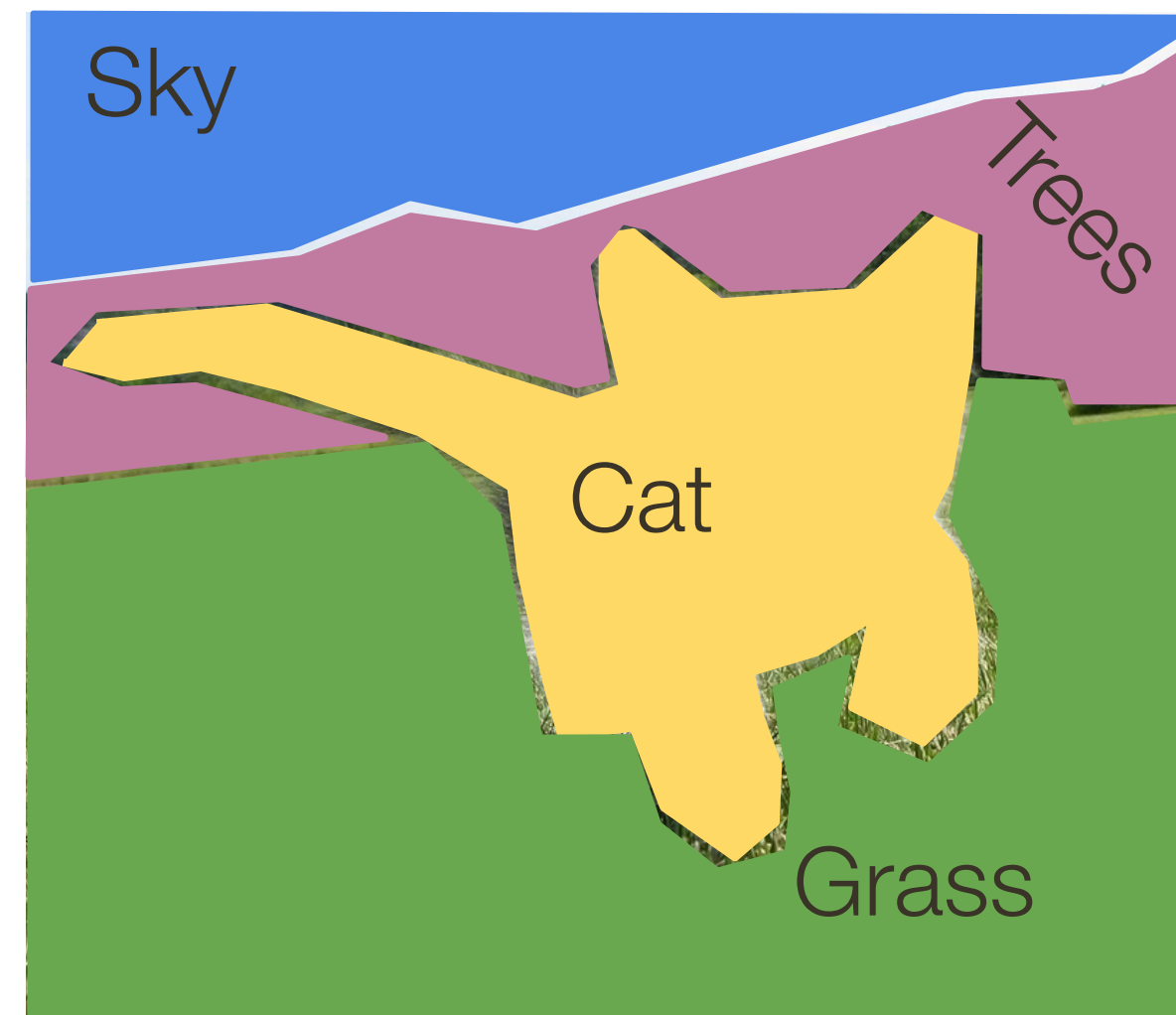
Horse  
Person





# Semantic Segmentation

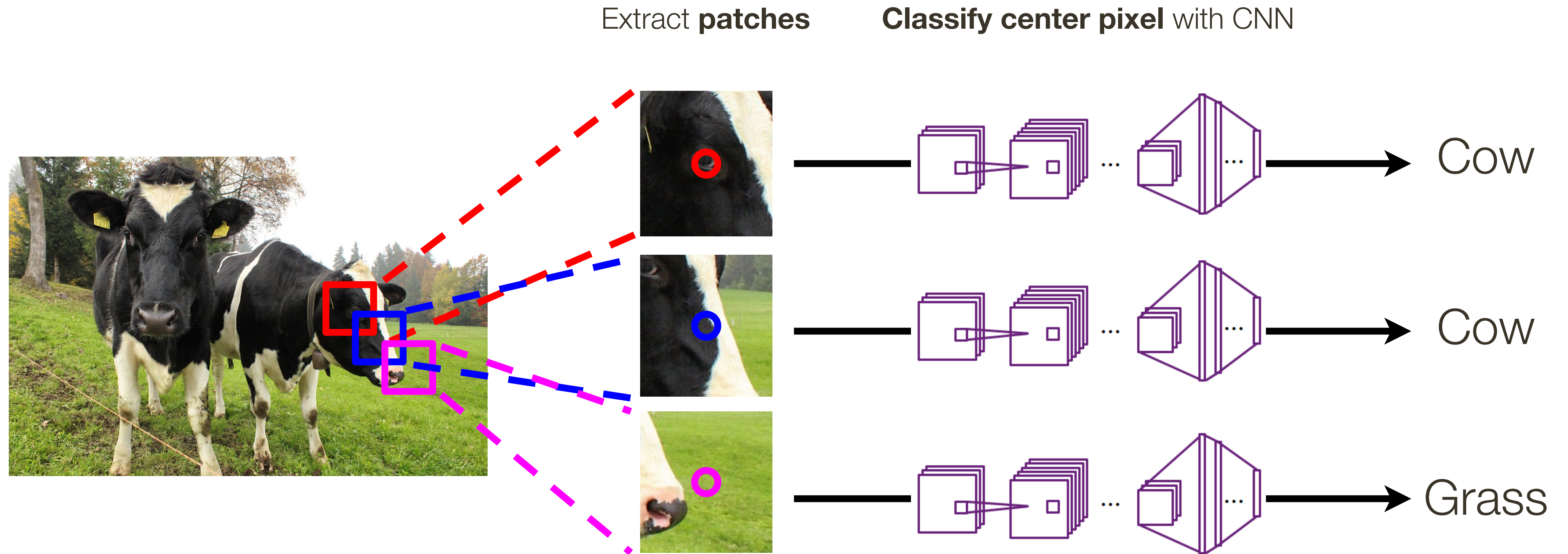
Label **every pixel** with a category label (without differentiating instances)





# Semantic Segmentation: Sliding Window

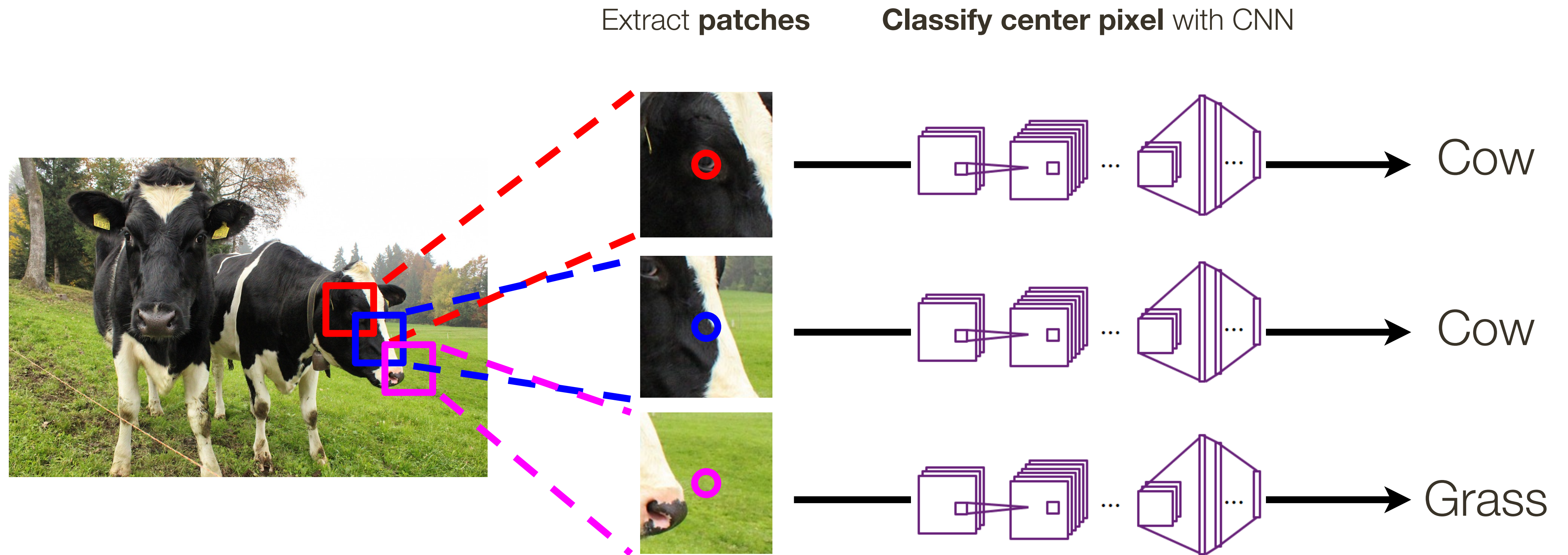
[ Farabet et al, TPAMI 2013 ]  
[ Pinheiro et al, ICML 2014 ]





# Semantic Segmentation: Sliding Window

[ Farabet et al, TPAMI 2013 ]  
[ Pinheiro et al, ICML 2014 ]

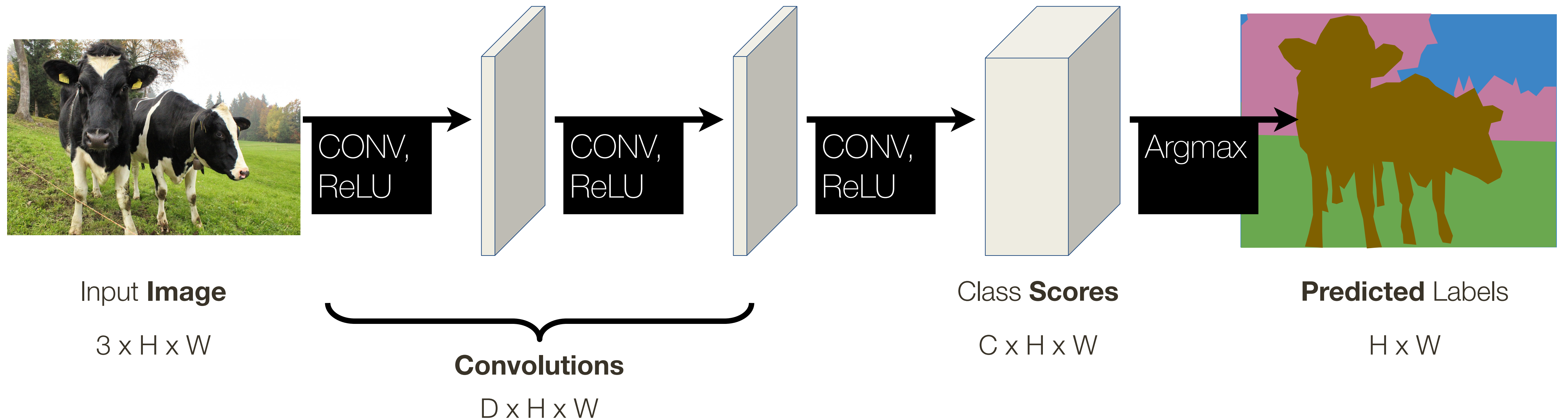


**Problem:** VERY inefficient, no reuse of computations for overlapping patches



# Semantic **Segmentation**: Fully Convolutional CNNs

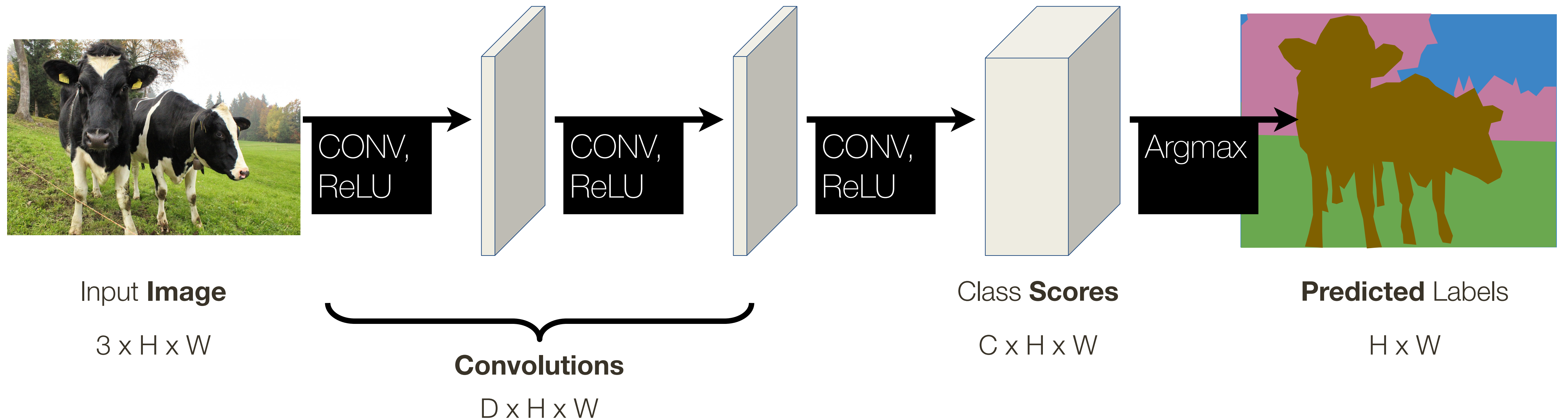
Design a network as a number of convolutional layers to make predictions for all pixels at once!





# Semantic **Segmentation**: Fully Convolutional CNNs

Design a network as a number of convolutional layers to make predictions for all pixels at once!



**Problem:** Convolutions at the original image scale will be very expensive

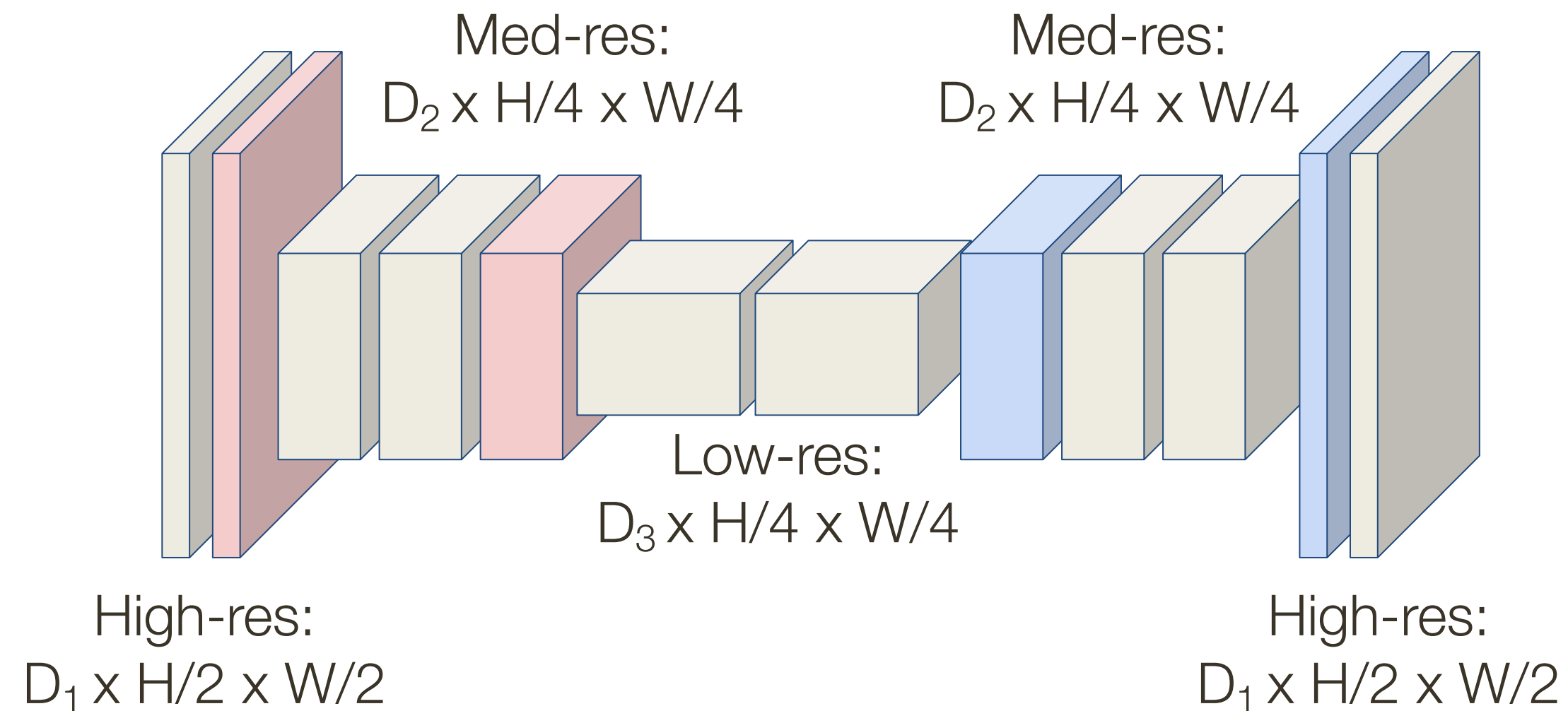
# Semantic Segmentation: Fully Convolutional CNNs

Design a network as a number of convolutional layers with **downsampling** and **upsampling** inside the network!



Input **Image**

$3 \times H \times W$



**Predicted Labels**

$H \times W$

[ Long et al, CVPR 2015 ]  
[ Noh et al, ICCV 2015 ]



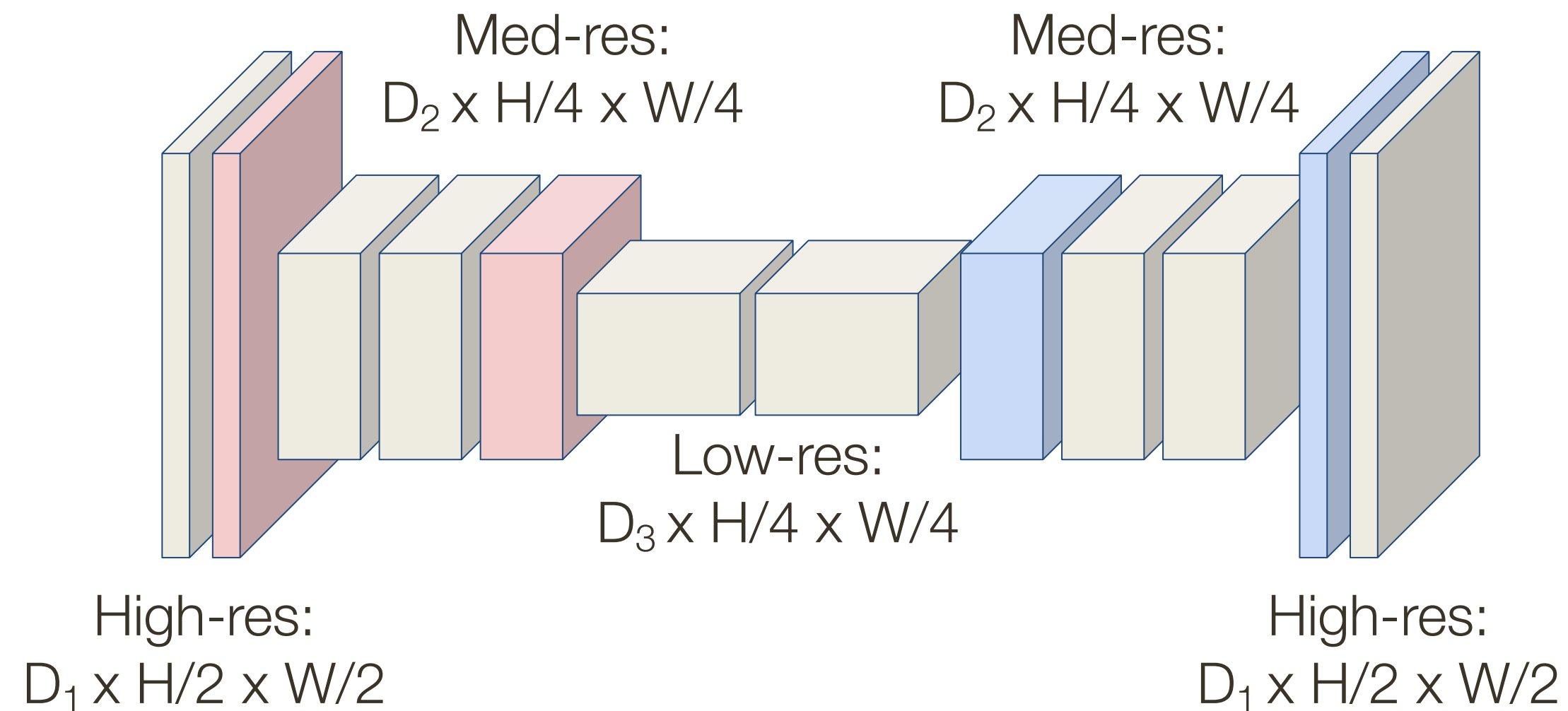
# Semantic Segmentation: Fully Convolutional CNNs

Design a network as a number of convolutional layers with **downsampling** and **upsampling** inside the network!



Input **Image**

$3 \times H \times W$



**Predicted Labels**

$H \times W$

**Downsampling** = Pooling

**Upsampling** = ???

[ Long et al, CVPR 2015 ]  
[ Noh et al, ICCV 2015 ]

# In-network **Up Sampling** (a.k.a “Unpooling”)

Nearest Neighbor

1	2
3	4



1	1		2	2
1	1		2	2
-----			-----	
3	3		4	4
3	3		4	4

**Input:** 2 x 2

**Output:** 4 x 4



# In-network **Up Sampling** (a.k.a “Unpooling”)

Nearest Neighbor

1	2
3	4



1	1	2	2
1	1	2	2
3	3	4	4
3	3	4	4

**Input:** 2 x 2

**Output:** 4 x 4

“Bed of Nails”

1	2
3	4



1	0	2	0
0	0	0	0
3	0	4	0
0	0	0	0

**Input:** 2 x 2

**Output:** 4 x 4

\* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

# In-network **Up Sampling:** Max Unpooling

### Max Pooling

Remember which element was max!

