CPSC 340:
Machine Learning and Data Mining

Principal Component Analysis

Summer 2021

Admin

- Assignment 5 due Friday at 11:55pm
  - Extended by 12+ hours
  - We will cover all relevant details today
- This lecture will be 75 minutes long
- Office hours and tutorials had low attendance
  - I promise they save you TONS of time in the long run
  - We have 10+ office hours per week
  - Going to a few office hours will probably get most of the work out of the way

In This Lecture

1. MAP Estimation
2. Wrapping Up Part 3
3. Principal Component Analysis

Last Time: MLE of Gaussian Likelihood

- Let's assume that $y_i = w^T x_i + \varepsilon_i$, with ε_i following standard normal:

$$p(\varepsilon_i) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\varepsilon_i^2}{2}\right)$$

also known as "Gaussian" distribution

- This leads to a Gaussian likelihood for example 'i' of the form:

$$p(y_i \mid x_i, w) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(w^T x_i - y_i)^2}{2}\right)$$

- Finding MLE (minimizing NLL) is least squares:

[1] $f(w) = -\sum_{i=1}^{n} \log(p(y_i \mid w, x_i))$

[2] $= -\sum_{i=1}^{n} \log\left(\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(w^T x_i - y_i)^2}{2}\right)\right)$

[3] $= -\sum_{i=1}^{n} \left[\log\left(\frac{1}{\sqrt{2\pi}}\right) + \log\left(\exp\left(-\frac{(w^T x_i - y_i)^2}{2}\right)\right)\right]$

constant in 'w'

operations cancel

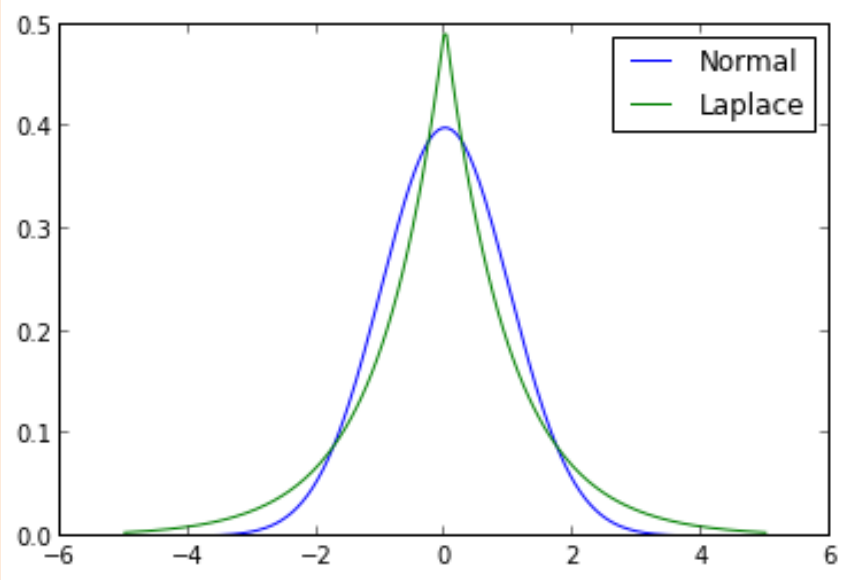
[4] $= -\sum_{i=1}^{n} \left[(\text{constant}) - \frac{1}{2}(w^T x_i - y_i)^2\right]$

[5] $= (\text{constant}) + \frac{1}{2}\sum_{i=1}^{n} (w^T x_i - y_i)^2$

[6] $= (\text{constant}) + \frac{1}{2}\|Xw - y\|^2$

"Heavy" Tails vs. "Light" Tails

- We know that L1-norm is more robust than L2-norm.
  - What does this mean in terms of probabilities?



Here "fail" means "mass of the distribution away from the mean"

  - Gaussian has "light tails": assumes everything is close to mean.
  - Laplace has "heavy tails": assumes some data is far from mean.
  - Student 't' is even more heavy-tailed/robust, but NLL is non-convex.

http://austinrochford.com/posts/2013-09-02-prior-distributions-for-bayesian-regression-using-pymc.html

Last Time: MLE of Sigmoid Likelihood

- For IID regression problems the conditional NLL can be written:

$$[1] \quad \underbrace{-\log(p(y \mid X, w))}_{NLL} = -\log\Big(\underbrace{\prod_{i=1}^{n} p(y_i \mid x_i, w)}_{\text{IID assumption}}\Big) = \underbrace{-\sum_{i=1}^{n}}_{\text{log turns product into sum}} \log(p(y_i \mid x_i, w))$$

- Logistic regression assumes sigmoid($w^T x_i$) conditional likelihood:

$$[2] \quad p(y_i \mid x_i, w) = h(y_i w^T x_i) \quad \text{where} \quad h(z_i) = \frac{1}{1+\exp(-z_i)}$$

- Plugging in the sigmoid likelihood, the NLL is the logistic loss:

$$[3] \quad NLL(w) = -\sum_{i=1}^{n} \log\left(\frac{1}{1+\exp(-y_i w^T x_i)}\right) = \sum_{i=1}^{n} \log(1+\exp(-y_i w^T x_i))$$

(since $\log(1) = 0$)

MLE Interpretation of Logistic Regression

- Instead of "smooth convex approximation of 0-1 loss", we now have that logistic regression is doing MLE in a probabilistic model.
  - "Maximize +1-ness of +1 examples and -1-ness of -1 examples"
  - The training and prediction would be the same as before.
    - We still minimize the logistic loss in terms of 'w'.
  - But MLE justifies using sigmoid with learned w to get +1-ness:

$$p(y_i \mid x_i, w) = \frac{1}{1 + \exp(-y_i w^T x_i)}$$

  - Softmax function and softmax loss are also connected via NLL
    - See Piazza for derivations

Coming Up Next

MAXIMUM A POSTERIORI ESTIMATION

Maximum Likelihood Estimation and Overfitting

- In our abstract setting with data D the MLE is:

$$\hat{w} \in \operatorname{argmax}_w \{p(D|w)\}$$

- But conceptually MLE is a bit weird:

$$P(D|\cdot): \mathbb{R}^d \to [0,1]$$

  - "Find the 'w' that makes 'D' have the highest probability given 'w'."