1	2	6	3
3	5	2	1
1	2	2	1
7	3	4	8

**Input:** 4 x 4



5	6
7	8

**Output:** 2 x 2

...  
Rest of the network

### Max Unpooling

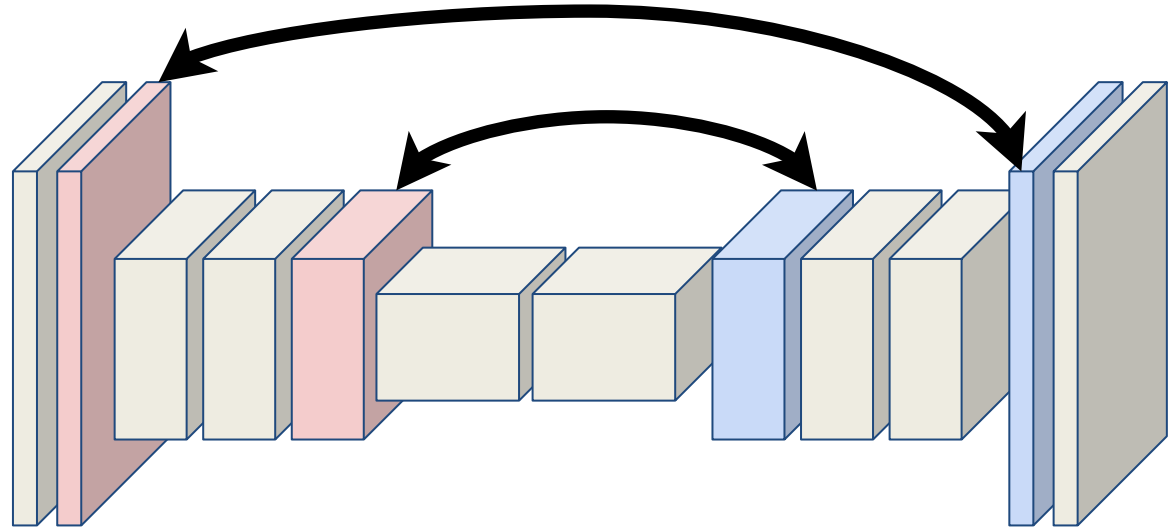
Use positions from pooling layer

1	2
3	4

**Input:** 2 x 2

0	0	2	0
0	1	0	0
0	0	0	0
3	0	0	4

**Output:** 4 x 4



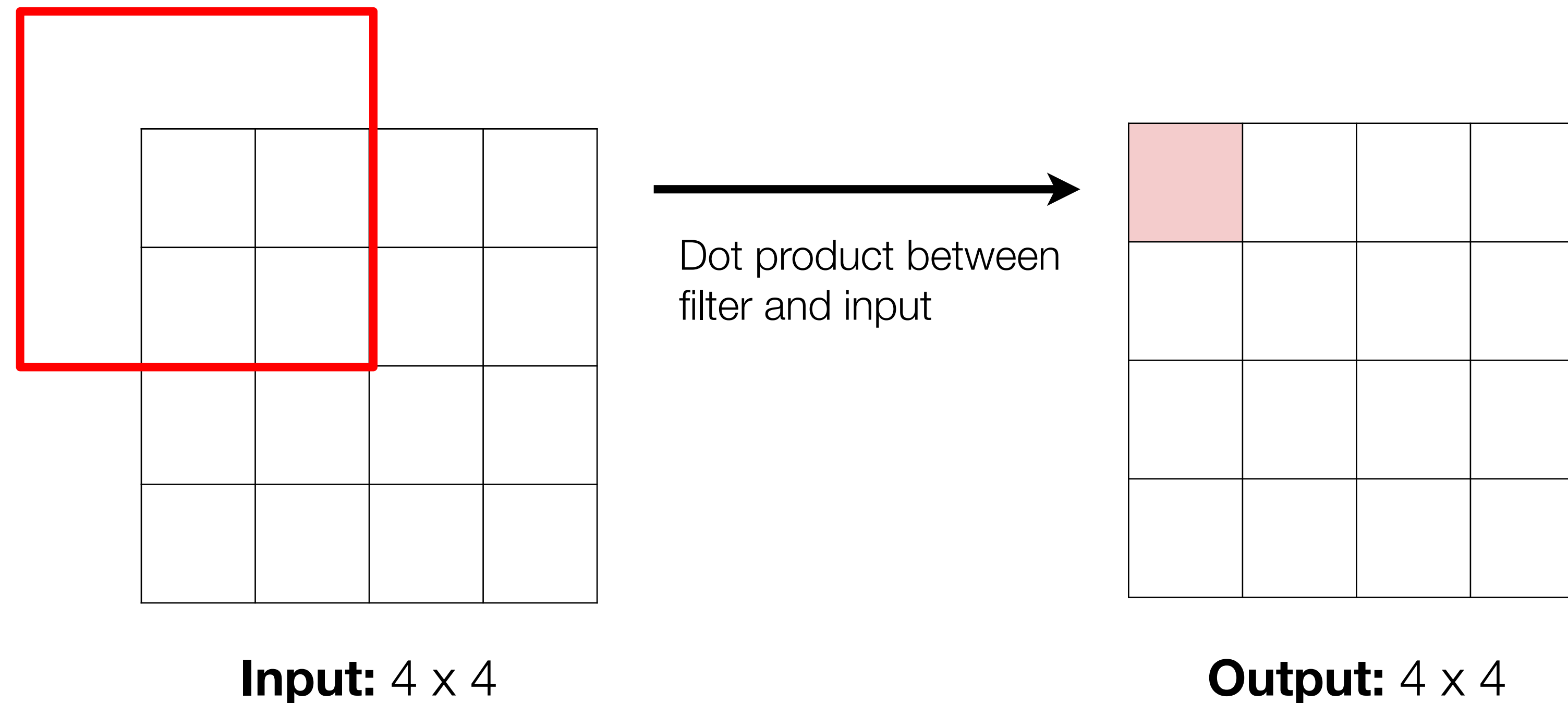
Corresponding pairs of downsampling and upsampling layers

\* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**



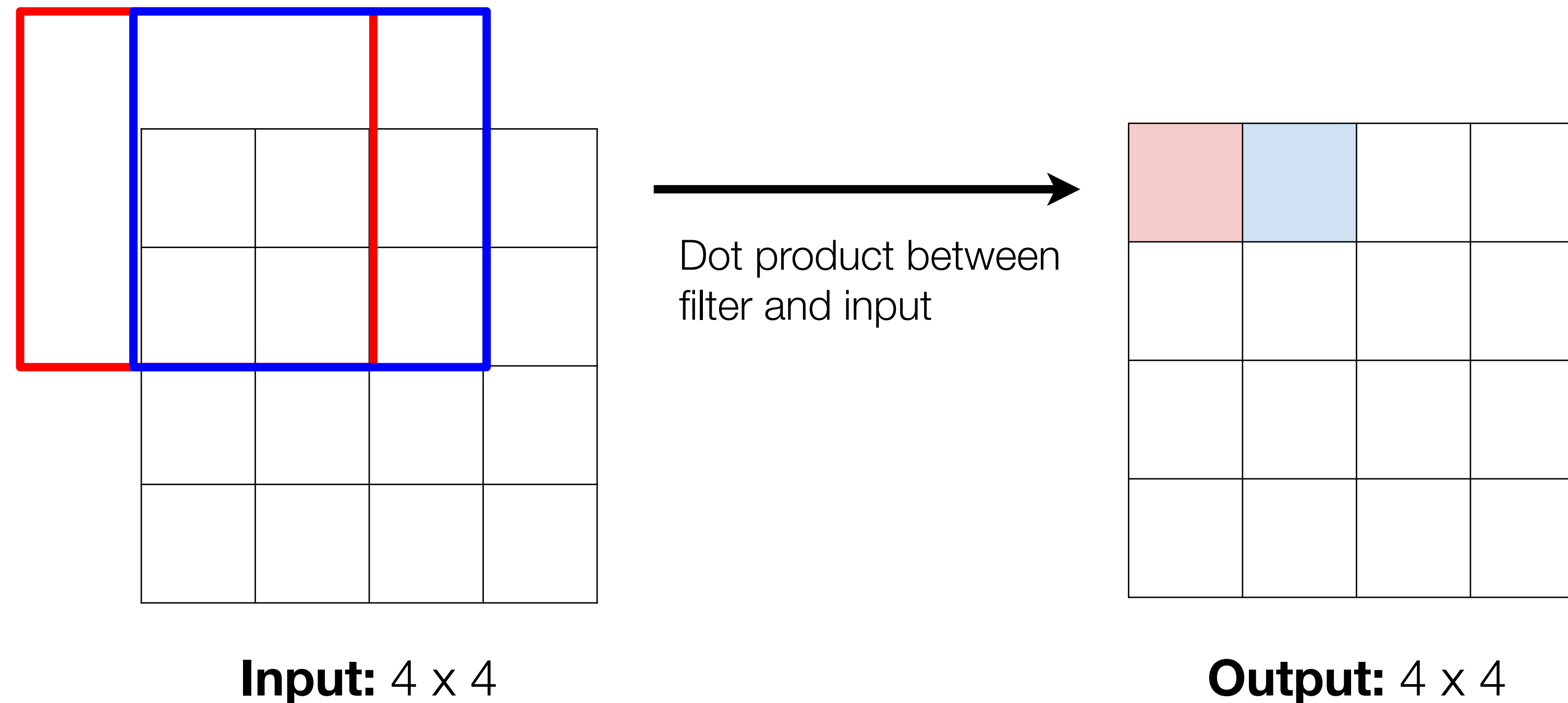
# In-network **Up Sampling:** Transpose Convolution

**Recall:** Normal 3 x 3 convolution, stride 1 pad 1



# In-network **Up Sampling:** Transpose Convolution

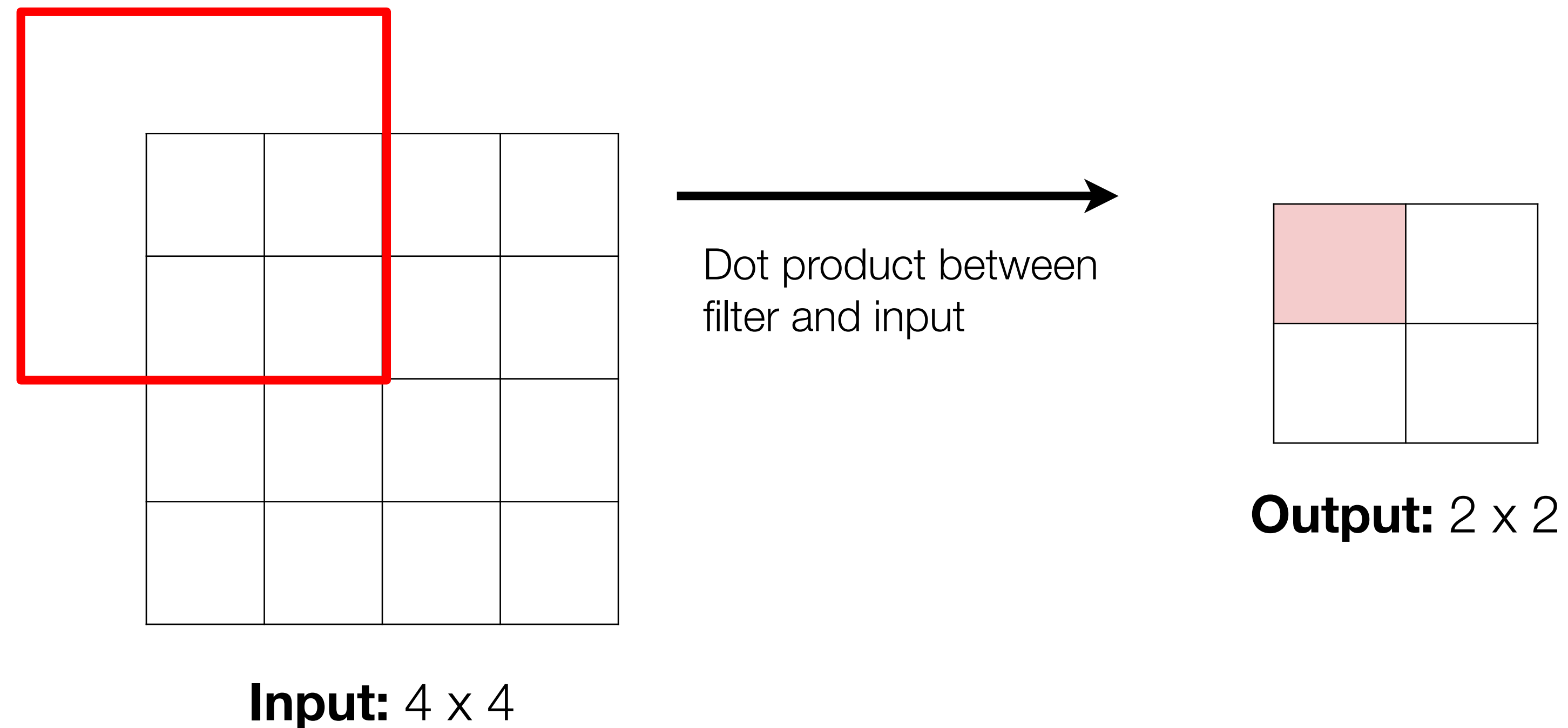
**Recall:** Normal 3 x 3 convolution, stride 1 pad 1





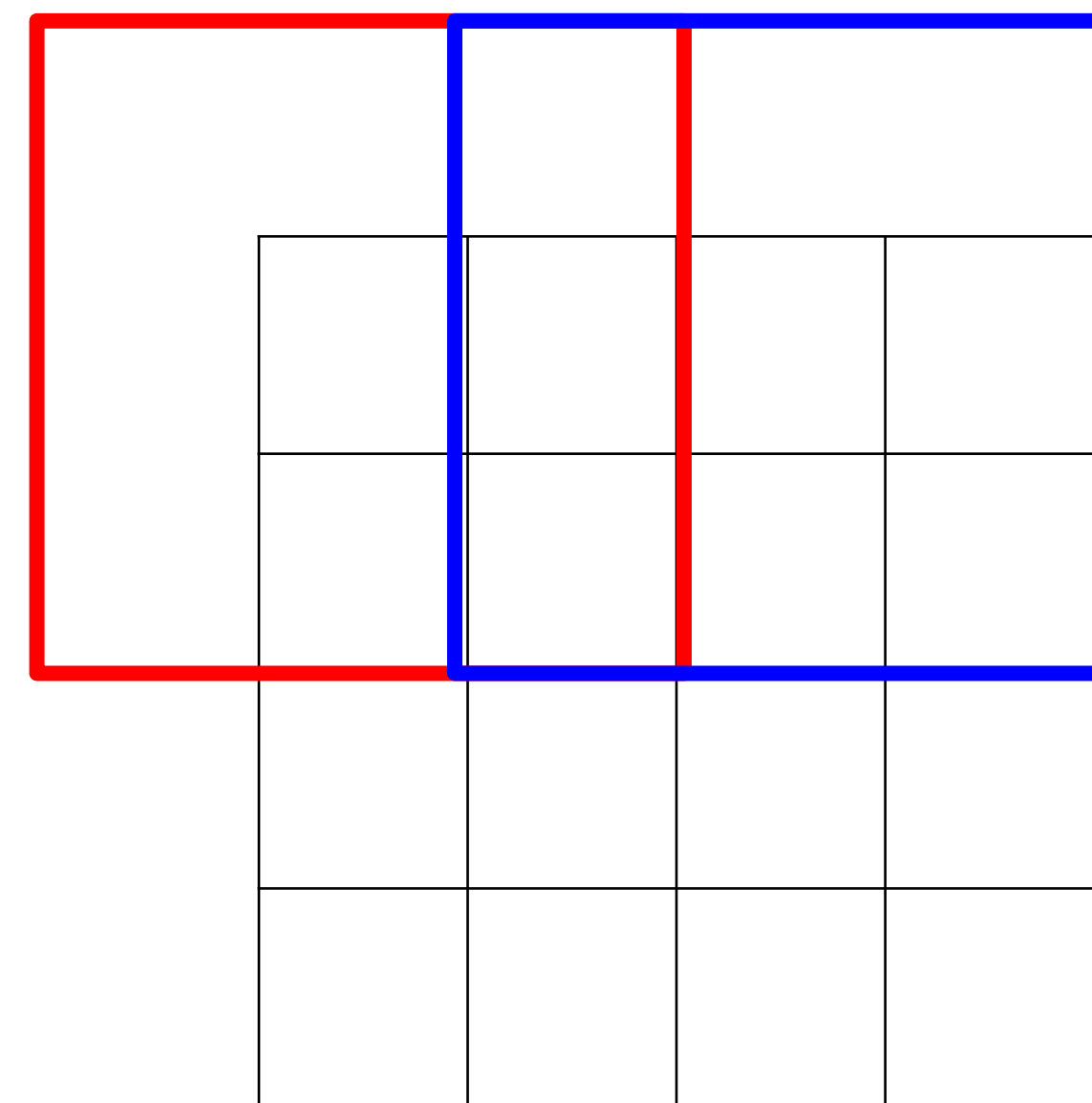
# In-network **Up Sampling:** Transpose Convolution

**Recall:** Normal 3 x 3 convolution, stride 2 pad 1



# In-network **Up Sampling:** Transpose Convolution

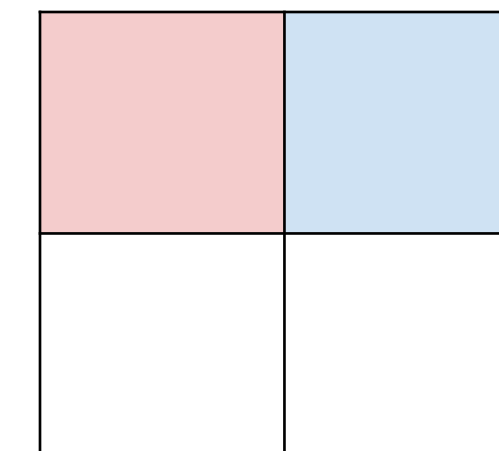
**Recall:** Normal 3 x 3 convolution, stride 2 pad 1



**Input:** 4 x 4



Dot product between  
filter and input



**Output:** 2 x 2

Filter moves 2 pixels in the **input** for every one  
pixel in the **output**

Stride gives ratio in movement in input vs output



# In-network **Up Sampling:** Transpose Convolution

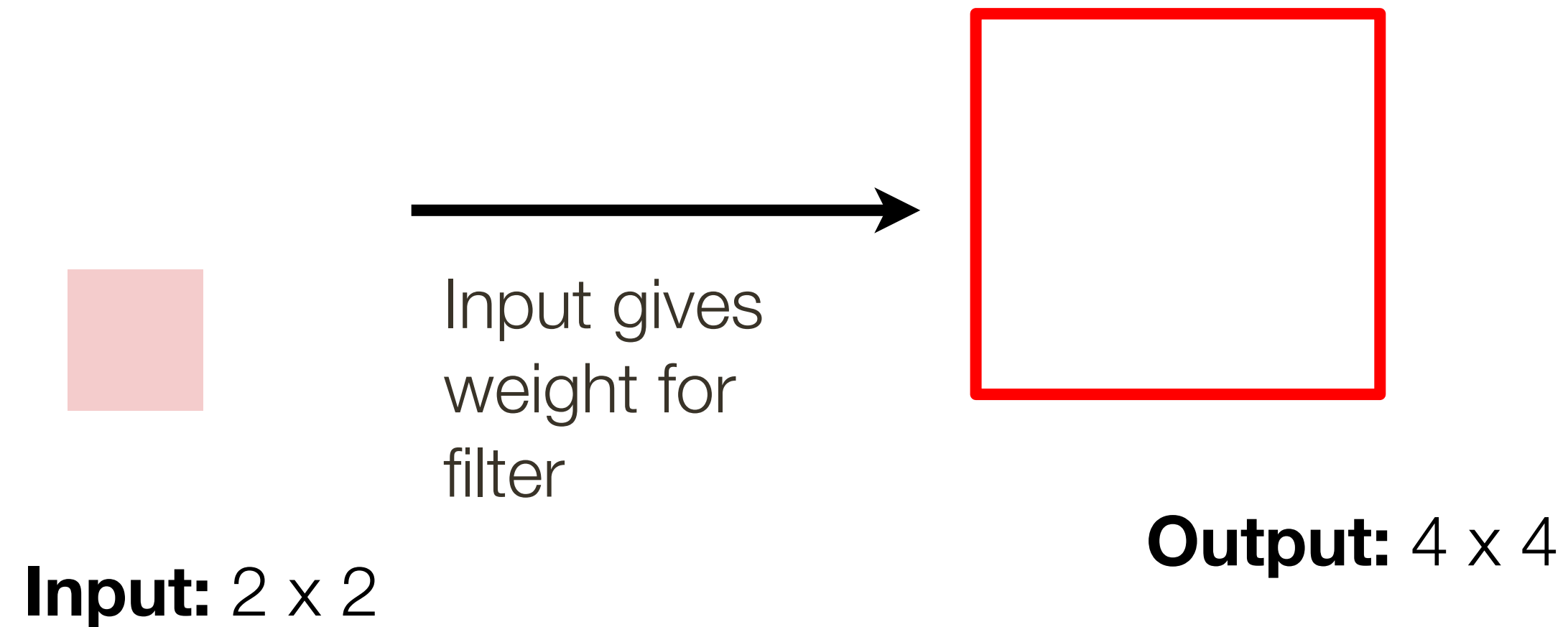
3 x 3 **transpose** convolution, stride 2 pad 1

**Input:** 2 x 2

**Output:** 4 x 4

# In-network **Up Sampling**: Transpose Convolution

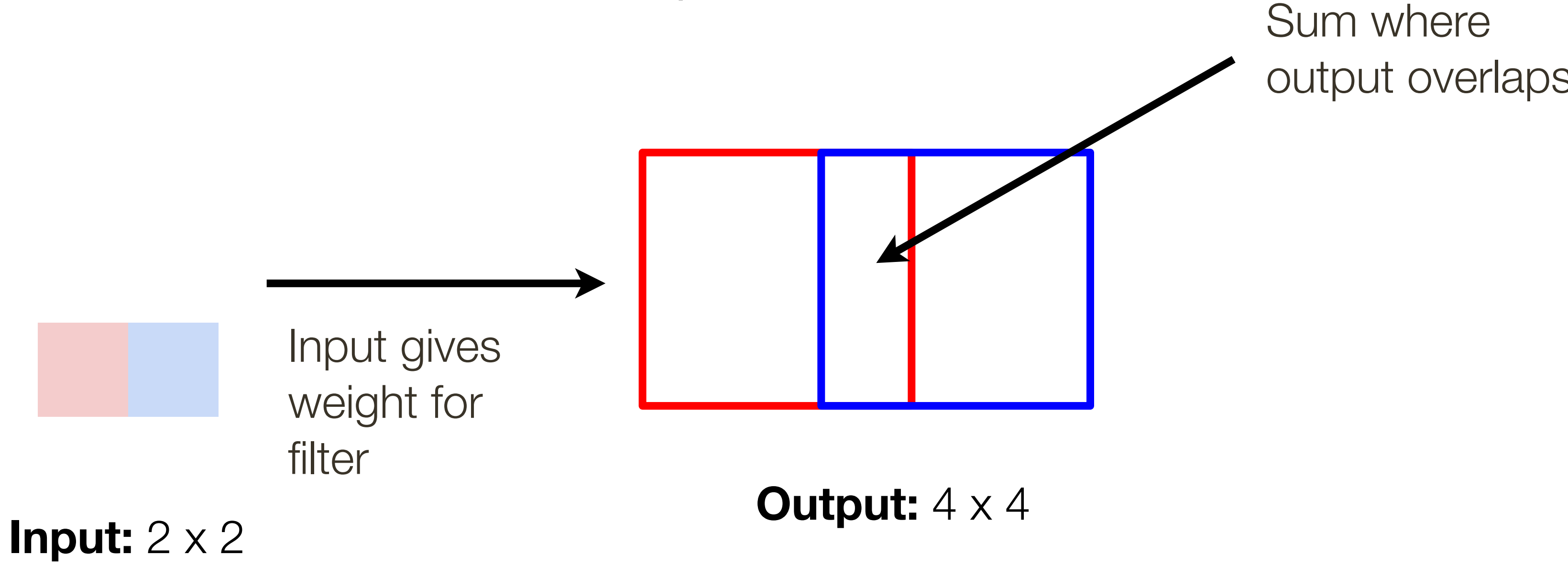
3 x 3 **transpose** convolution, stride 2 pad 1





# In-network **Up Sampling:** Transpose Convolution

3 x 3 **transpose** convolution, stride 2 pad 1

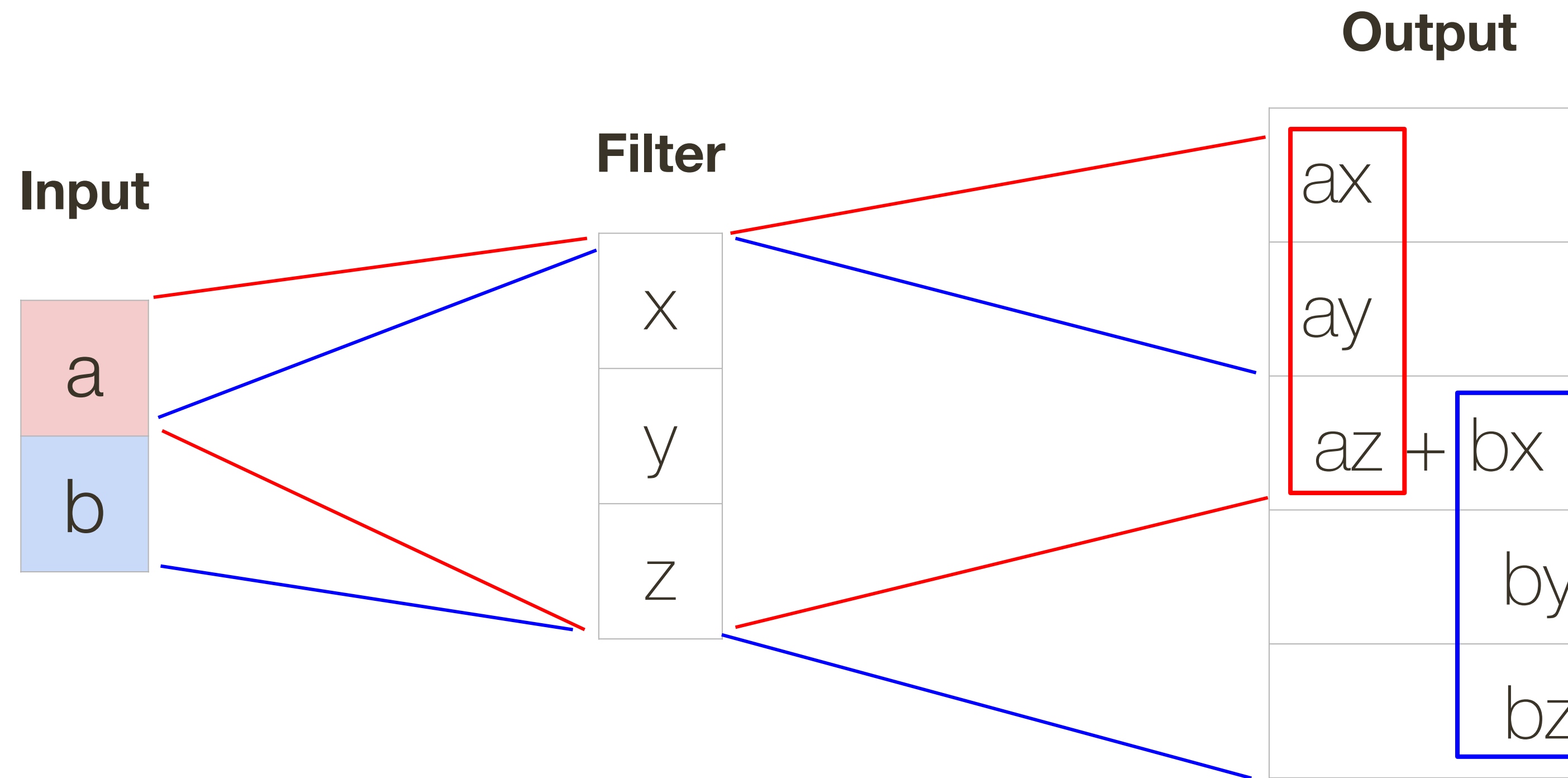


Filter moves 2 pixels in the **output** for every one pixel in the **input**

Stride gives ratio in movement in output vs input

\* slide from Fei-Dei Li, Justin Johnson, Serena Yeung, **cs231n Stanford**

# Transpose Convolution: 1-D Example

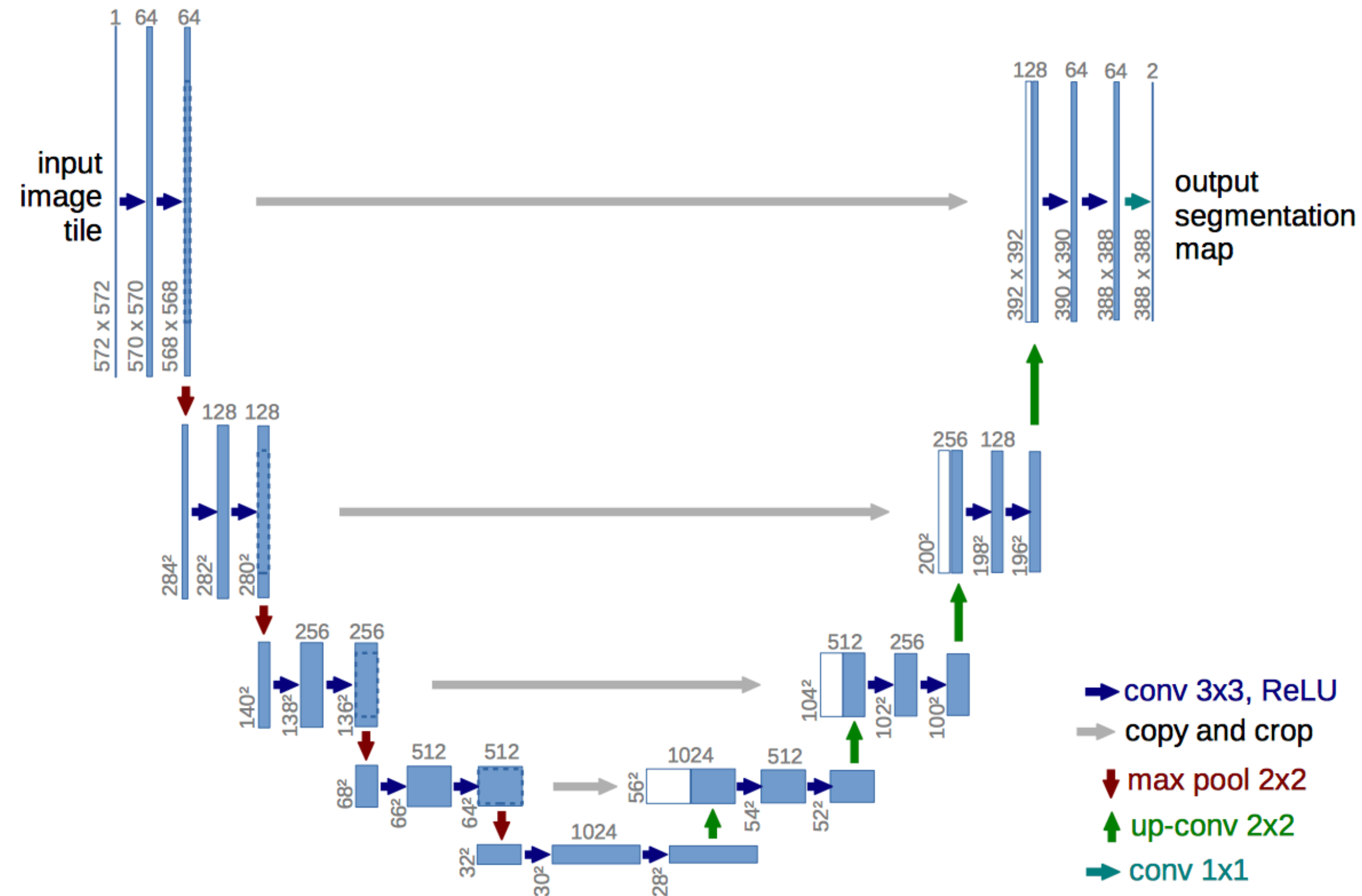


Output contains copies of the filter weighted multiplied by the input, summing at overlaps in the output



# U-Net Architecture

## ResNet-like Fully convolutional CNN



# Computer **Vision Problems** (no language for now)

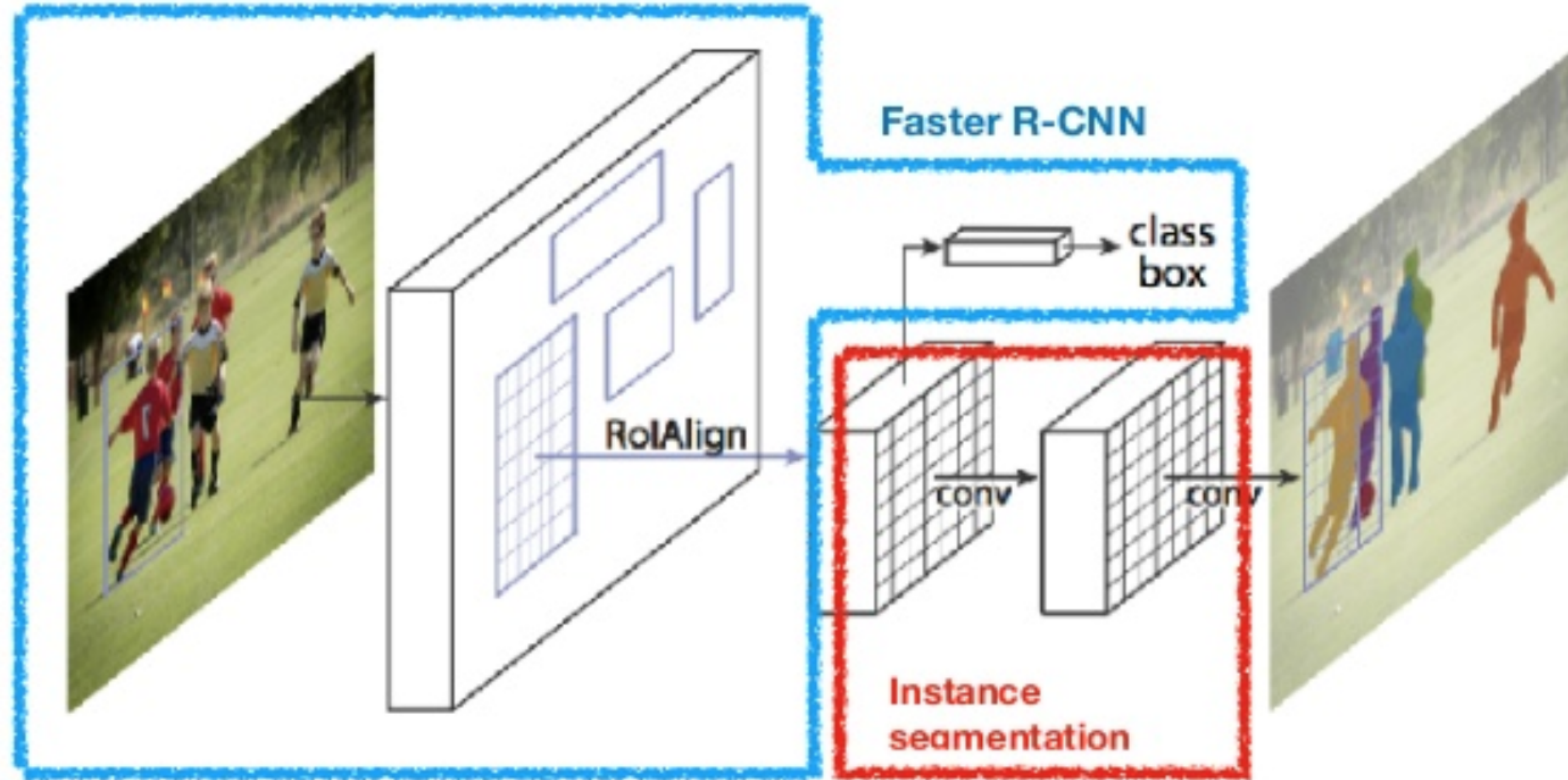
## Instance Segmentation



Horse1  
Horse2  
Person1  
Person2



# Mask R-CNN









# Summary

Common types of layers:

1. **Convolutional** Layer
  - Parameters define a set of learnable filters
2. **Pooling** Layer
  - Performs a downsampling along the spatial dimensions
3. **Fully-Connected** Layer
  - As in a regular neural network

Each layer accepts an input 3D volume and transforms it to an output 3D volume through a differentiable function

# Summary

The parameters of a neural network are learned using **backpropagation**, which computes gradients via recursive application of the chain rule

A **convolutional neural network** assumes inputs are images, and constrains the network architecture to reduce the number of parameters

A **convolutional layer** applies a set of learnable filters

A **pooling layer** performs spatial downsampling

A **fully-connected** layer is the same as in a regular neural network

Convolutional neural networks can be seen as learning a hierarchy of filters



# Attention Layer

## Inputs:

Query vectors:  $\mathbf{Q}$  (Shape:  $N_Q \times D_Q$ )

Input vectors:  $\mathbf{X}$  (Shape:  $N_X \times D_X$ )

Key matrix:  $\mathbf{W}_K$  (Shape:  $D_X \times D_Q$ )

Value matrix:  $\mathbf{W}_V$  (Shape:  $D_X \times D_V$ )

## Computation:

Key vectors:  $\mathbf{K} = \mathbf{XW}_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $\mathbf{V} = \mathbf{XW}_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $\mathbf{E} = \mathbf{QK}^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (\mathbf{Q}_i \cdot \mathbf{K}_j) / \sqrt{D_Q}$

Attention weights:  $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$  (Shape:  $N_Q \times N_X$ )

Output vectors:  $\mathbf{Y} = \mathbf{AV}$  (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} \mathbf{V}_j$

$X_1$

$X_2$

$X_3$

$Q_1$

$Q_2$

$Q_3$

$Q_4$

# Attention Layer

## Inputs:

Query vectors:  $Q$  (Shape:  $N_Q \times D_Q$ )

Input vectors:  $X$  (Shape:  $N_X \times D_X$ )

Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ )

Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

## Computation:

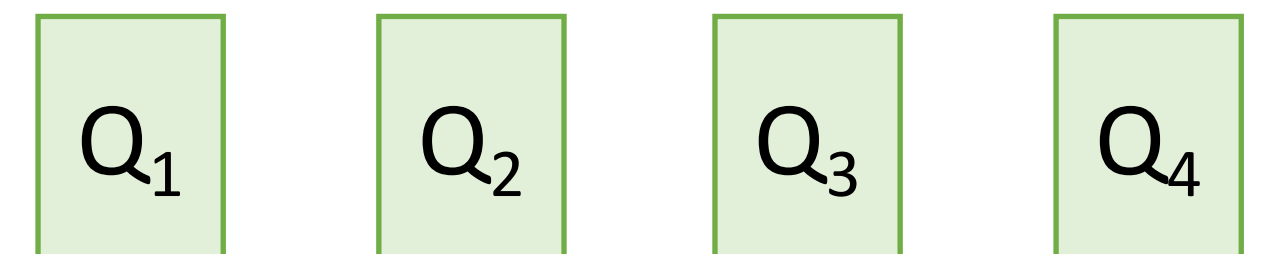
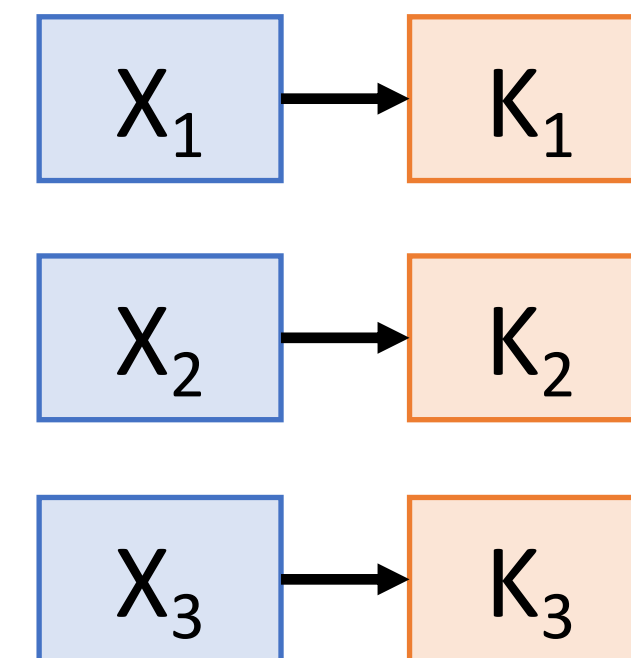
Key vectors:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

Attention weights:  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_Q \times N_X$ )

Output vectors:  $Y = AV$  (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$





# Attention Layer

## Inputs:

Query vectors:  $\mathbf{Q}$  (Shape:  $N_Q \times D_Q$ )

Input vectors:  $\mathbf{X}$  (Shape:  $N_X \times D_X$ )

Key matrix:  $\mathbf{W}_K$  (Shape:  $D_X \times D_Q$ )

Value matrix:  $\mathbf{W}_V$  (Shape:  $D_X \times D_V$ )

## Computation:

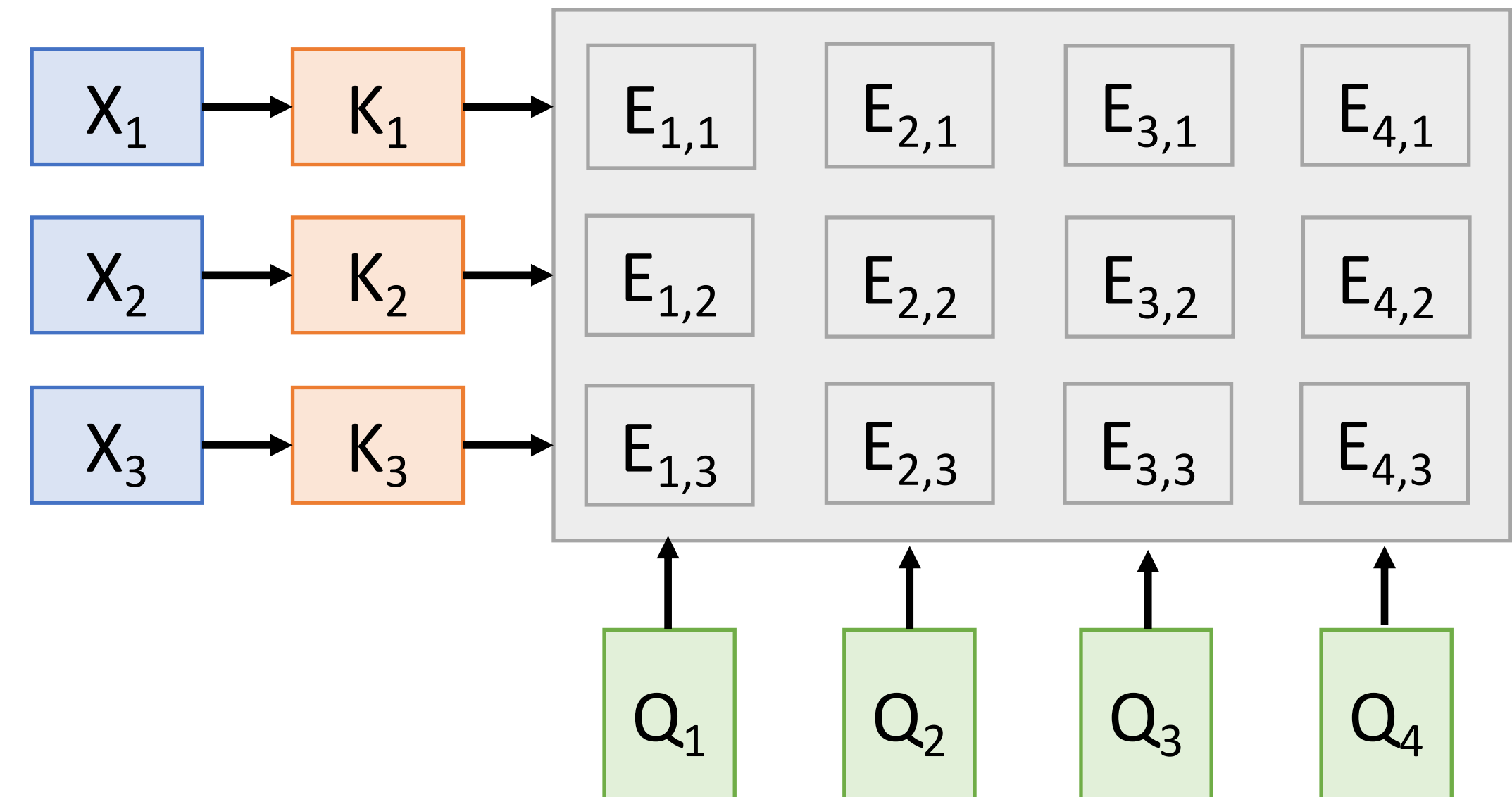
Key vectors:  $\mathbf{K} = \mathbf{XW}_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $\mathbf{V} = \mathbf{XW}_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $\mathbf{E} = \mathbf{QK}^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (\mathbf{Q}_i \cdot \mathbf{K}_j) / \sqrt{D_Q}$