- And MLE often leads to overfitting:
  - Data could be very likely for some very unlikely 'w'.
  - For example, a complex model that overfits by memorizing the data.

- What we really want: $P(\cdot|D): \mathbb{R}^d \to [0,1]$
  - "Find the 'w' that has the highest probability given the data D."

Maximum a Posteriori (MAP) Estimation

- Maximum a posteriori (MAP) estimate maximizes the reverse probability:

$$\hat{w} \in \underset{w}{\operatorname{argmax}} \{ p(w \mid D) \}$$

  - This is what we want: the probability of 'w' given our data.

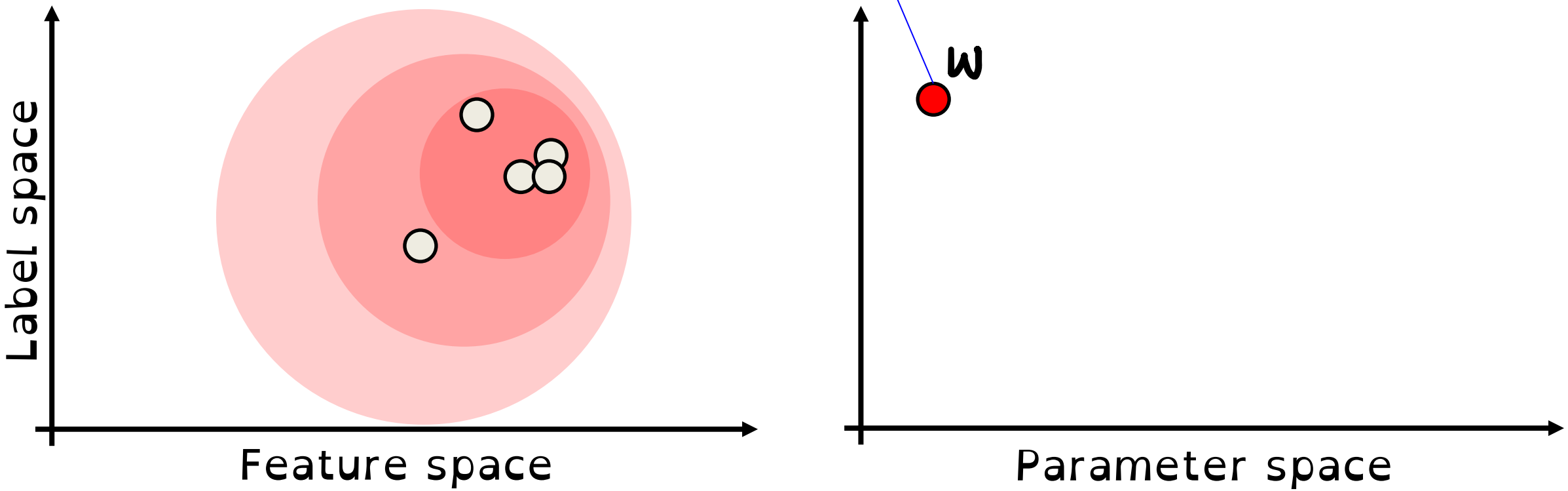
- MLE and MAP are connected by Bayes rule:

$$\underbrace{p(w \mid D)}_{\text{posterior}} = \frac{p(D \mid w) p(w)}{p(D)} \propto \underbrace{p(D \mid w)}_{\text{likelihood}} \underbrace{p(w)}_{\text{prior}}$$

- So MAP maximizes the likelihood p(D|w) times the prior p(w):
  - Prior is our "belief" that 'w' is correct before seeing data.
  - Prior can reflect that complex models are likely to overfit.

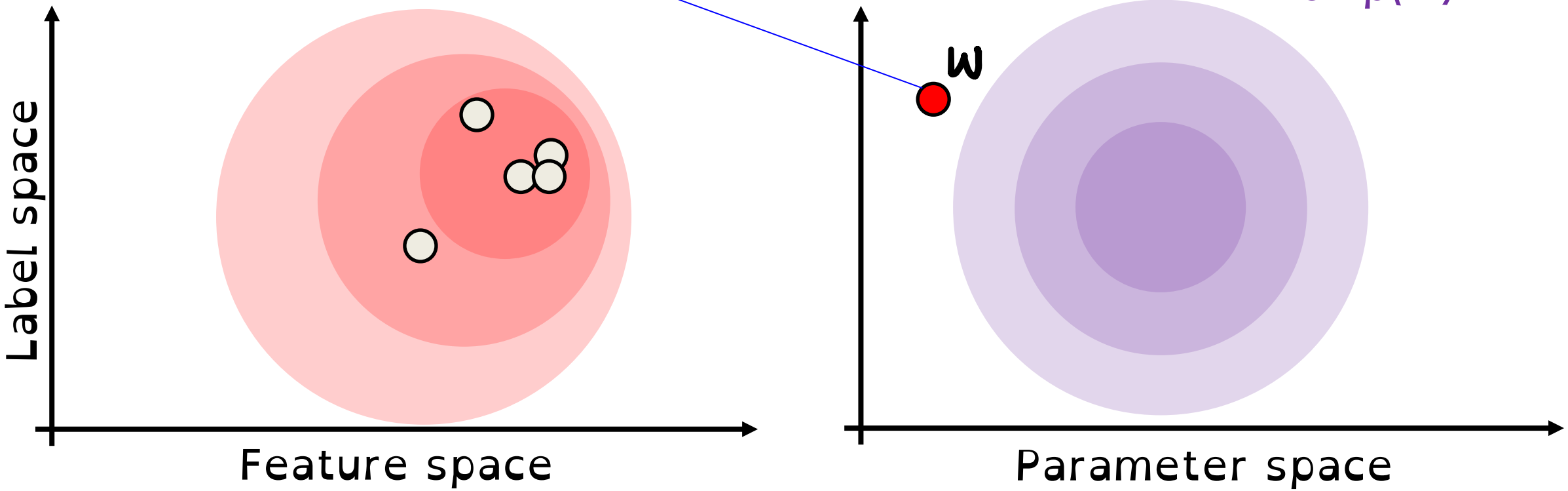
"Prior"

Q: What if this is overfitting?
How do we discourage w from going here?



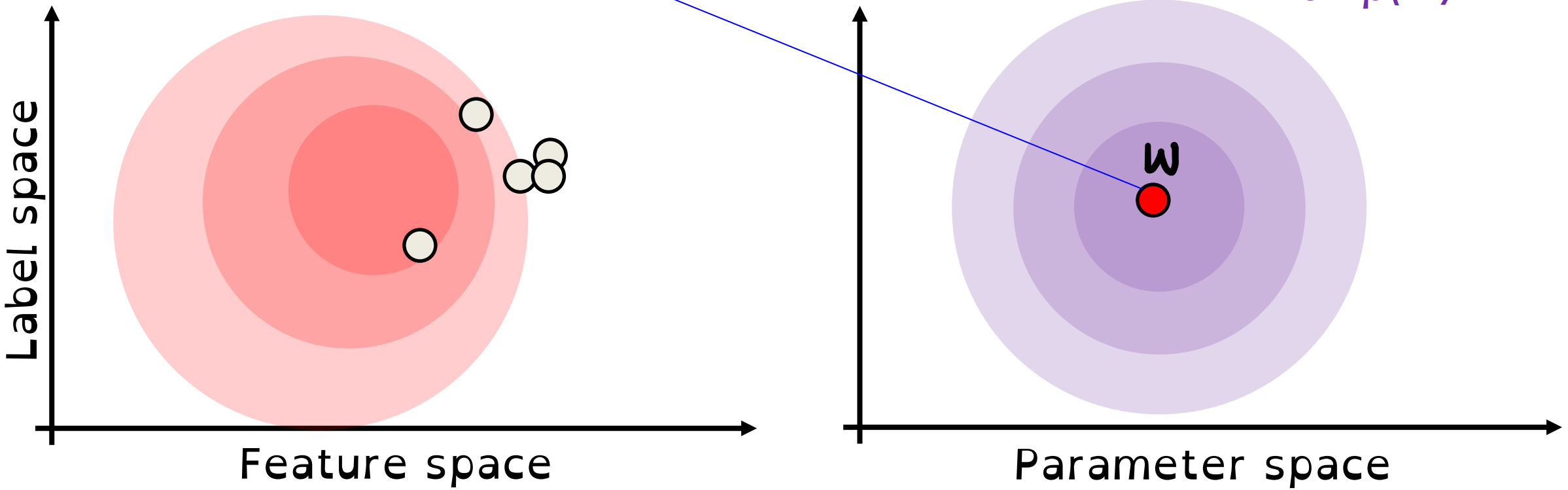
"Prior"

Q: Is this a good value according to the prior?



“Prior”

Q: Is this a good value according to the prior?



Q: Haven’t we seen a similar concept before?

MAP Estimation and Regularization

- From Bayes rule, the MAP estimate with IID examples D_i is:

$$\hat{w} \in \operatorname*{argmax}_w \{p(w|D)\} \equiv \operatorname*{argmax}_w \left\{\prod_{i=1}^{n}[p(D_i|w)]p(w)\right\}$$

- By again taking the negative of the logarithm as before we get:

$$\hat{w} \in \operatorname*{argmin}_w \left\{-\sum_{i=1}^{n}[\log(p(D_i|w))] - \log(p(w))\right\}$$

loss regularizer

- So we can view the negative log-prior as a regularizer:
  - Many regularizers are equivalent to negative log-priors.

L2-Regularization and MAP Estimation

- We obtain L2-regularization under an independent Gaussian assumption:

[1] Assume each w_j comes from a Gaussian with mean 0 and variance $1/\lambda$

- This implies that:

[2] $$p(w) = \prod_{j=1}^{d} p(w_j) \propto \prod_{j=1}^{d} \exp\left(-\frac{\lambda}{2} w_j^2\right) = \exp\left(-\frac{\lambda}{2} \sum_{j=1}^{d} w_j^2\right)$$

Gaussian assumption; independence; $e^{\alpha} e^{\beta} = e^{\alpha+\beta}$

- So we have that:

[3] $$-\log(p(w)) = -\log\left(\exp\left(-\frac{\lambda}{2}\|w\|^2\right)\right) + (\text{constant}) = \frac{\lambda}{2}\|w\|^2 + (\text{constant})$$

- With this prior, the MAP estimate with IID training examples would be

[4] $$\hat{w} \in \operatorname{argmin}\left\{-\log(p(y|X,w)) - \log(p(w))\right\} \equiv \operatorname{argmin}_w \left\{-\sum_{i=1}^{n}\left[\log(p(y_i|x_i,w))\right] + \frac{\lambda}{2}\|w\|^2\right\}$$

MAP Estimation and Regularization

- MAP estimation gives link between probabilities and loss functions.
  - Gaussian likelihood ($\sigma = 1$) + Gaussian prior gives L2-regularized least squares.

If $p(y_i \mid x_i, w) \propto \exp\left(-\frac{(w^T x_i - y_i)^2}{2}\right)$ $p(w_j) \propto \exp\left(-\frac{\lambda}{2} w_j^2\right)$

then MAP estimation is equivalent to minimizing $f(w) = \frac{1}{2}\|Xw - y\|^2 + \frac{\lambda}{2}\|w\|^2$

  - Laplace likelihood ($\sigma = 1$) + Gaussian prior give L2-regularized robust regression:

If $p(y_i \mid x_i, w) \propto \exp\left(-|w^T x_i - y_i|\right)$ $p(w) \propto \exp\left(-\frac{\lambda}{2} w_j^2\right)$

then MAP estimation is equivalent to minimizing $f(w) = \|Xw - y\|_1 + \frac{\lambda}{2}\|w\|^2$

  - As 'n' goes to infinity, effect of prior/regularizer goes to zero.
  - Unlike with MLE, the choice of σ changes the MAP solution for these models.

Summary of MAP Estimation

- Many of our loss functions and regularizers have probabilistic interpretations.
  - Laplace likelihood leads to absolute error.
  - Laplace prior leads to L1-regularization.

- The choice of likelihood corresponds to the choice of loss.
  - Our assumptions about how the y_i-values can come from the x_i and 'w'.

- The choice of prior corresponds to the choice of regularizer.
  - Our assumptions about which 'w' values are plausible.