Attention weights:  $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$  (Shape:  $N_Q \times N_X$ )

Output vectors:  $\mathbf{Y} = \mathbf{AV}$  (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} \mathbf{V}_j$



# Attention Layer

## Inputs:

Query vectors:  $Q$  (Shape:  $N_Q \times D_Q$ )

Input vectors:  $X$  (Shape:  $N_X \times D_X$ )

Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ )

Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

## Computation:

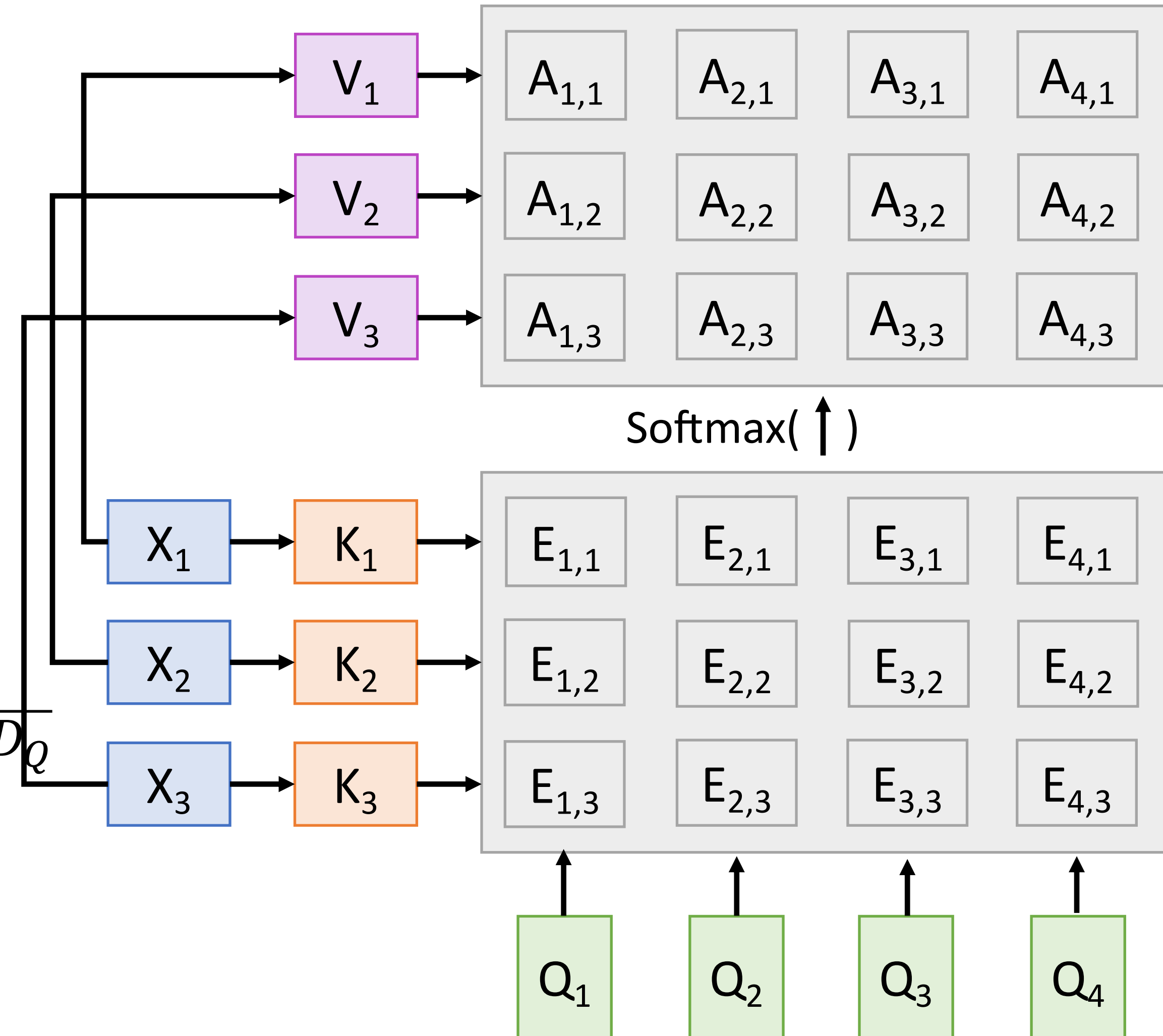
Key vectors:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

Attention weights:  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_Q \times N_X$ )

Output vectors:  $Y = AV$  (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$





# Attention Layer

## Inputs:

Query vectors:  $Q$  (Shape:  $N_Q \times D_Q$ )

Input vectors:  $X$  (Shape:  $N_X \times D_X$ )

Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ )

Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

## Computation:

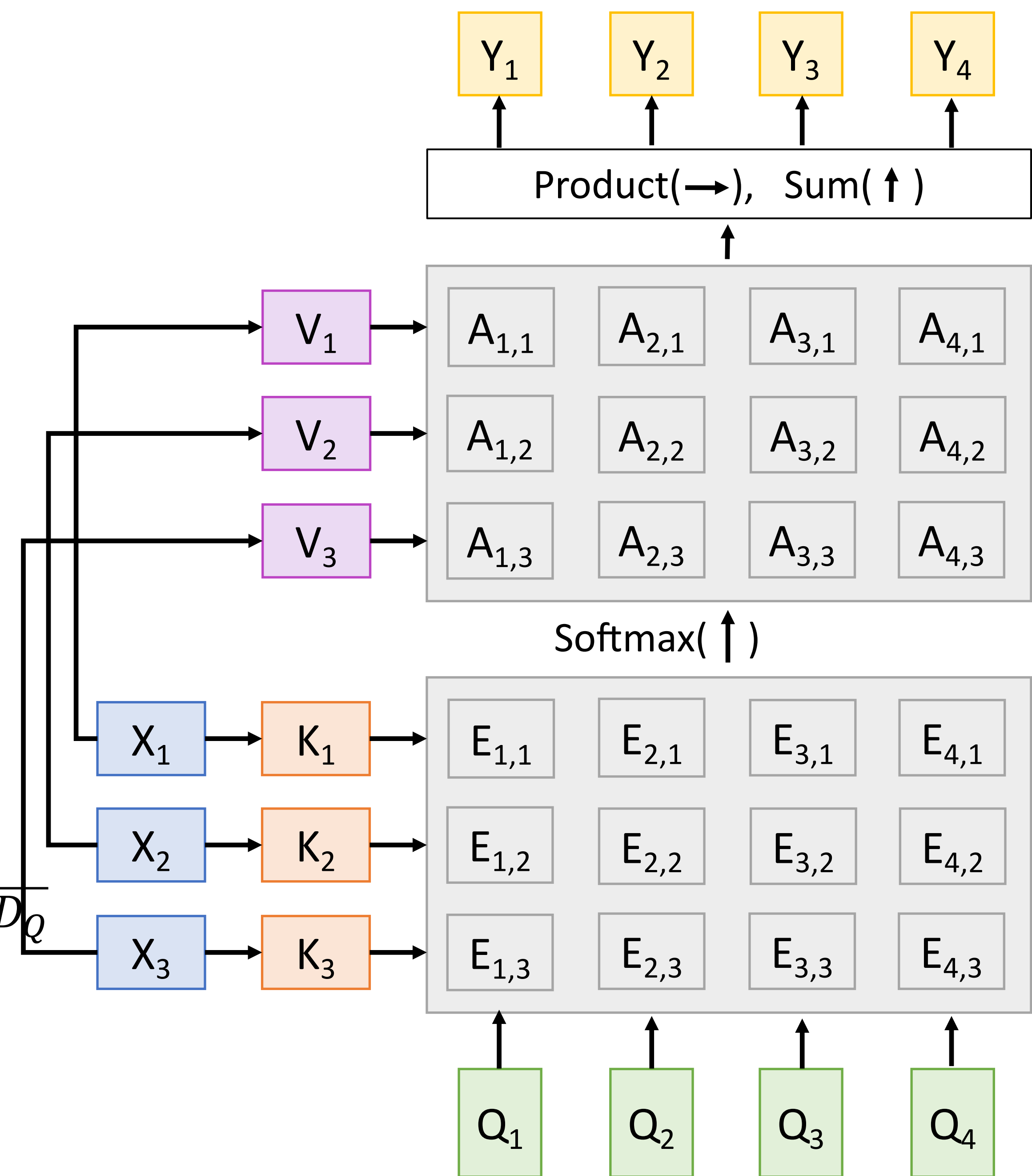
Key vectors:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

Attention weights:  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_Q \times N_X$ )

Output vectors:  $Y = AV$  (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$



# Self-attention Layer

One **query** per **input vector**

## Inputs:

**Query vectors:**  $\mathbf{Q}$  (Shape:  $N_Q \times D_Q$ )

**Input vectors:**  $\mathbf{X}$  (Shape:  $N_X \times D_X$ )

**Key matrix:**  $\mathbf{W}_K$  (Shape:  $D_X \times D_Q$ )

**Value matrix:**  $\mathbf{W}_V$  (Shape:  $D_X \times D_V$ )

## Computation:

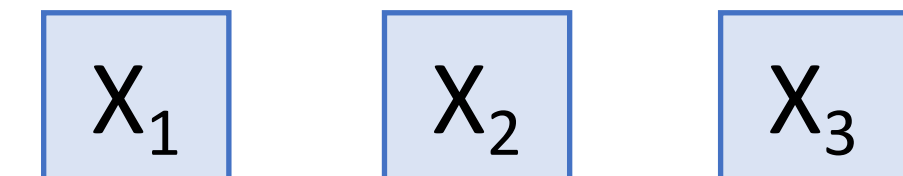
**Key vectors:**  $\mathbf{K} = \mathbf{XW}_K$  (Shape:  $N_X \times D_Q$ )

**Value Vectors:**  $\mathbf{V} = \mathbf{XW}_V$  (Shape:  $N_X \times D_V$ )

**Similarities:**  $\mathbf{E} = \mathbf{QK}^T / \sqrt{D_Q}$  (Shape:  $N_Q \times N_X$ )  $E_{i,j} = (\mathbf{Q}_i \cdot \mathbf{K}_j) / \sqrt{D_Q}$

**Attention weights:**  $\mathbf{A} = \text{softmax}(\mathbf{E}, \text{dim}=1)$  (Shape:  $N_Q \times N_X$ )

**Output vectors:**  $\mathbf{Y} = \mathbf{AV}$  (Shape:  $N_Q \times D_V$ )  $Y_i = \sum_j A_{i,j} \mathbf{V}_j$





# Self-attention Layer

One **query** per **input vector**

## Inputs:

**Input vectors:**  $X$  (Shape:  $N_x \times D_x$ )

**Key matrix:**  $W_k$  (Shape:  $D_x \times D_Q$ )

**Value matrix:**  $W_v$  (Shape:  $D_x \times D_V$ )

**Query matrix:**  $W_Q$  (Shape:  $D_x \times D_Q$ )

## Computation:

**Query vectors:**  $Q = XW_Q$

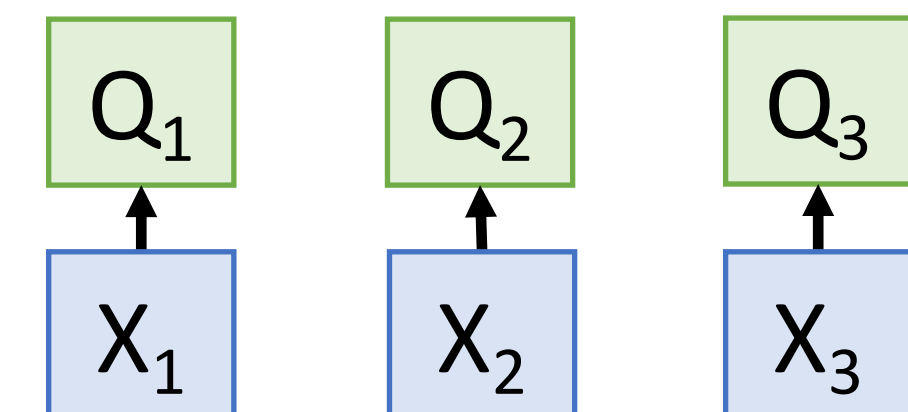
**Key vectors:**  $K = XW_k$  (Shape:  $N_x \times D_Q$ )

**Value Vectors:**  $V = XW_v$  (Shape:  $N_x \times D_V$ )

**Similarities:**  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_x \times N_x$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

**Attention weights:**  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_x \times N_x$ )

**Output vectors:**  $Y = AV$  (Shape:  $N_x \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$



# Self-attention Layer

One **query** per **input vector**

## Inputs:

Input vectors:  $X$  (Shape:  $N_x \times D_x$ )

Key matrix:  $W_k$  (Shape:  $D_x \times D_Q$ )

Value matrix:  $W_v$  (Shape:  $D_x \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_x \times D_Q$ )

## Computation:

Query vectors:  $Q = XW_Q$

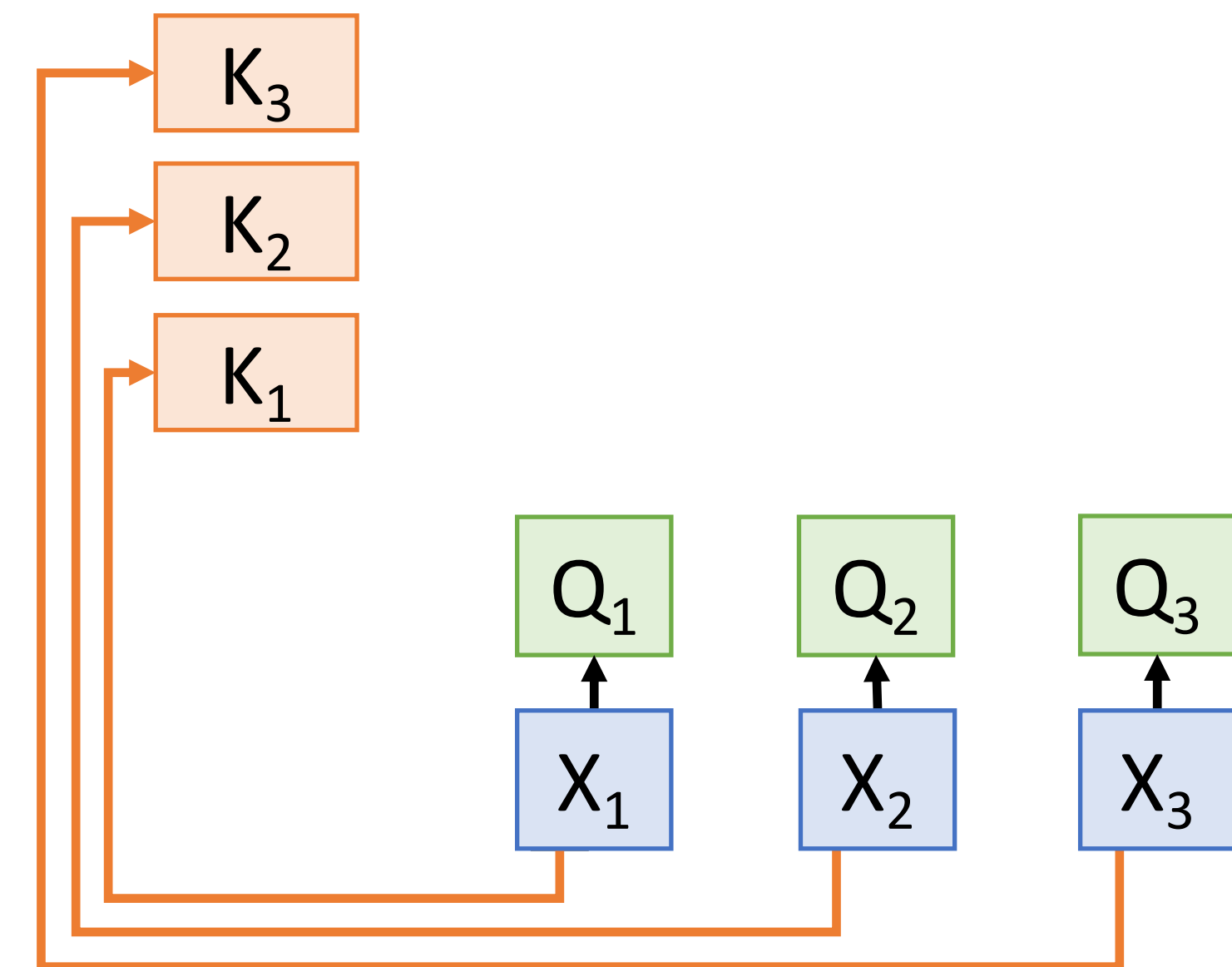
Key vectors:  $K = XW_k$  (Shape:  $N_x \times D_Q$ )

Value Vectors:  $V = XW_v$  (Shape:  $N_x \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_x \times N_x$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

Attention weights:  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_x \times N_x$ )

Output vectors:  $Y = AV$  (Shape:  $N_x \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$





# Self-attention Layer

One **query** per **input vector**

## Inputs:

**Input vectors:**  $X$  (Shape:  $N_x \times D_x$ )

**Key matrix:**  $W_k$  (Shape:  $D_x \times D_Q$ )

**Value matrix:**  $W_v$  (Shape:  $D_x \times D_v$ )

**Query matrix:**  $W_Q$  (Shape:  $D_x \times D_Q$ )

## Computation:

**Query vectors:**  $Q = XW_Q$

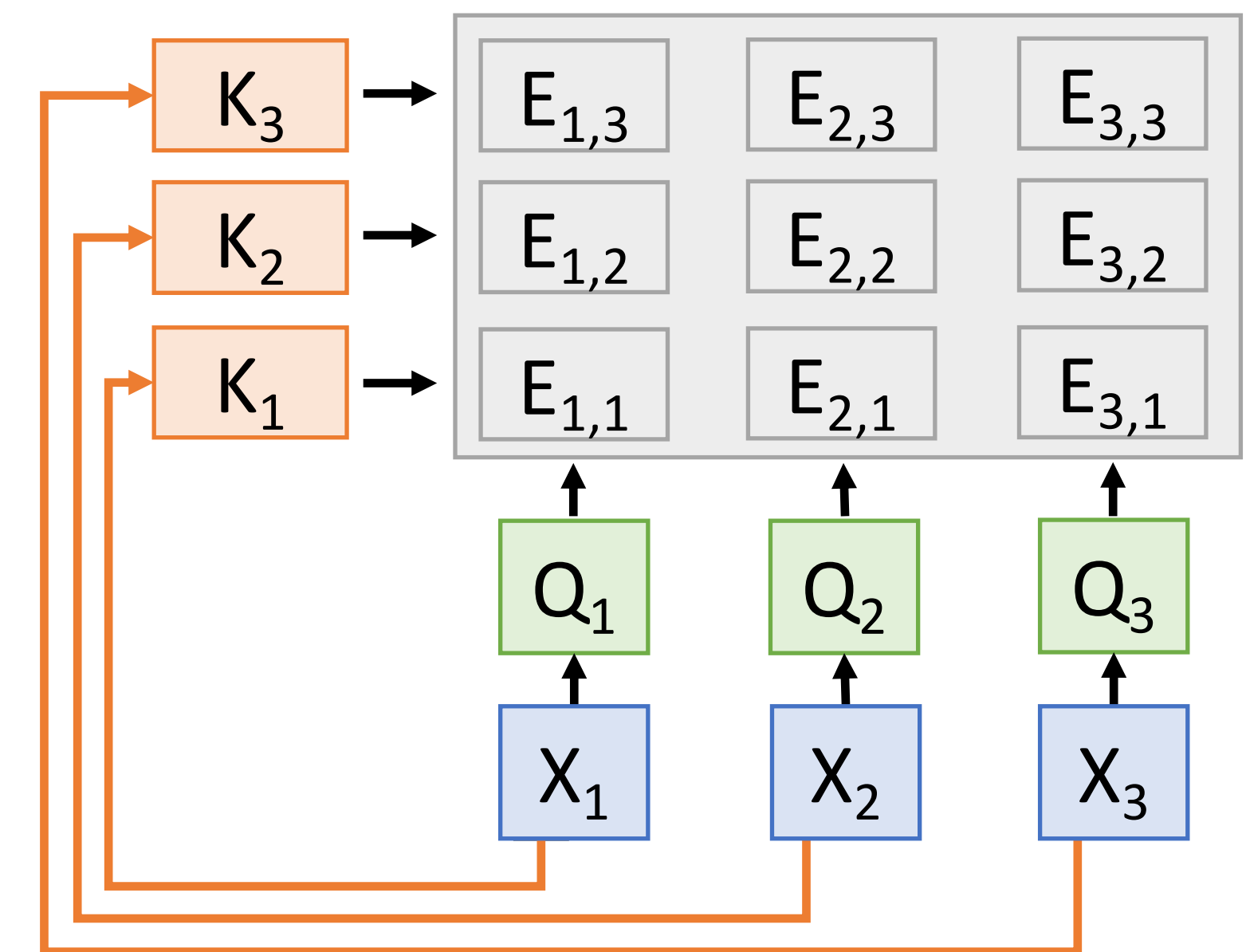
**Key vectors:**  $K = XW_k$  (Shape:  $N_x \times D_Q$ )

**Value Vectors:**  $V = XW_v$  (Shape:  $N_x \times D_v$ )

**Similarities:**  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_x \times N_x$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