Regularizing Other Models

- We can view priors in other models as regularizers.

- Remember the problem with MLE for naïve Bayes:
  - The MLE of p('lactase' = 1| 'spam') is: count(spam,lactase)/count(spam).
  - But this caused problems if count(spam,lactase) = 0.

- Our solution was Laplace smoothing:
  - Add "+1" to our estimates: (count(spam,lactase)+1)/(counts(spam)+2).
  - This corresponds to a "Beta" prior so Laplace smoothing is a regularizer.

Why do we care about MLE and MAP?

- Unified way of thinking about many of our tricks?
  - Probabilitic interpretation of logistic loss.
  - Laplace smoothing and L2-regularization are doing the same thing.

- Remember our two ways to reduce overfitting in complicated models:
  - Model averaging (ensemble methods).
  - Regularization (linear models).

- "Fully"-Bayesian methods (CPSC 440) combine both of these.
  - Average over all models, weighted by posterior (including regularizer).
  - Can use extremely-complicated models without overfitting.

Losses for Other Discrete Labels

- MLE/MAP gives loss for classification with basic labels:
  - Least squares and absolute loss for regression.
  - Logistic regression for binary labels {“spam”, “not spam”}.
  - Softmax regression for multi-class {“spam”, “not spam”, “important”}.

- But MLE/MAP lead to losses with other discrete labels (bonus):
  - Ordinal: {1 star, 2 stars, 3 stars, 4 stars, 5 stars}.
  - Counts: 602 ‘likes’.
  - Survival rate: 60% of patients were still alive after 3 years.
  - Unbalanced classes: 99.9% of examples are classified as +1.
- Define likelihood of labels, and use NLL as the loss function.

- We can also use ratios of probabilities to define more losses (bonus):
  - Binary SVMs, multi-class SVMs, and “pairwise preferences” (ranking) models.

End of Part 3: Linear Models

End of Part 3: Key Concepts

- Linear models predict based on linear combination(s) of features:

$$w^T x_i = w_1 x_{i1} + w_2 x_{i2} + \cdots + w_d x_{id}$$

- We model non-linear effects using a change of basis:
  - Replace d-dimensional x_i with k-dimensional z_i and use $v^T z_i$.
  - Examples include polynomial basis and (non-parametric) RBFs.

- Regression is supervised learning with continuous labels.
  - Logical error measure for regression is squared error:

  - Can be solved as a system of linear equations.

$$f(w) = \frac{1}{2}\|Xw - y\|^2$$

End of Part 3: Key Concepts

- Gradient descent finds local minimum of smooth objectives.
  - Converges to a global optimum for convex functions.
  - Can use smooth approximations (Huber, log-sum-exp)

- Stochastic gradient methods allow huge/infinite 'n'.
  - Though very sensitive to the step-size.

- Kernels let us use similarity between examples, instead of features.
  - Lets us use some exponential- or infinite-dimensional features.

- Feature selection is a messy topic.
  - Classic method is forward selection based on L0-norm.
  - L1-regularization simultaneously regularizes and selects features.

End of Part 3: Key Concepts

- We can reduce over-fitting by using regularization:

$$f(w) = \frac{1}{2}\|Xw - y\|^2 + \frac{\lambda}{2}\|w\|^2$$

- Squared error is not always right measure:
  - Absolute error is less sensitive to outliers.
  - Logistic loss and hinge loss are better for binary y_i.
  - Softmax loss is better for multi-class y_i.
- MLE/MAP perspective:
  - We can view loss as log-likelihood and regularizer as log-prior.
  - Allows us to define losses based on probabilities.

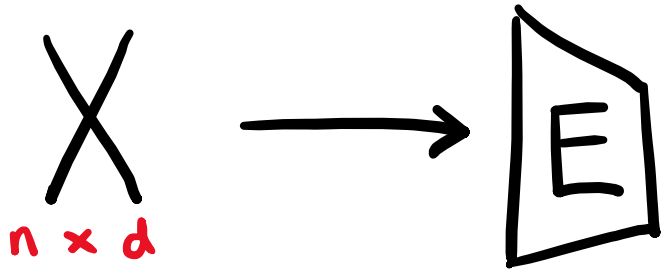
The Story So Far...

- Part 1: Supervised Learning.
  - Methods based on counting and distances.
- Part 2: Unsupervised Learning.
  - Methods based on counting and distances.
- Part 3: Supervised Learning (just finished).
  - Methods based on linear models and gradient descent.
- Part 4: Unsupervised Learning (starting now).
  - Methods based on linear models and gradient descent.

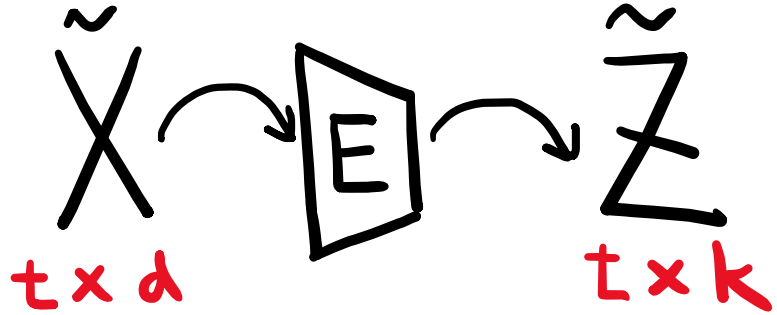
Part 4: Latent Factor Models

The "Encoder Learning Problem"

- The Encoder Learning Problem
  - Input: Feature matrix 'X'
  - Output: An "encoder" model that can transform examples



- The Encoding Problem
  - Input: A test example $\tilde{x}_i$ and a learned encoder model
  - Output: The "encoded" example $\tilde{z}_i$