**Attention weights:**  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_x \times N_x$ )

**Output vectors:**  $Y = AV$  (Shape:  $N_x \times D_v$ )  $Y_i = \sum_j A_{i,j} V_j$



# Self-attention Layer

One **query** per **input vector**

## Inputs:

**Input vectors:**  $X$  (Shape:  $N_x \times D_x$ )

**Key matrix:**  $W_k$  (Shape:  $D_x \times D_Q$ )

**Value matrix:**  $W_v$  (Shape:  $D_x \times D_V$ )

**Query matrix:**  $W_Q$  (Shape:  $D_x \times D_Q$ )

## Computation:

**Query vectors:**  $Q = XW_Q$

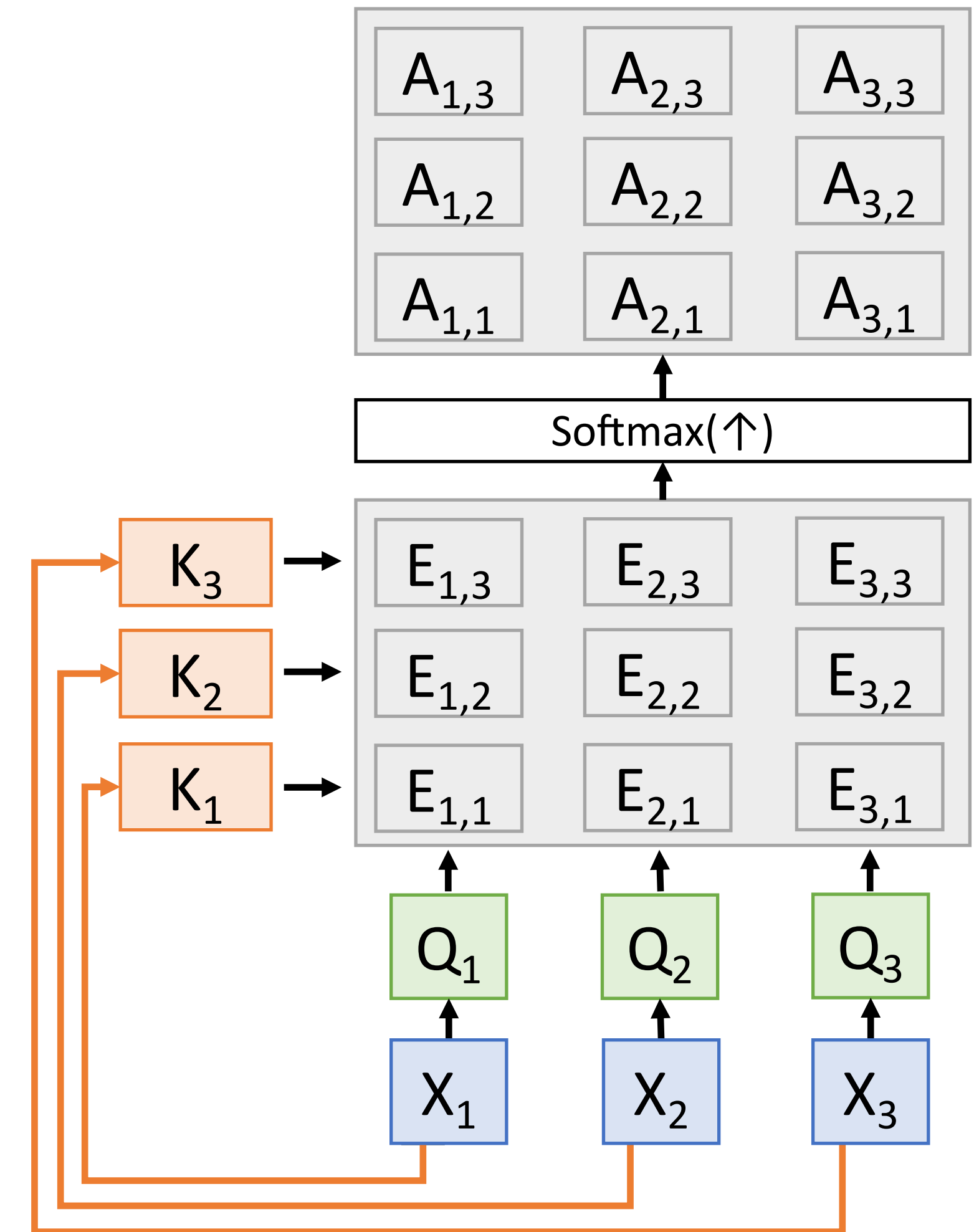
**Key vectors:**  $K = XW_k$  (Shape:  $N_x \times D_Q$ )

**Value Vectors:**  $V = XW_v$  (Shape:  $N_x \times D_V$ )

**Similarities:**  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_x \times N_x$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

**Attention weights:**  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_x \times N_x$ )

**Output vectors:**  $Y = AV$  (Shape:  $N_x \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$





# Self-attention Layer

One **query** per **input vector**

## Inputs:

**Input vectors:**  $X$  (Shape:  $N_x \times D_x$ )

**Key matrix:**  $W_k$  (Shape:  $D_x \times D_Q$ )

**Value matrix:**  $W_v$  (Shape:  $D_x \times D_V$ )

**Query matrix:**  $W_Q$  (Shape:  $D_x \times D_Q$ )

## Computation:

**Query vectors:**  $Q = XW_Q$

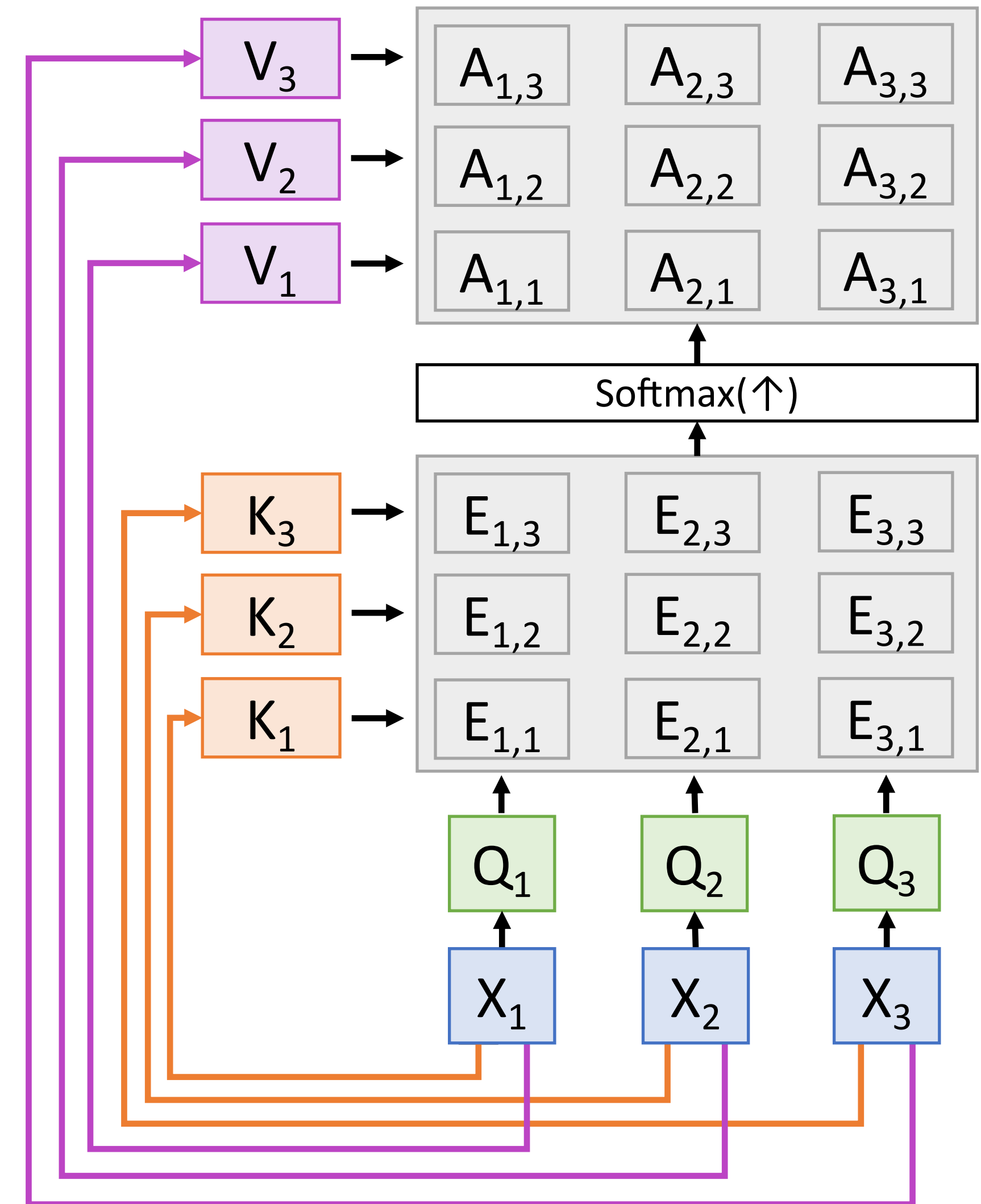
**Key vectors:**  $K = XW_k$  (Shape:  $N_x \times D_Q$ )

**Value Vectors:**  $V = XW_v$  (Shape:  $N_x \times D_V$ )

**Similarities:**  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_x \times N_x$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

**Attention weights:**  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_x \times N_x$ )

**Output vectors:**  $Y = AV$  (Shape:  $N_x \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$



# Self-attention Layer

One **query** per **input vector**

## Inputs:

Input vectors:  $X$  (Shape:  $N_x \times D_x$ )

Key matrix:  $W_k$  (Shape:  $D_x \times D_Q$ )

Value matrix:  $W_v$  (Shape:  $D_x \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_x \times D_Q$ )

## Computation:

Query vectors:  $Q = XW_Q$

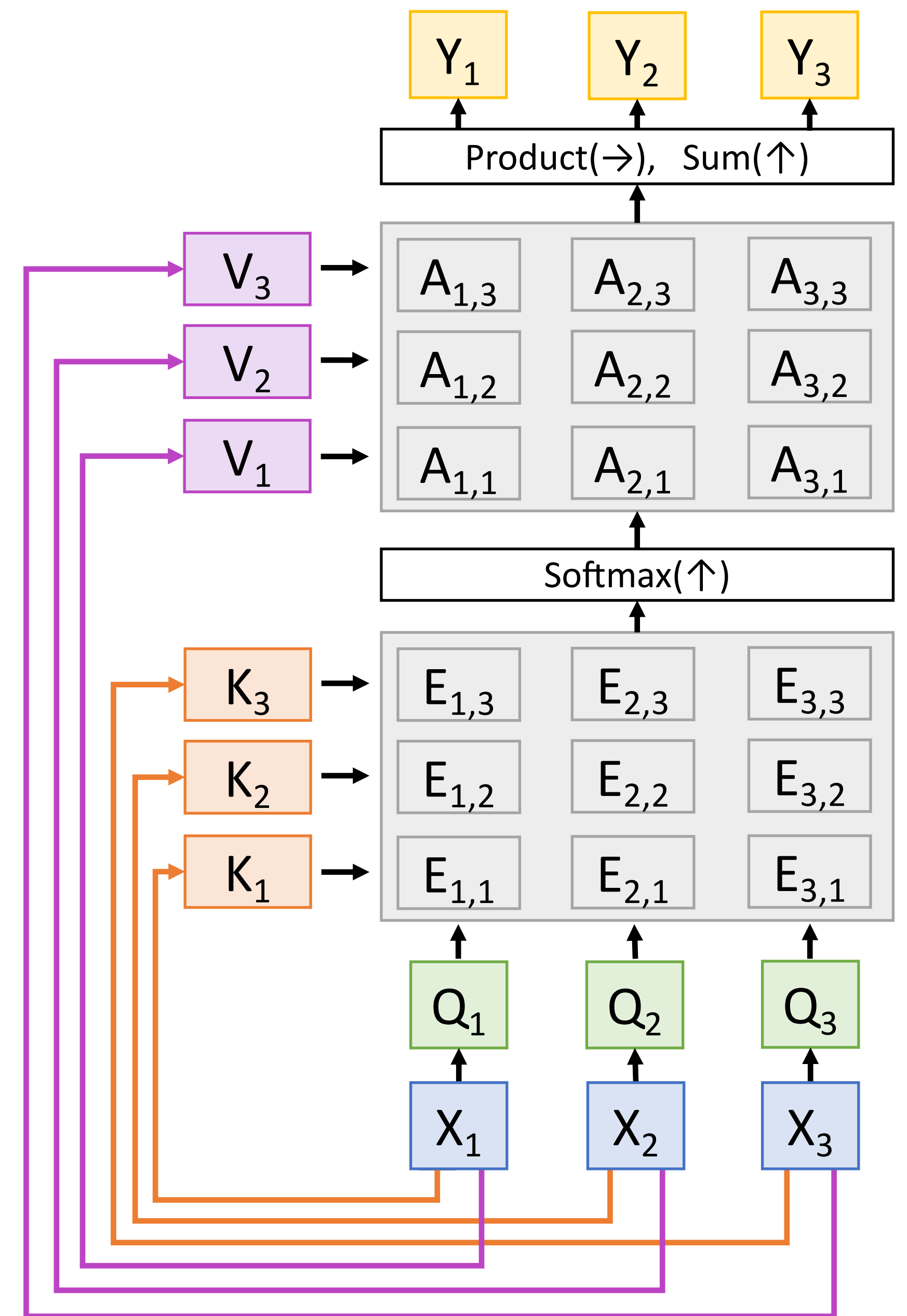
Key vectors:  $K = XW_k$  (Shape:  $N_x \times D_Q$ )

Value Vectors:  $V = XW_v$  (Shape:  $N_x \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_x \times N_x$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

Attention weights:  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_x \times N_x$ )

Output vectors:  $Y = AV$  (Shape:  $N_x \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$





# Self-attention Layer

## Inputs:

Input vectors:  $X$  (Shape:  $N_x \times D_x$ )

Key matrix:  $W_k$  (Shape:  $D_x \times D_Q$ )

Value matrix:  $W_v$  (Shape:  $D_x \times D_v$ )

Query matrix:  $W_Q$  (Shape:  $D_x \times D_Q$ )

## Computation:

Query vectors:  $Q = XW_Q$

Key vectors:  $K = XW_k$  (Shape:  $N_x \times D_Q$ )

Value Vectors:  $V = XW_v$  (Shape:  $N_x \times D_v$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_x \times N_x$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

Attention weights:  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_x \times N_x$ )

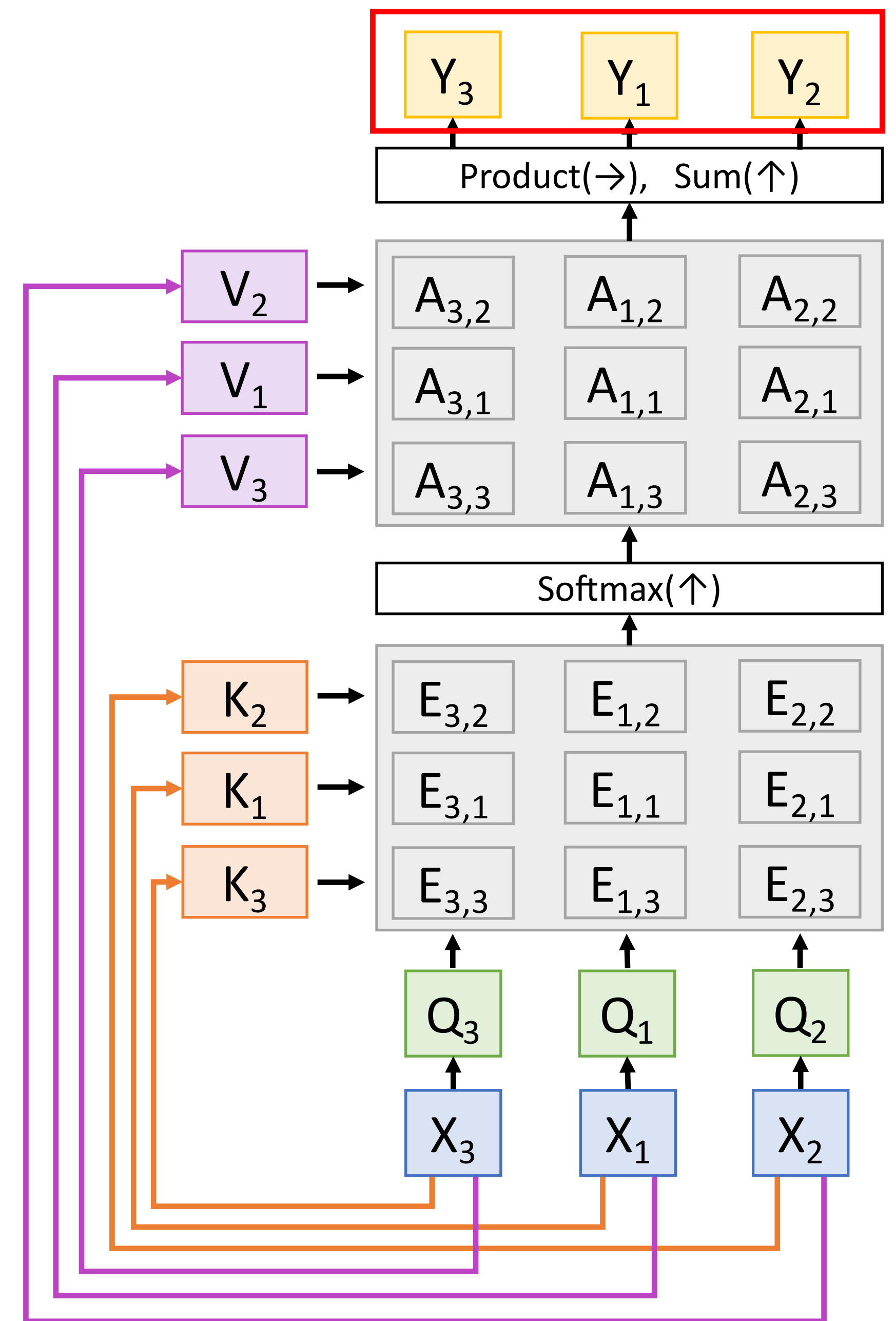
Output vectors:  $Y = AV$  (Shape:  $N_x \times D_v$ )  $Y_i = \sum_j A_{i,j} V_j$

Consider **permuting** the input vectors:

Outputs will be the same, but **permuted**

Self-attention layer is **Permutation Equivariant**  
 $f(s(x)) = s(f(x))$

Self-Attention layer works on **sets** of vectors



# Multi-head Self-attention Layer

## Inputs:

Input vectors:  $X$  (Shape:  $N_X \times D_X$ )

Key matrix:  $W_K$  (Shape:  $D_X \times D_Q$ )

Value matrix:  $W_V$  (Shape:  $D_X \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_X \times D_Q$ )

## Computation:

Query vectors:  $Q = XW_Q$

Key vectors:  $K = XW_K$  (Shape:  $N_X \times D_Q$ )

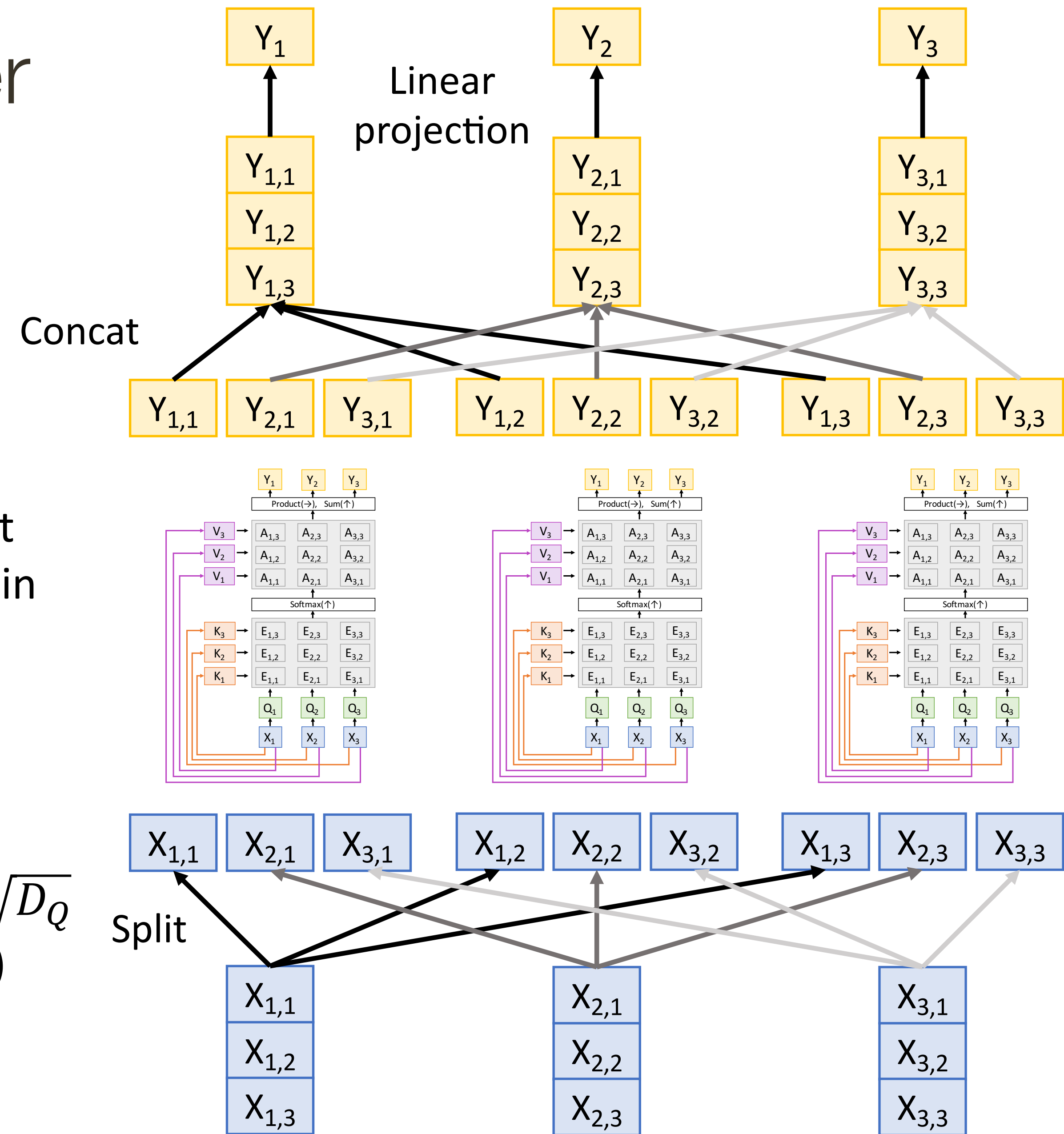
Value Vectors:  $V = XW_V$  (Shape:  $N_X \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_X \times N_X$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

Attention weights:  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_X \times N_X$ )

Output vectors:  $Y = AV$  (Shape:  $N_X \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$

Use H independent  
“Attention Heads” in  
parallel



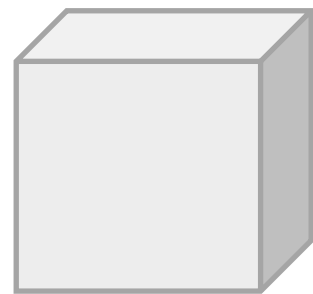
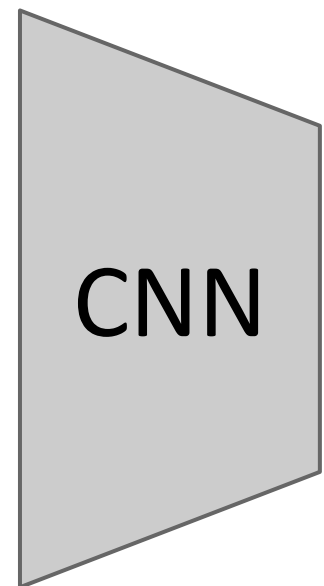