Motivation: Human vs. Machine Perception

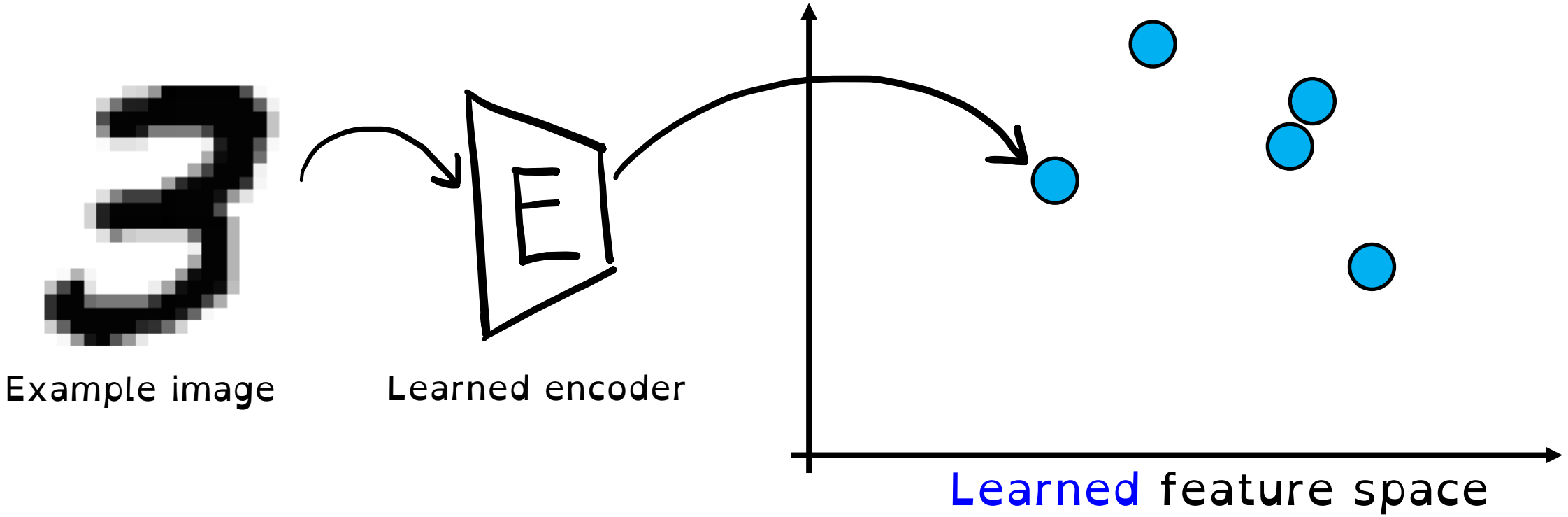
- Huge difference between what we see and what computer sees:

What we see:

What the computer "sees":

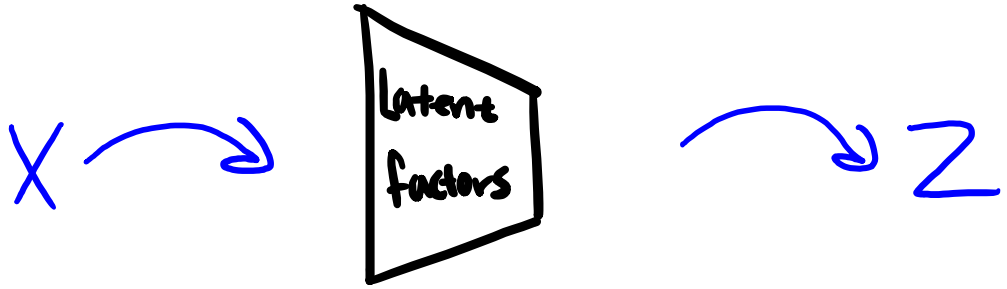
- But maybe images shouldn't be written as combinations of pixels.
  - Can we learn a better representation?
  - In other words, can we learn good features?

Encoding Images



- This is like feature engineering!
  - Find a better feature space for analyzing the data
- But now, we're learning the features!

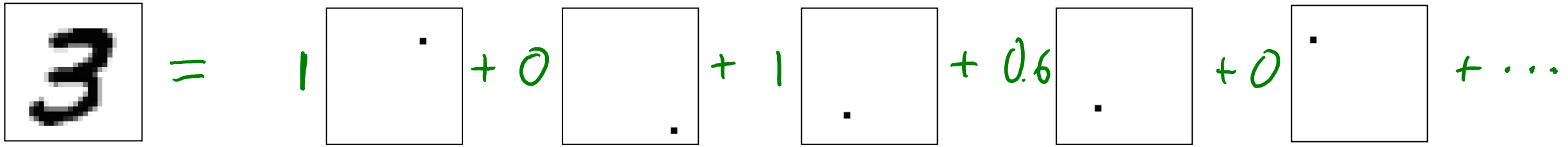
Part 4: Latent-Factor Models



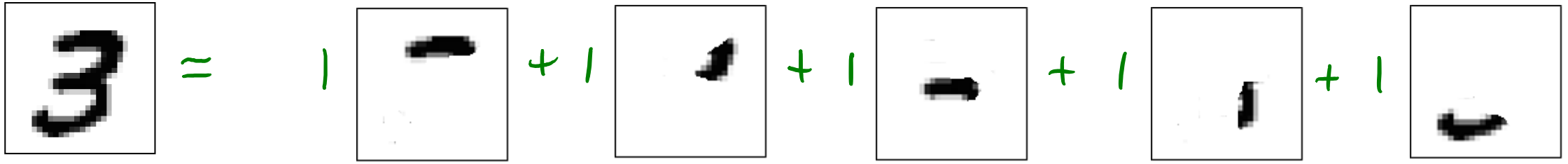
- Part 4 is about learning the encoder from data.
- Our encoders are linear models that use "latent factors"

Motivation: Pixels vs. Parts

- Can view 28x28 image as weighted sum of "single pixel on" images:



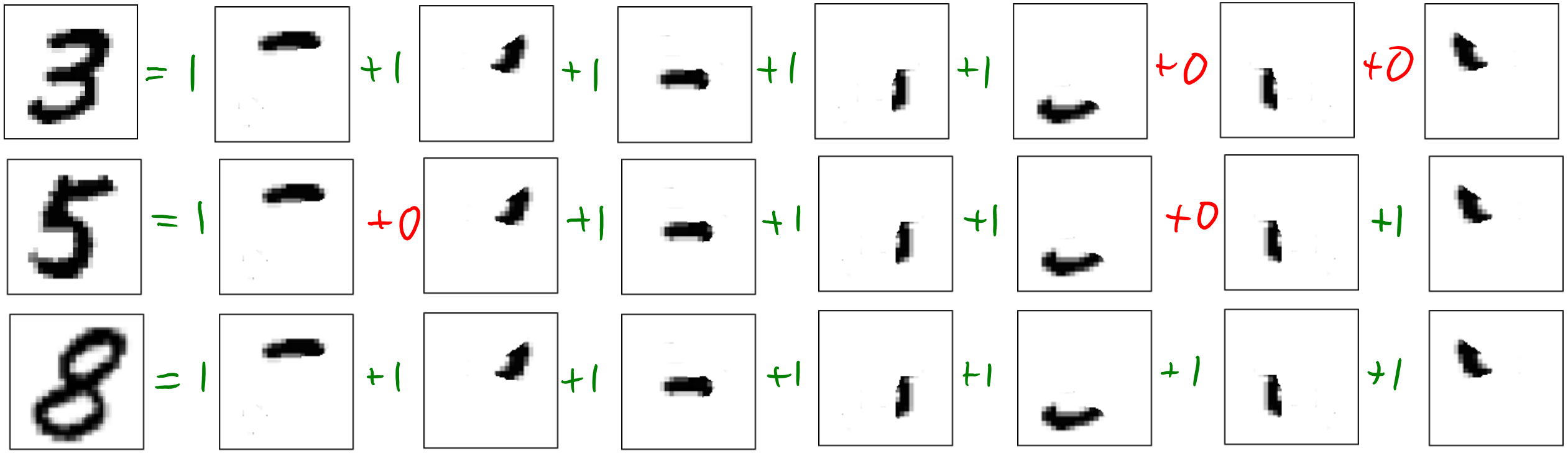
  - We have one image/feature for each pixel.
  - The weights specify "how much of this pixel is in the image".
    - A weight of zero means that pixel is white, a weight of 1 means it's black.
- This is non-intuitive, isn't a "3" made of small number of "parts"?



  - Now the weights are "how much of this part is in the image".

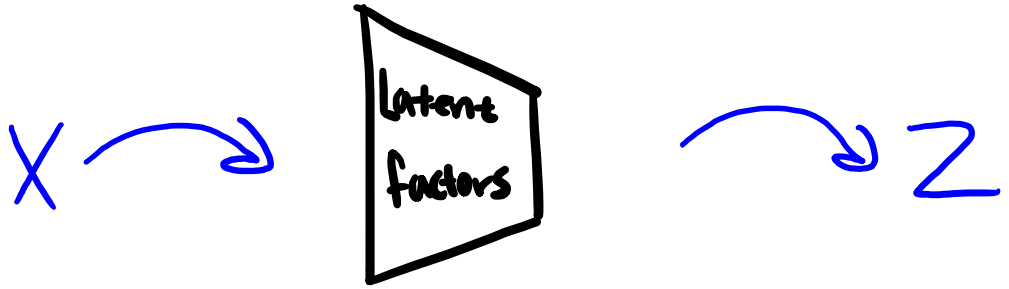
Motivation: Pixels vs. Parts

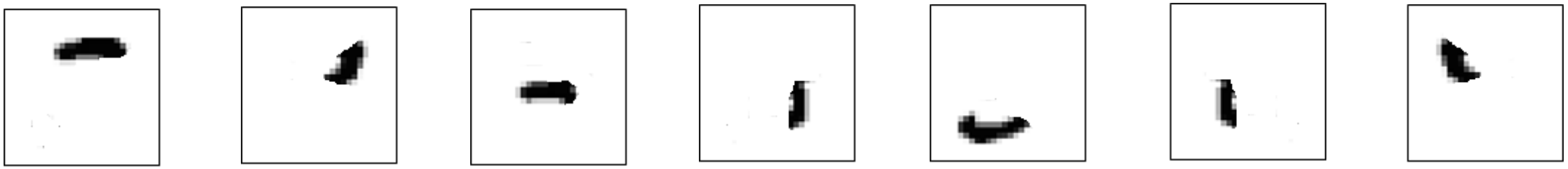
- We could represent other digits as different combinations of "parts":



- Consider replacing images x_i by the weights z_i of the different parts:
  - The 784-dimensional x_i for the "5" image is replaced by 7 numbers: z_i = [1 0 1 1 1 0 1].
  - Features like this could make learning much easier.

Why Latent Factors?





- These “parts” are called “factors”
- Factors are learned from data

Latent Factor Models are Useful

- Supervised learning:
  - we could use learned features as input features.
- Outlier detection:
  - example might be an outlier if isn't a combination of usual parts.
- Dimension reduction:
  - compress data into limited number of "part weights".
- Visualization:
  - if we have only 2 "part weights", we can view data as a scatterplot.
- Interpretation:
  - we can try and figure out what the "parts" represent.

Coming Up Next

PRINCIPAL COMPONENT ANALYSIS INTRO

Principal Component Analysis (PCA) Applications

- Principal component analysis (PCA) has been invented many times:

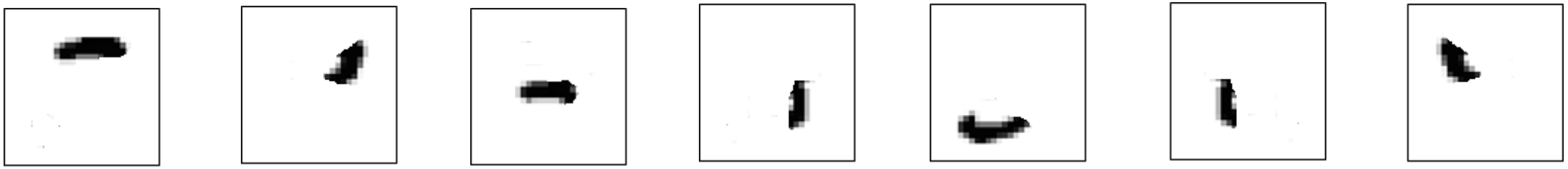
PCA was invented in 1901 by Karl Pearson,[1] as an analogue of the principal axis theorem in mechanics; it was later independently developed (and named) by Harold Hotelling in the 1930s.[2] Depending on the field of application, it is also named the discrete Kosambi-Karhunen–Loève transform (KLT) in signal processing, the Hotelling transform in multivariate quality control, proper orthogonal decomposition (POD) in mechanical engineering, singular value decomposition (SVD) of **X** (Golub and Van Loan, 1983), eigenvalue decomposition (EVD) of $\mathbf{X}^T\mathbf{X}$ in linear algebra, factor analysis (for a discussion of the differences between PCA and factor analysis see Ch. 7 of [3]), Eckart–Young theorem (Harman, 1960), or Schmidt–Mirsky theorem in psychometrics, empirical orthogonal functions (EOF) in meteorological science, empirical eigenfunction decomposition (Sirovich, 1987), empirical component analysis (Lorenz, 1956), quasiharmonic modes (Brooks et al., 1988), spectral decomposition in noise and vibration, and empirical modal analysis in structural dynamics.