# CNN with Self-attention

Input Image

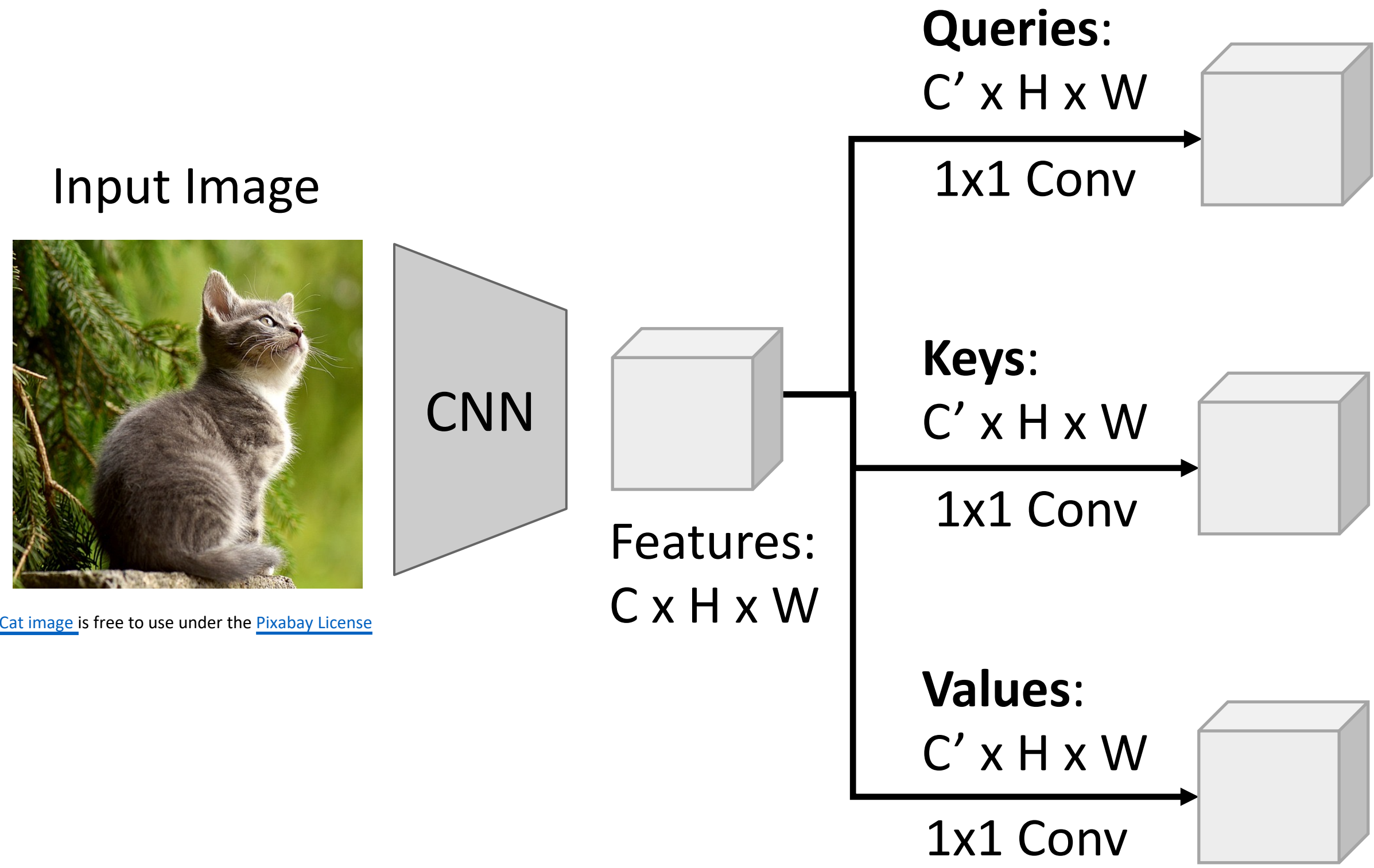


[Cat image](#) is free to use under the [Pixabay License](#)



Features:  
 $C \times H \times W$

# CNN with Self-attention

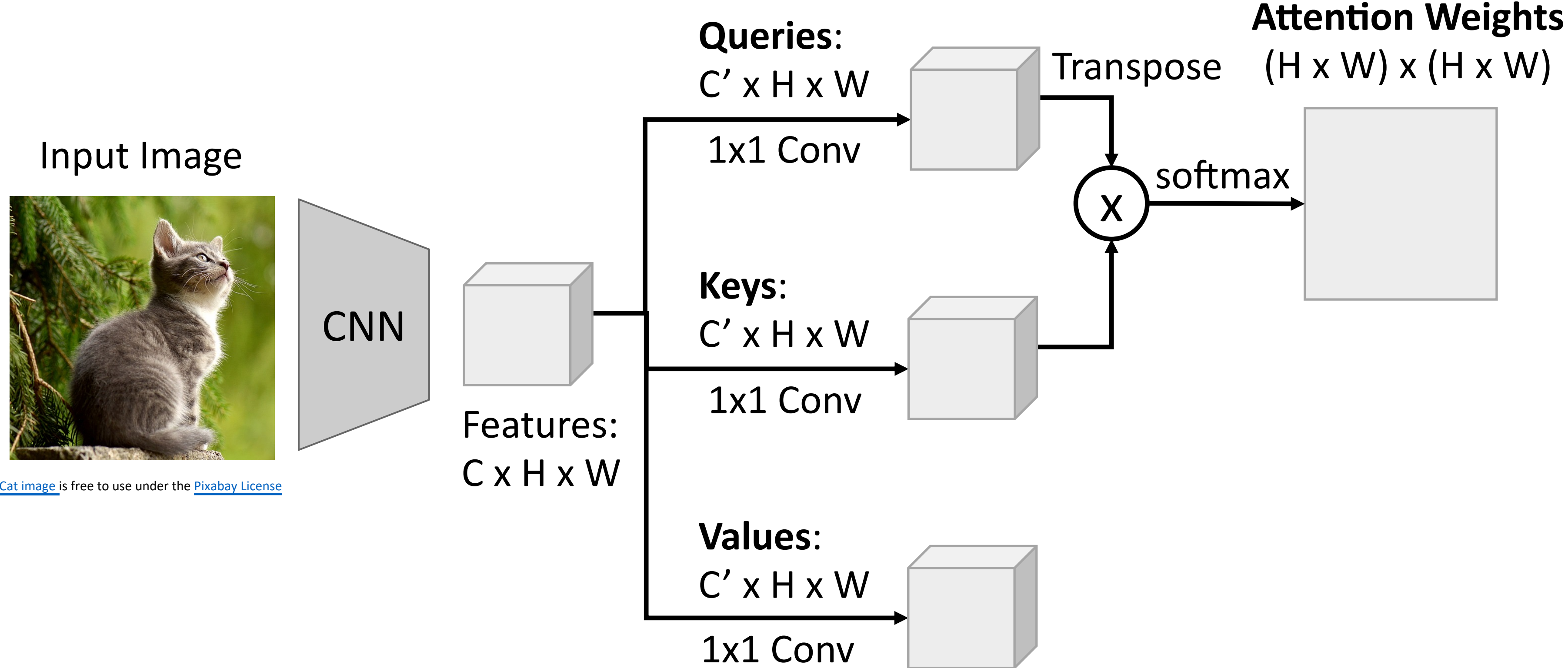


Zhang et al, "Self-Attention Generative Adversarial Networks", ICML 2018

[ slide from Justin Johnson, U Michigan ]



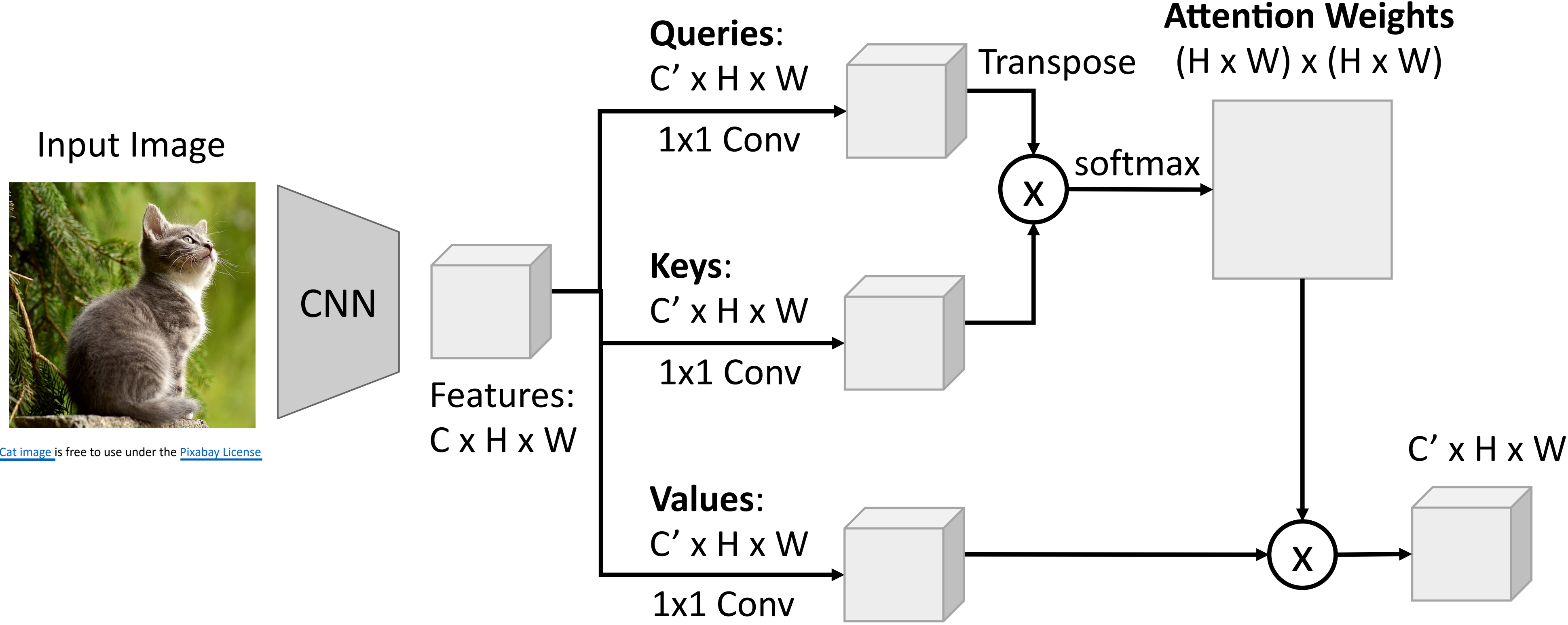
# CNN with Self-attention



Zhang et al, "Self-Attention Generative Adversarial Networks", ICML 2018

[ slide from Justin Johnson, U Michigan ]

# CNN with Self-attention

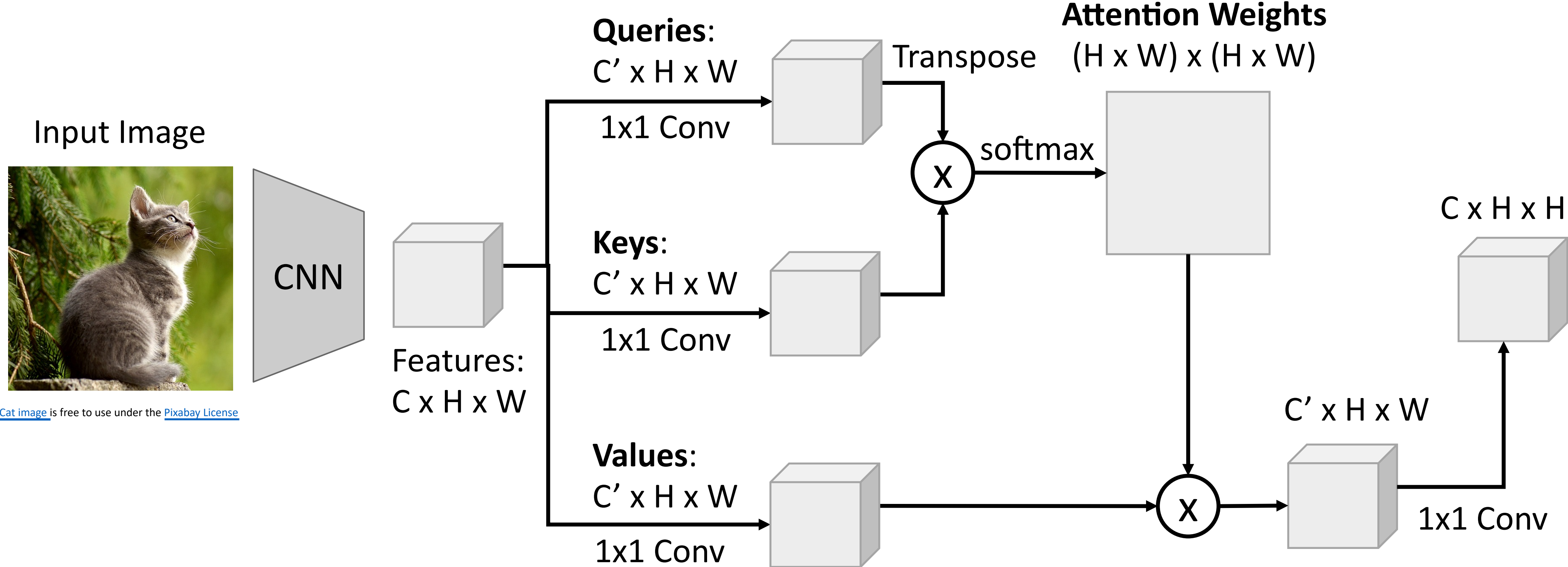


Zhang et al, "Self-Attention Generative Adversarial Networks", ICML 2018

[ slide from Justin Johnson, U Michigan ]



# CNN with Self-attention



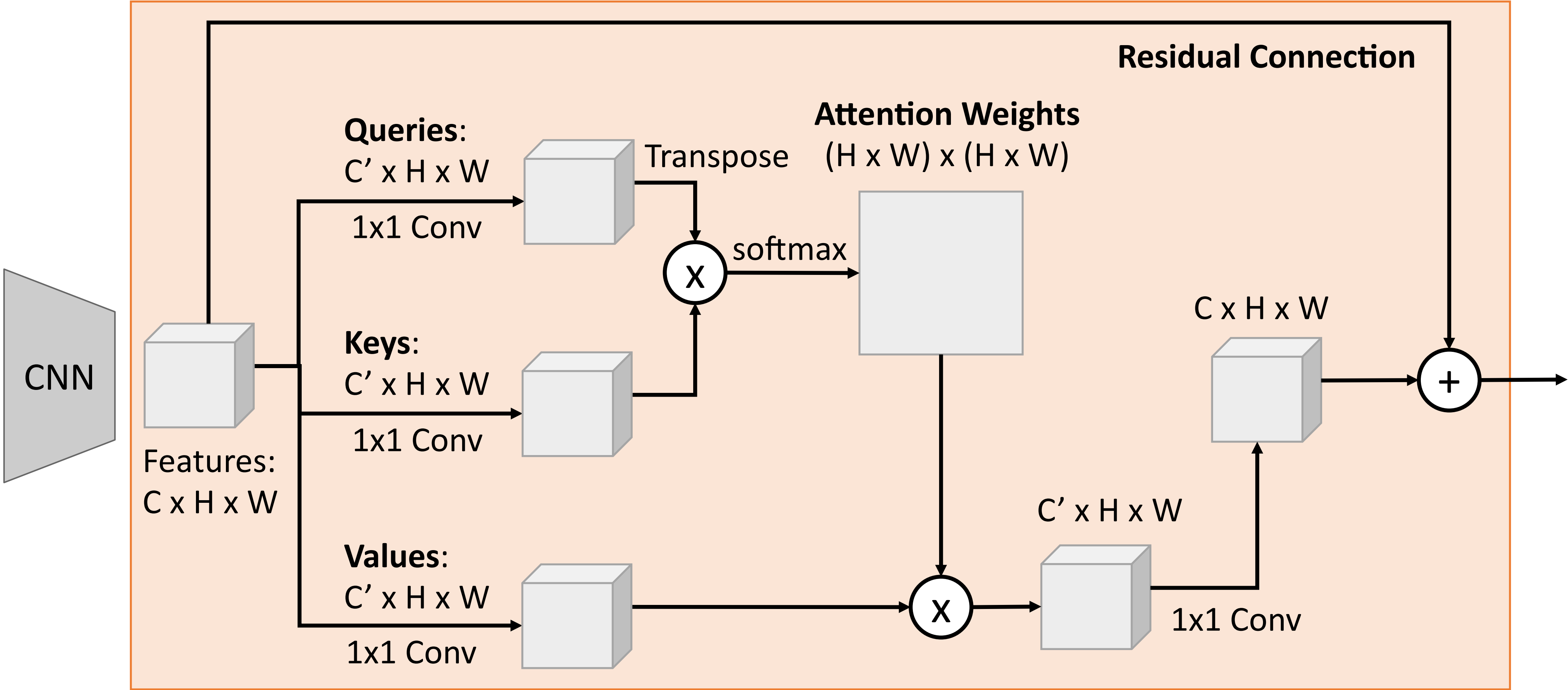
Zhang et al, "Self-Attention Generative Adversarial Networks", ICML 2018

[ slide from Justin Johnson, U Michigan ]

# CNN with Self-attention



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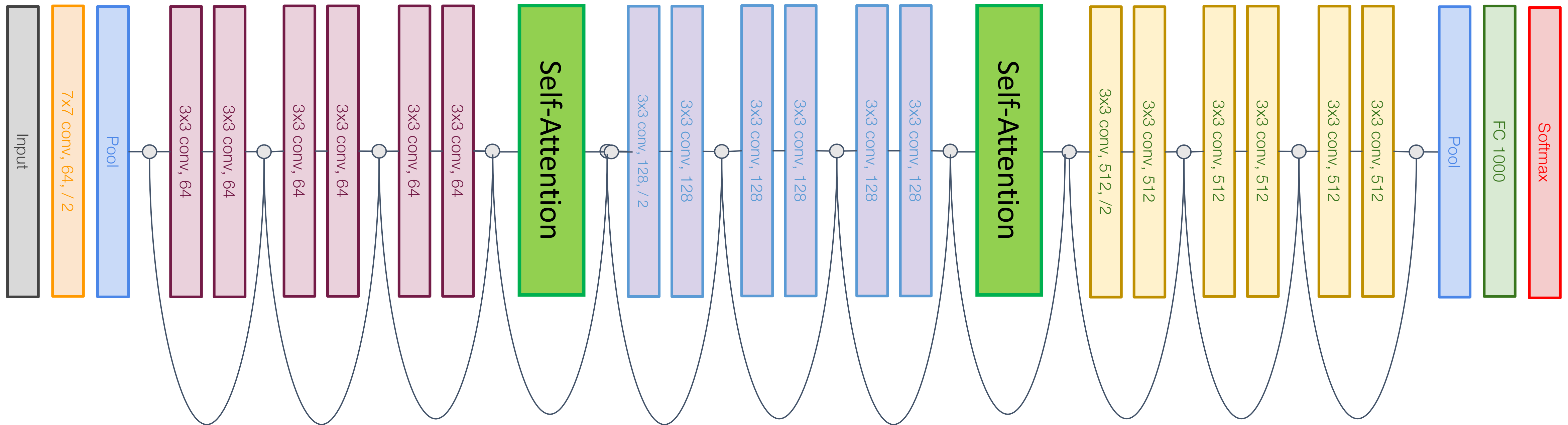


Self-Attention Module

Zhang et al, "Self-Attention Generative Adversarial Networks", ICML 2018

[ slide from Justin Johnson, U Michigan ]

# Attention with Existing CNNs



Zhang et al, "Self-Attention Generative Adversarial Networks", ICML 2018

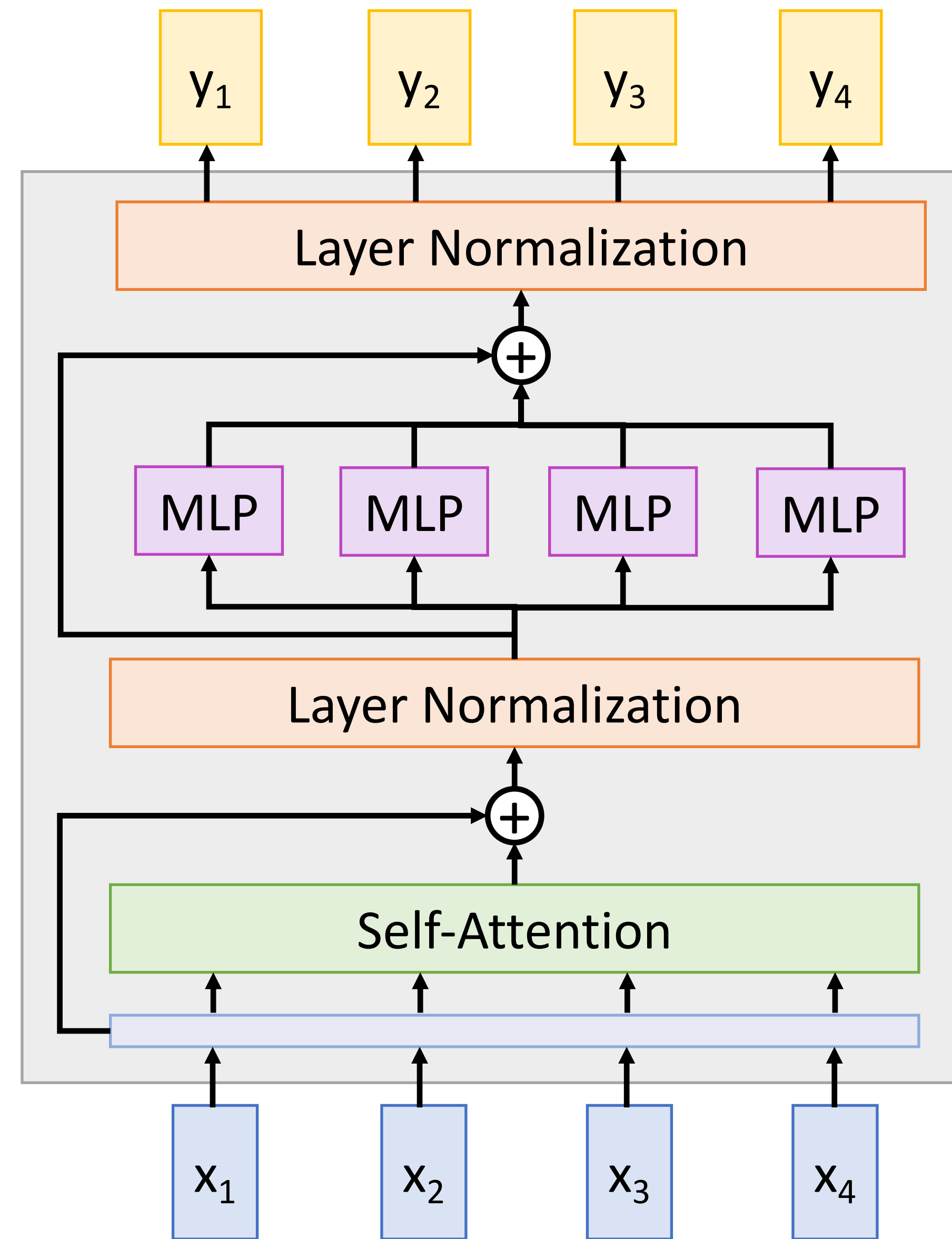
Wang et al, "Non-local Neural Networks", CVPR 2018



# Transformer

Transformer block inputs a set of vectors, outputs a set of vectors.

Vectors only communicate via (multiheaded) self-attention



# Transformer

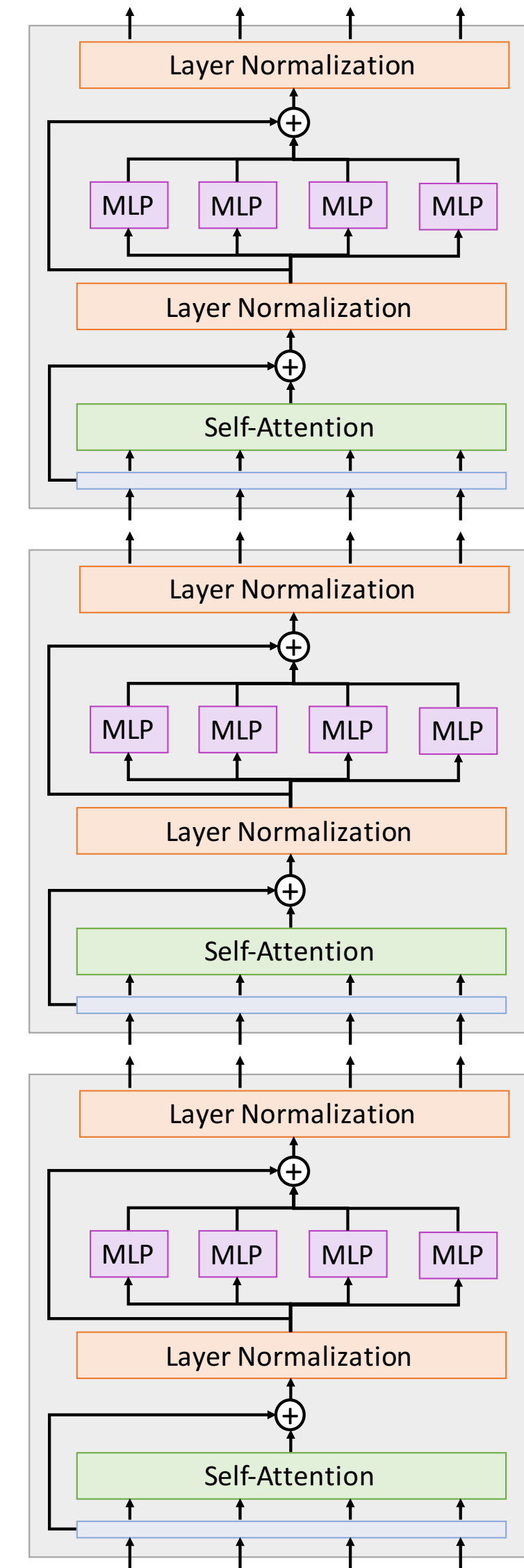
## Transformer Block:

**Input:** Set of vectors  $x$

**Output:** Set of vectors  $y$

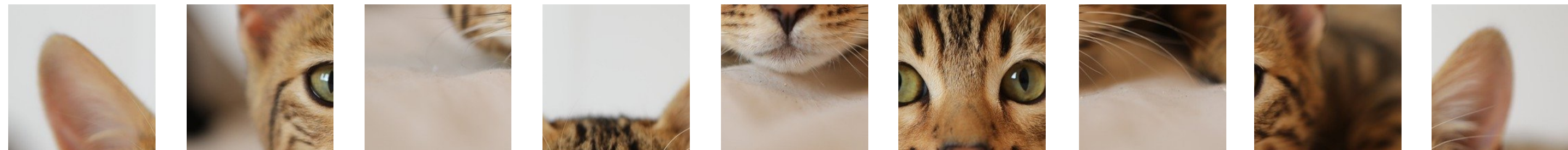
Hyperparameters:

- Number of blocks
- Number of heads per block
- Width (channels per head, FFN width)



# Transformer on Image Patches

N input patches, each  
of shape 3x16x16



Dosovitskiy et al, "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale", ICLR 2021

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use under a [Pixabay license](#)

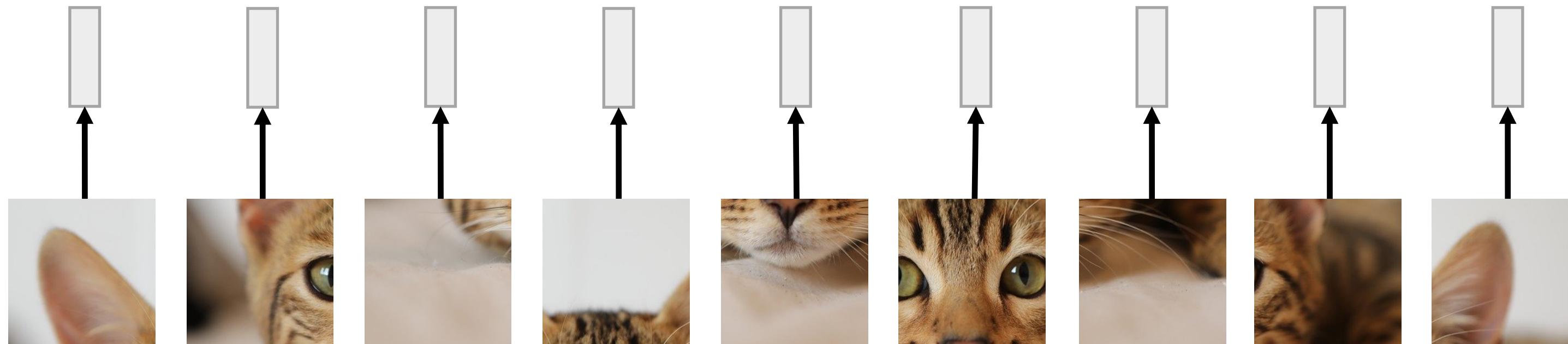
[ slide from Justin Johnson, U Michigan ]



# Transformer on Image Patches

Linear projection to  
D-dimensional vector

N input patches, each  
of shape 3x16x16



Dosovitskiy et al, "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale", ICLR 2021

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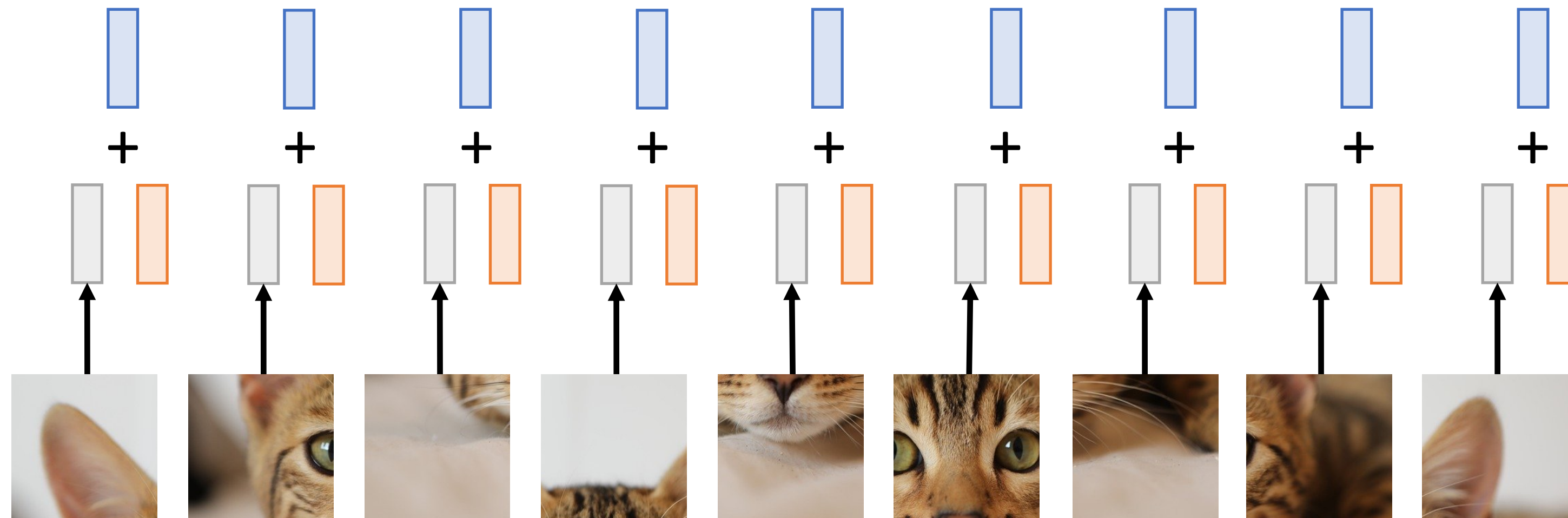
[ slide from Justin Johnson, U Michigan ]

# Transformer on Image Patches

Add positional embedding: learned D-dim vector per position

Linear projection to D-dimensional vector

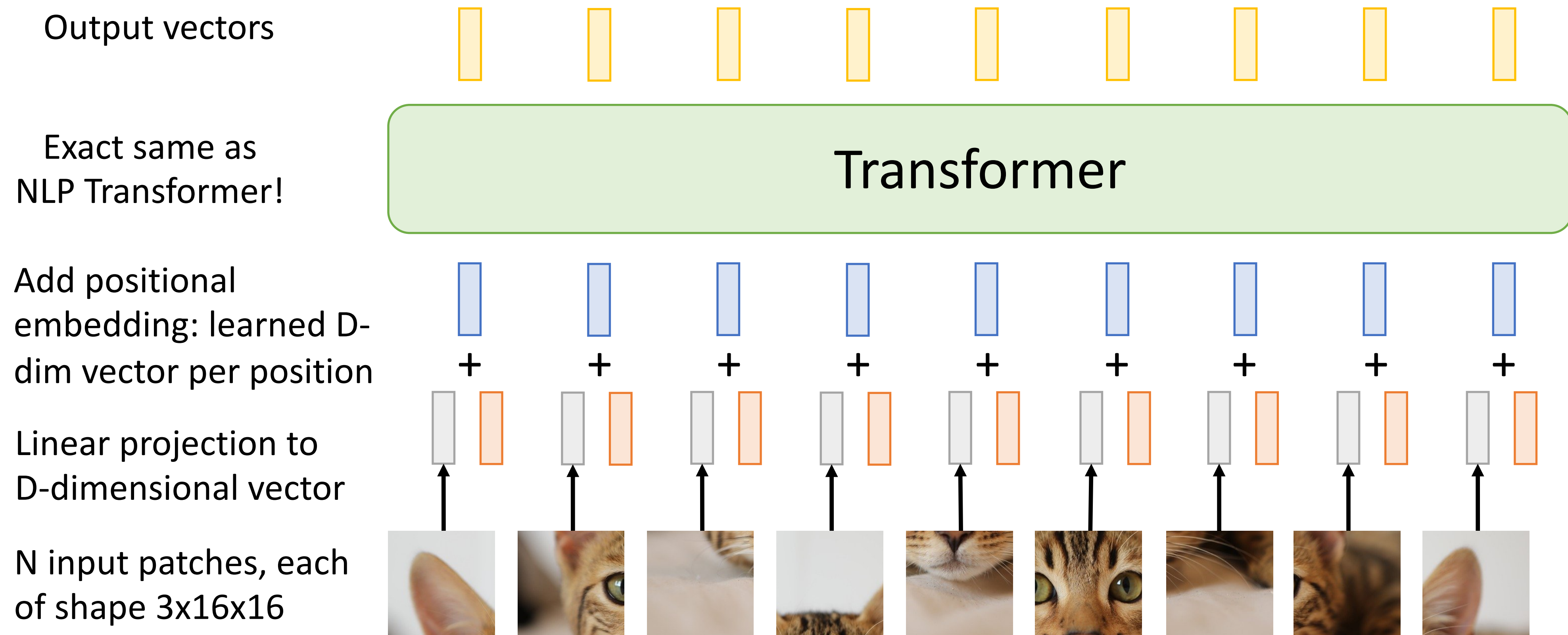
N input patches, each of shape 3x16x16



Dosovitskiy et al, "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale", ICLR 2021

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# Transformer on Image Patches

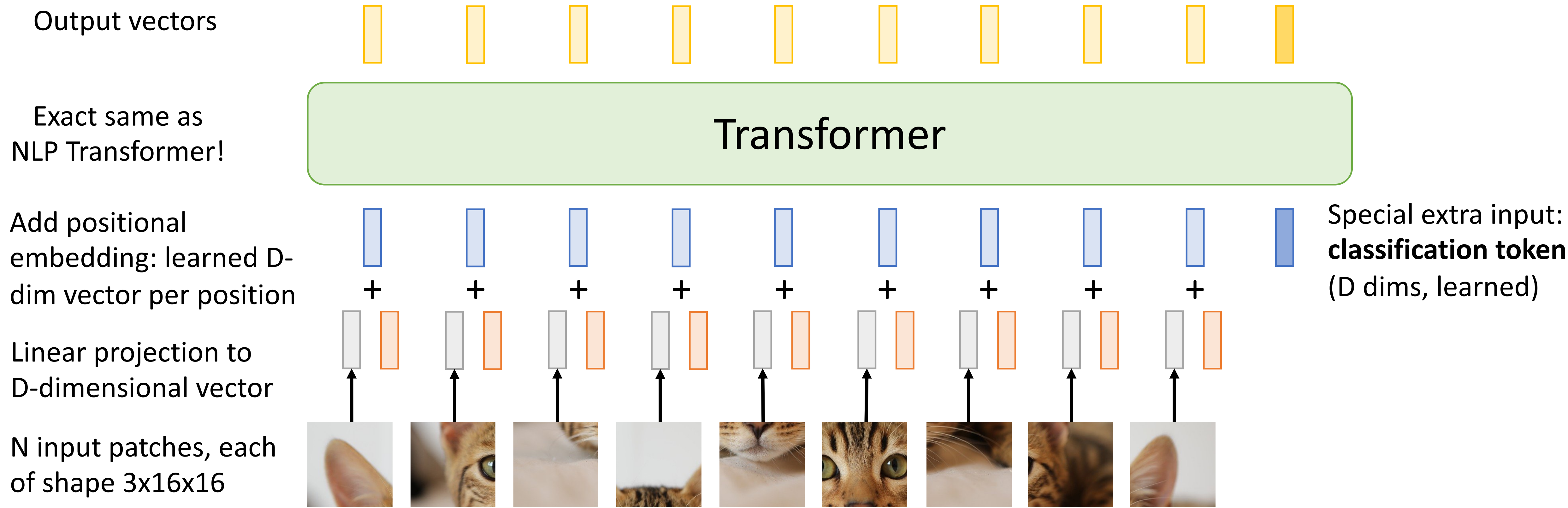


Dosovitskiy et al, "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale", ICLR 2021

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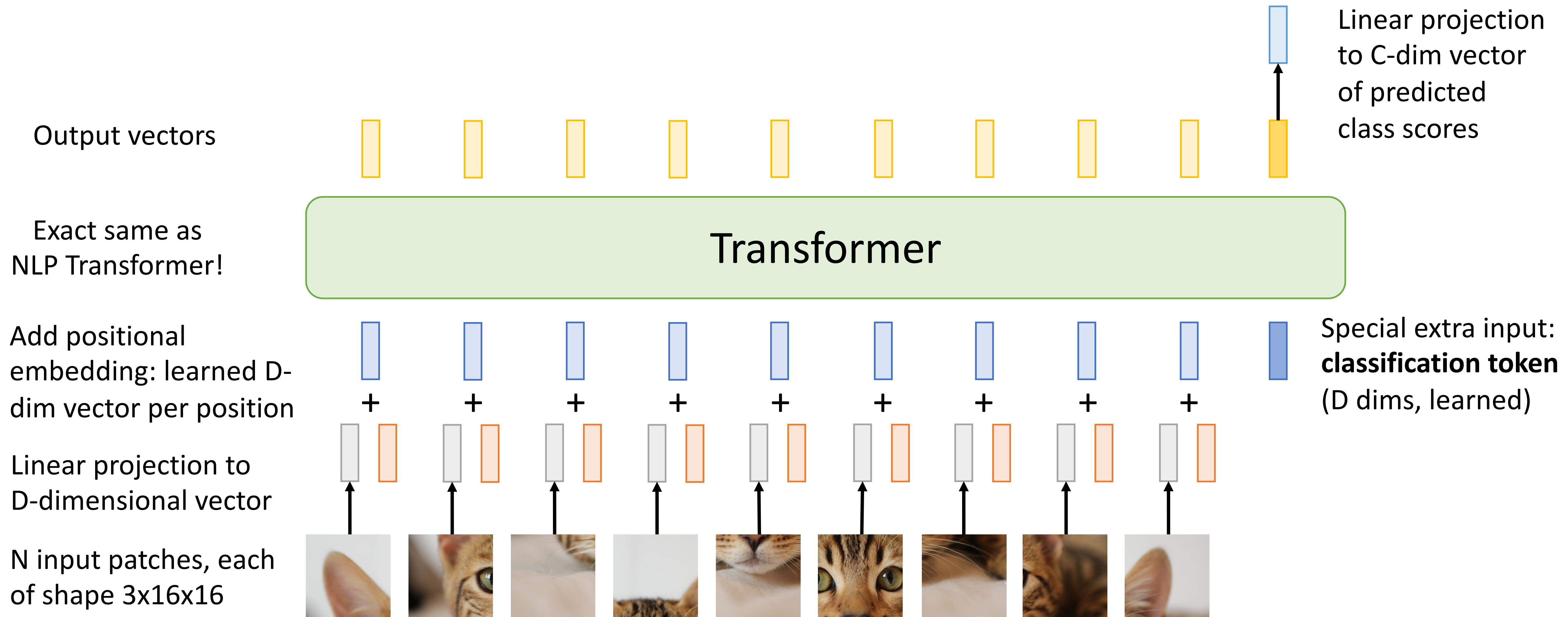
# Transformer on Image Patches



Dosovitskiy et al, "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale", ICLR 2021

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# Transformer on Image Patches



Dosovitskiy et al, "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale", ICLR 2021

[Cat image](#) is free for commercial use under a [Pixabay license](#)

# Vision Transformer (ViT)

Computer vision model  
with no convolutions!

Output vectors

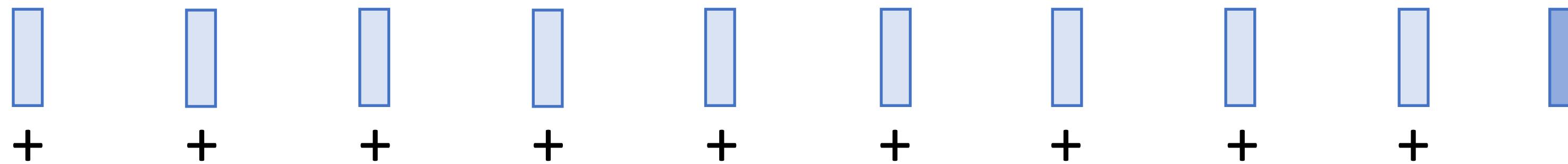


Linear projection  
to C-dim vector  
of predicted  
class scores

Exact same as  
NLP Transformer!