standard deviation of 3 in roughly the
(0.878, 0.478) direction and of 1 in th
orthogonal direction. The vectors
shown are the eigenvectors of the
covariance matrix scaled by the squa
root of the corresponding eigenvalue,
and shifted so their tails are at the
mean.

https://en.wikipedia.org/wiki/Principal_component_analysis

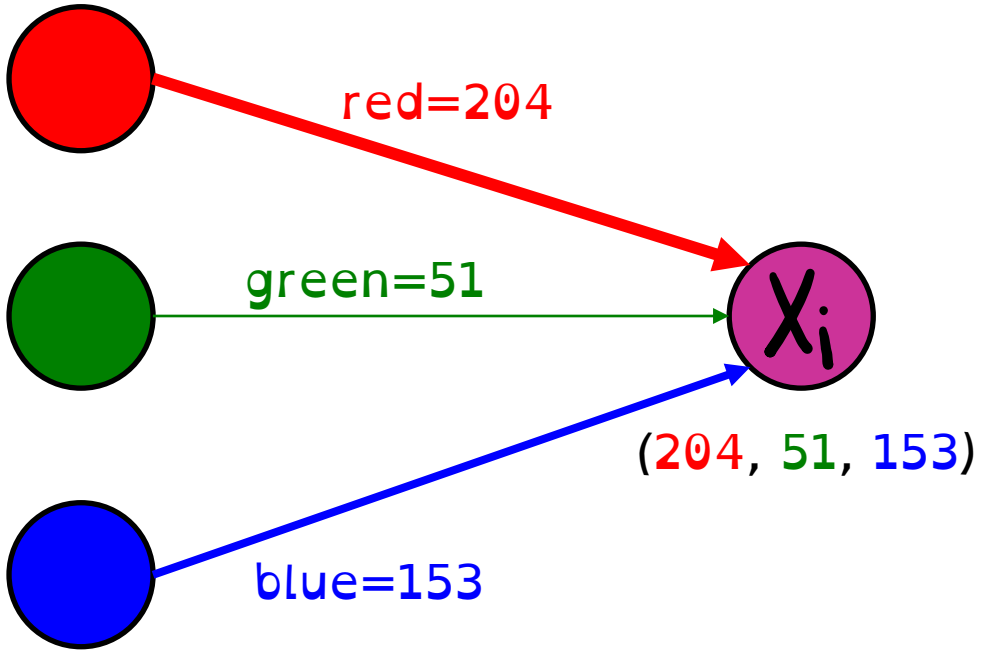
What is PCA?

- "PCA" := Principal Component Analysis
- An instance of latent factors model
  - Learn the factors from data → principal components (PCs)
  - Use factors to transform data



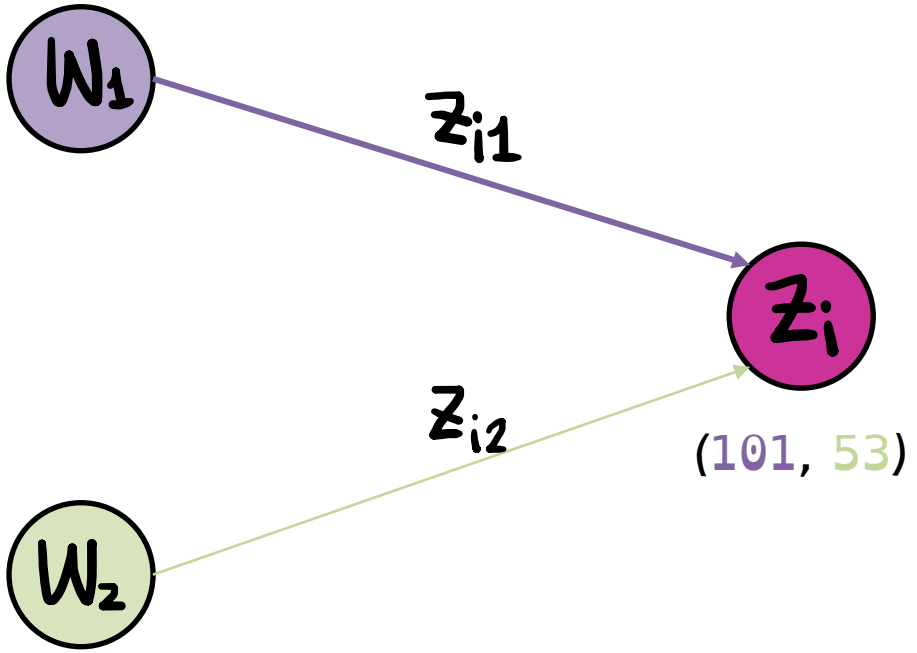
- Assumption: an example in feature space is a linear combination of factors