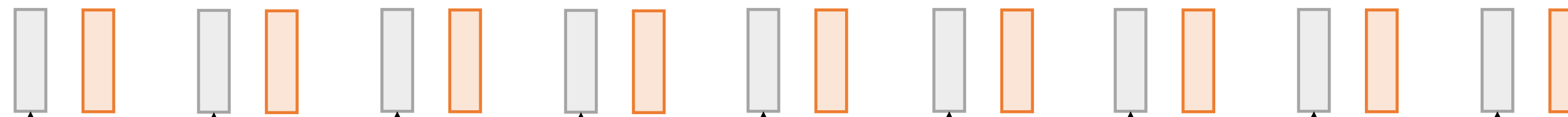
Transformer

Add positional  
embedding: learned D-  
dim vector per position

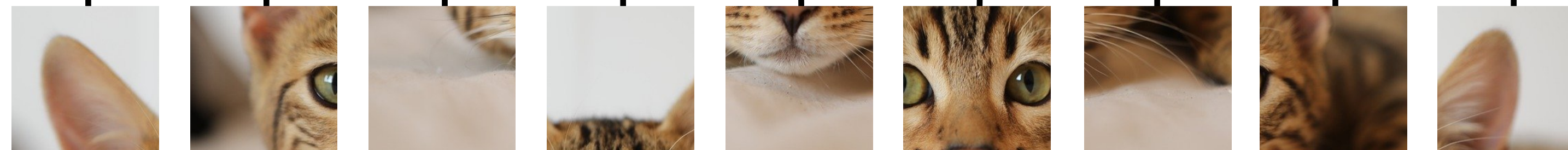


Special extra input:  
**classification token**  
(D dims, learned)

Linear projection to  
D-dimensional vector



N input patches, each  
of shape 3x16x16



Dosovitskiy et al, "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale", ICLR 2021

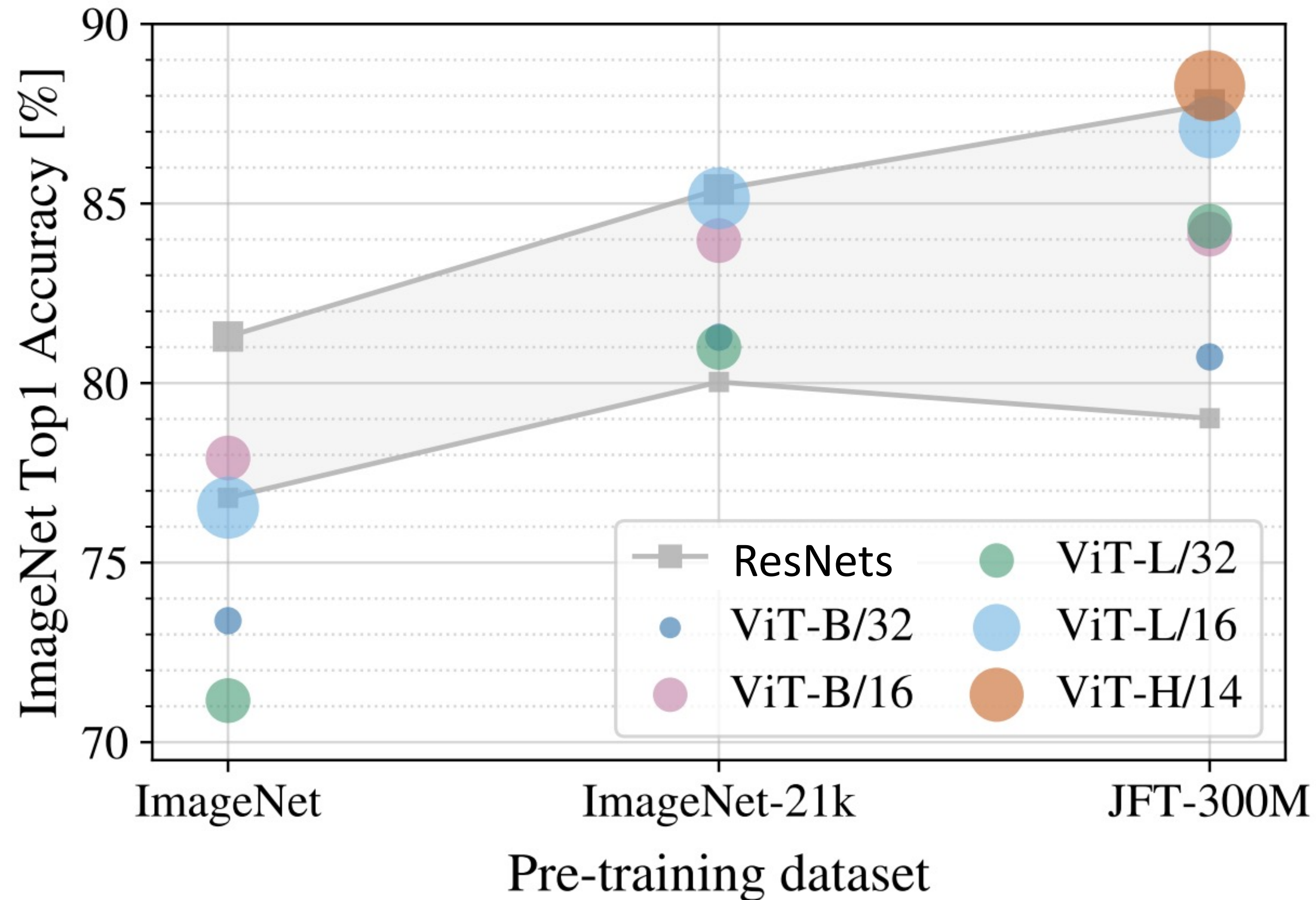
[Cat image](#) is free for commercial  
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# Vision Transformer (ViT) vs. ResNet

JFT-300M is an internal Google dataset with 300M labeled images

If you pretrain on JFT and finetune on ImageNet, large ViTs outperform large ResNets



B = Base  
L = Large  
H = Huge

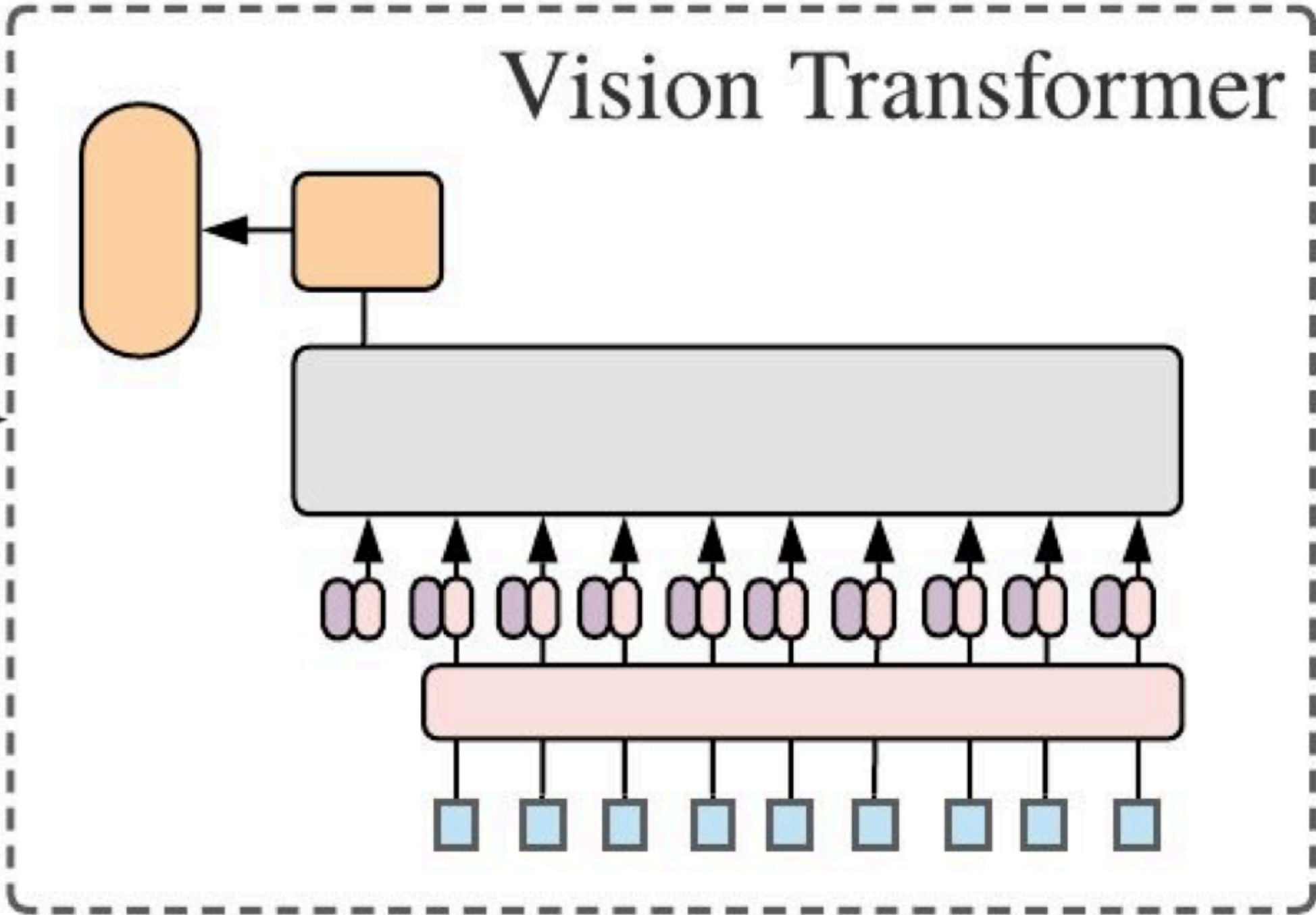
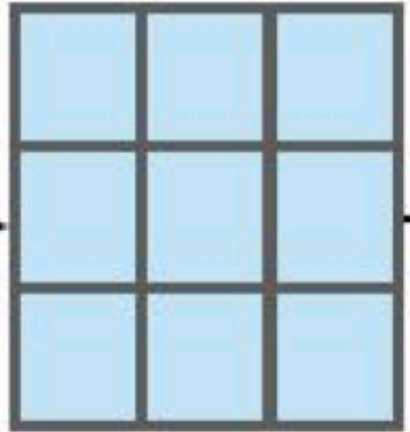
/32, /16, /14 is patch size; smaller patch size is a bigger model (more patches)

Dosovitskiy et al, "An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale", ICLR 2021

# ResNet-ViT Hybrid

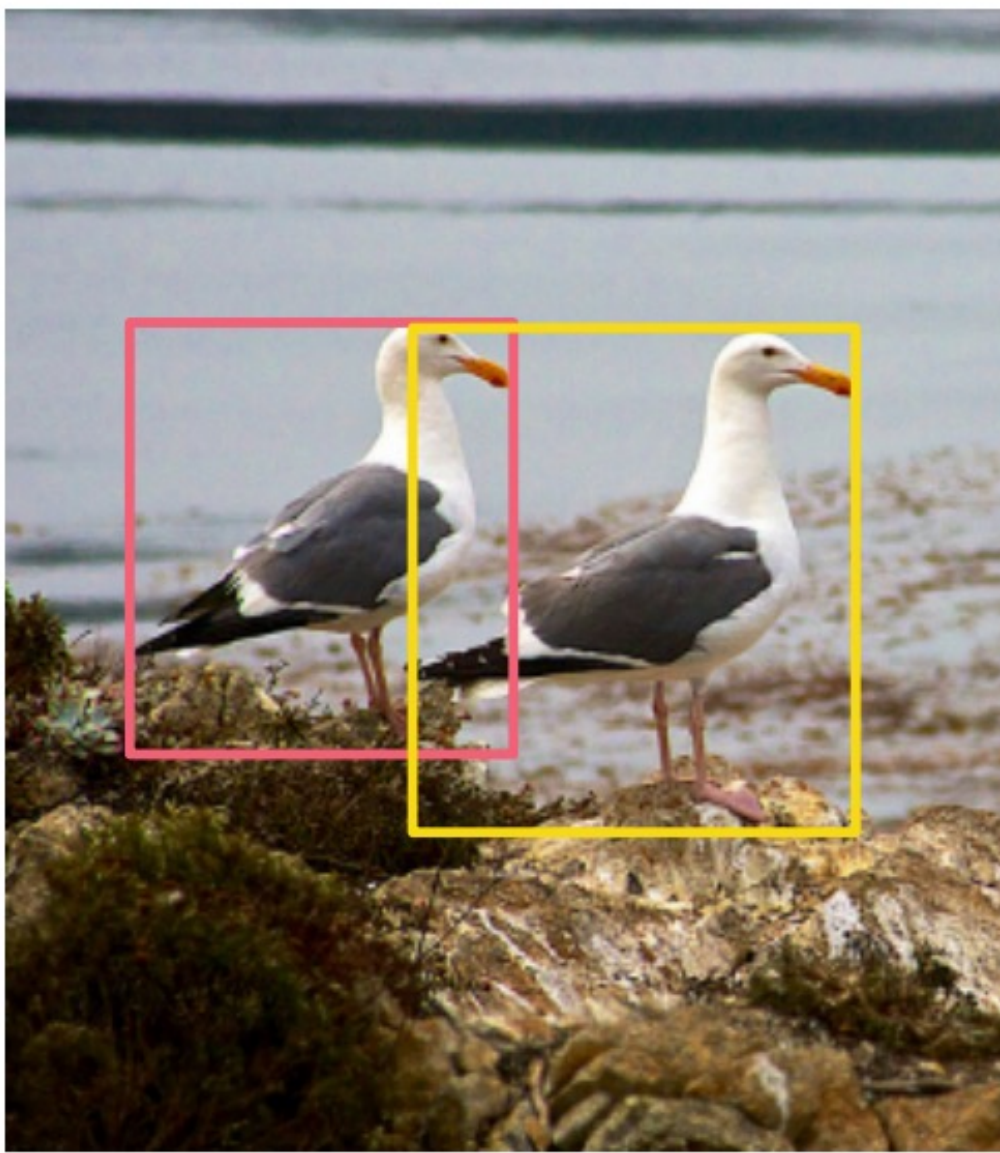
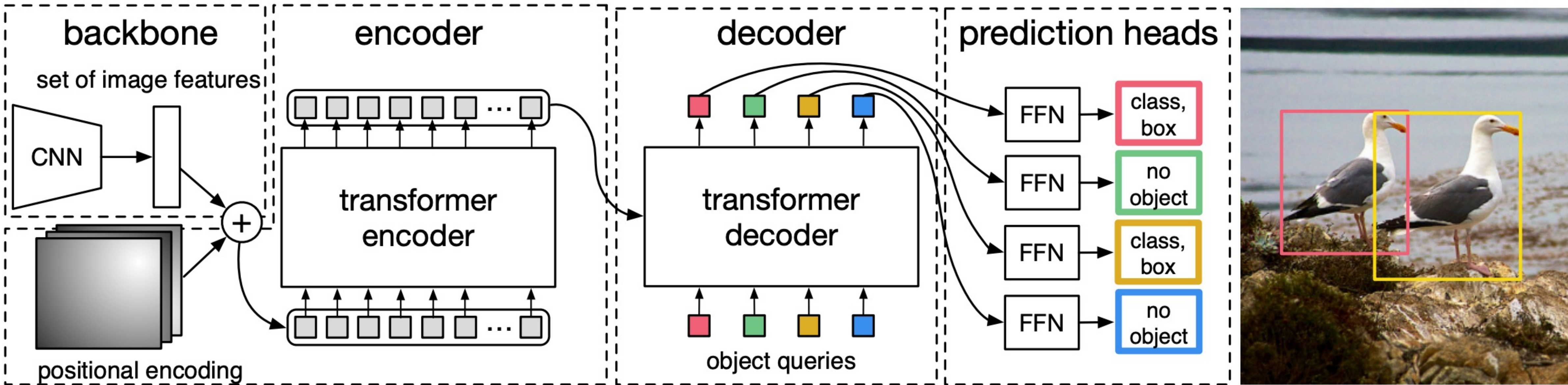
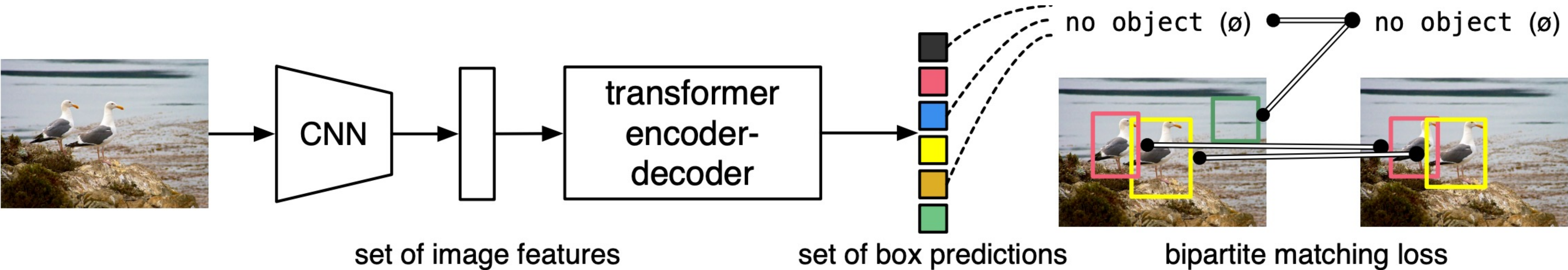


50 layer ResNet





# Object Detection with Transformers: DETR Model

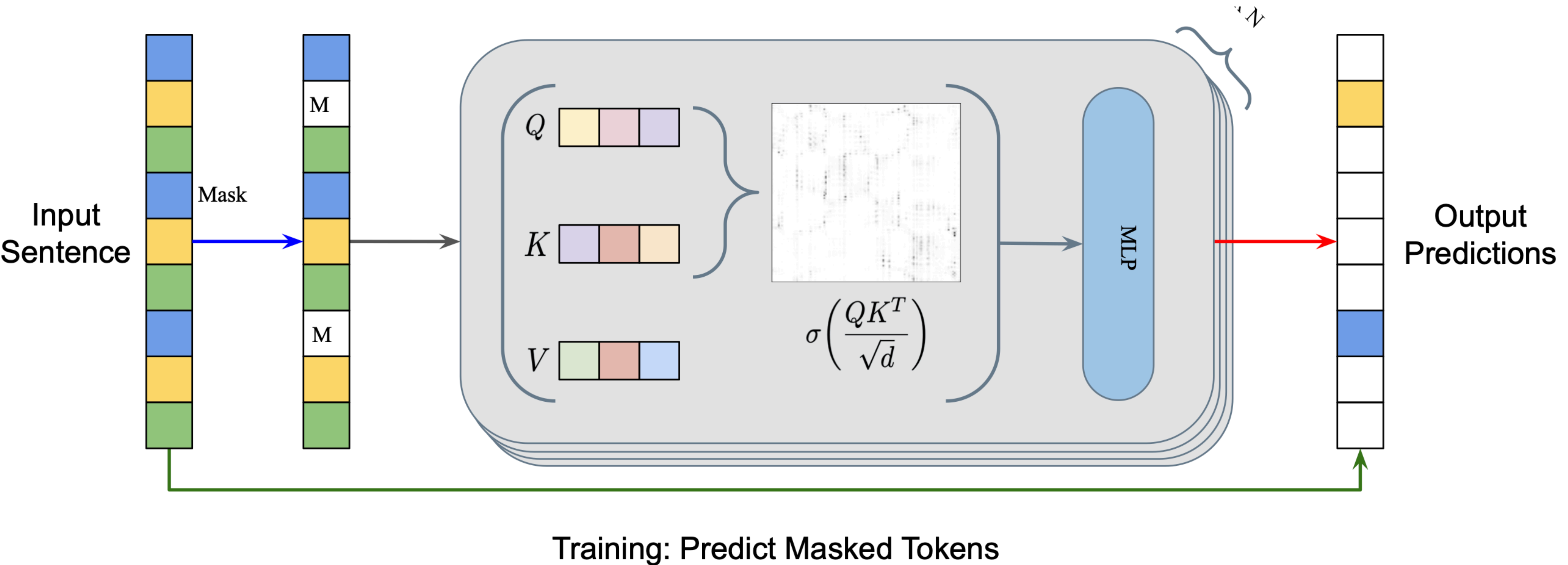


Carion et al, "End-to-End Object Detection with Transformers", ECCV 2020

[ slide from Justin Johnson, U Michigan ]



# Masked Modeling with Transformers (BERT, GPT, etc.)



$$\mathcal{L}_{\text{MLM}}(X; \theta) = \mathbb{E}_{x \sim X} \mathbb{E}_{\text{mask}} \sum_{i \in \text{mask}} \log p(x_i | x_j \notin \text{mask}; \theta)$$

(mask 15% at a time)

# Masked Self-Attention Layer

Don't let vectors "look ahead" in the sequence  
Used for language modeling (predict next word)

## Inputs:

Input vectors:  $X$  (Shape:  $N_x \times D_x$ )

Key matrix:  $W_k$  (Shape:  $D_x \times D_Q$ )

Value matrix:  $W_v$  (Shape:  $D_x \times D_V$ )

Query matrix:  $W_Q$  (Shape:  $D_x \times D_Q$ )

## Computation:

Query vectors:  $Q = XW_Q$

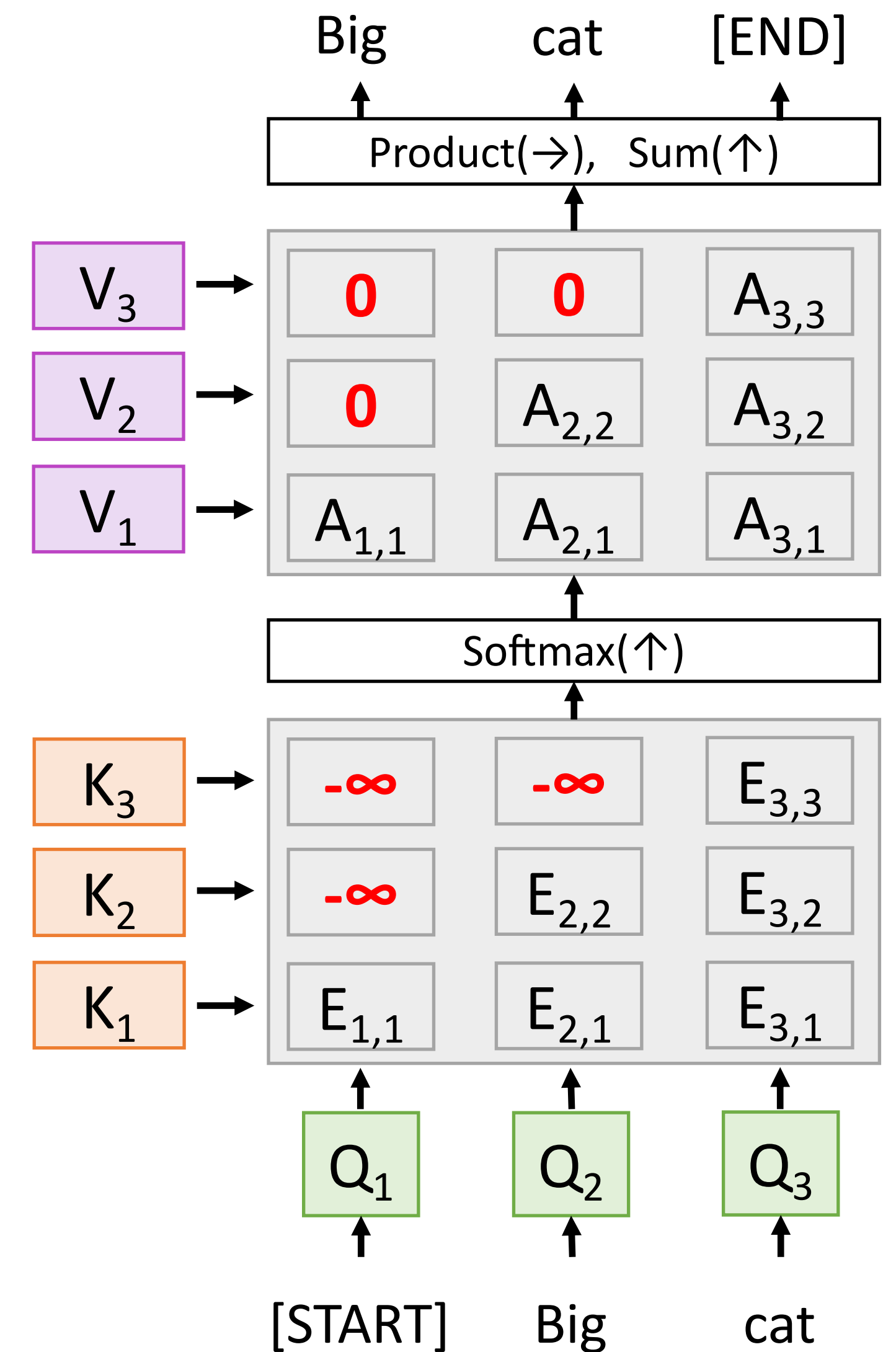
Key vectors:  $K = XW_k$  (Shape:  $N_x \times D_Q$ )

Value Vectors:  $V = XW_v$  (Shape:  $N_x \times D_V$ )

Similarities:  $E = QK^T / \sqrt{D_Q}$  (Shape:  $N_x \times N_x$ )  $E_{i,j} = (Q_i \cdot K_j) / \sqrt{D_Q}$

Attention weights:  $A = \text{softmax}(E, \text{dim}=1)$  (Shape:  $N_x \times N_x$ )

Output vectors:  $Y = AV$  (Shape:  $N_x \times D_V$ )  $Y_i = \sum_j A_{i,j} V_j$



Please fill out  
**Student Evaluations**  
(on Canvas)