Visualizing Factors



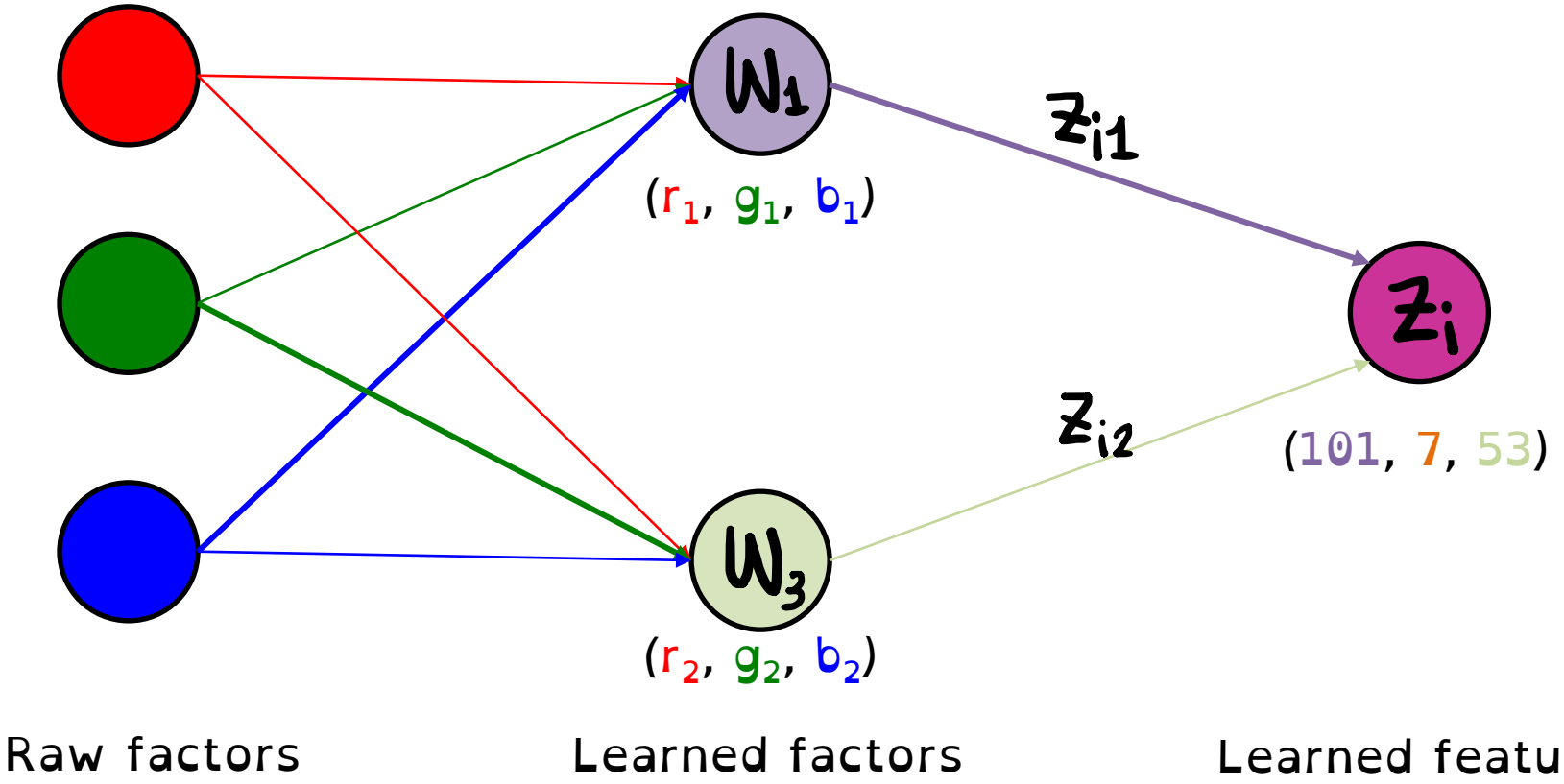
- (204, 51, 153) is the RGB value of this colour
- Given these RGB factors,
  the features are “factor-ness” scores

Visualizing Factors

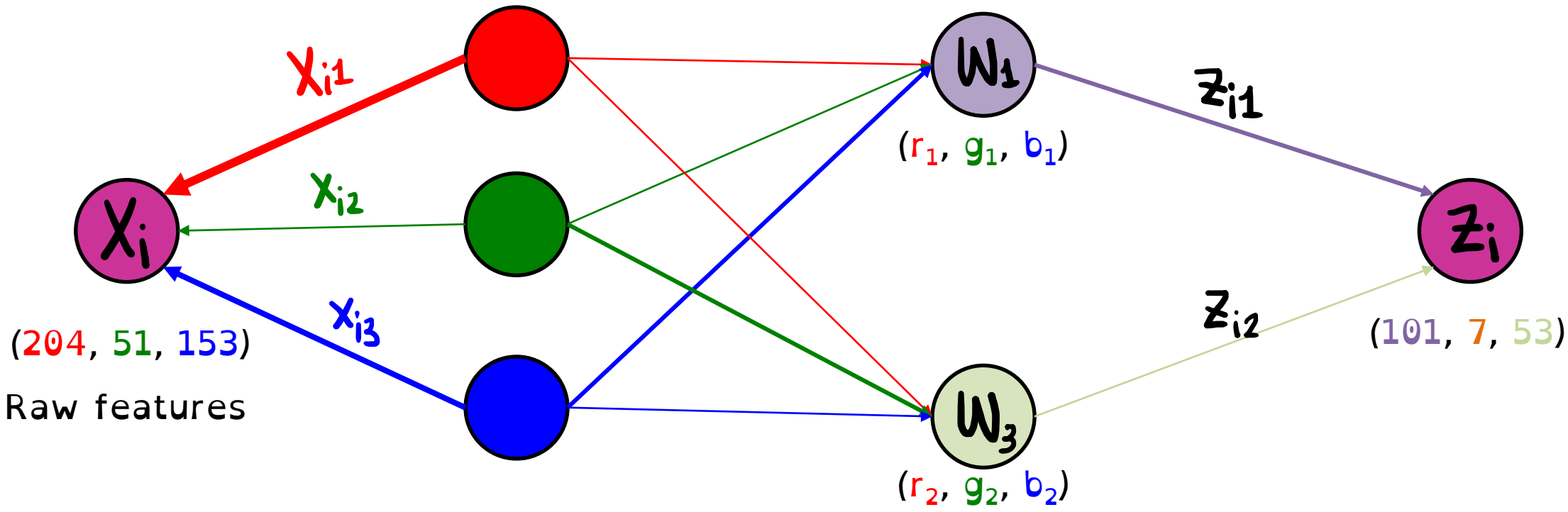


- (101, 53) is NOT the RGB value of this colour
- But it represents the influence of these colours!
- Idea: use (101, 53) as features
  - Given factors w_1, w_2, the edge weights are "factor-ness" scores

Visualizing Factors



Visualizing Factors



Facts:

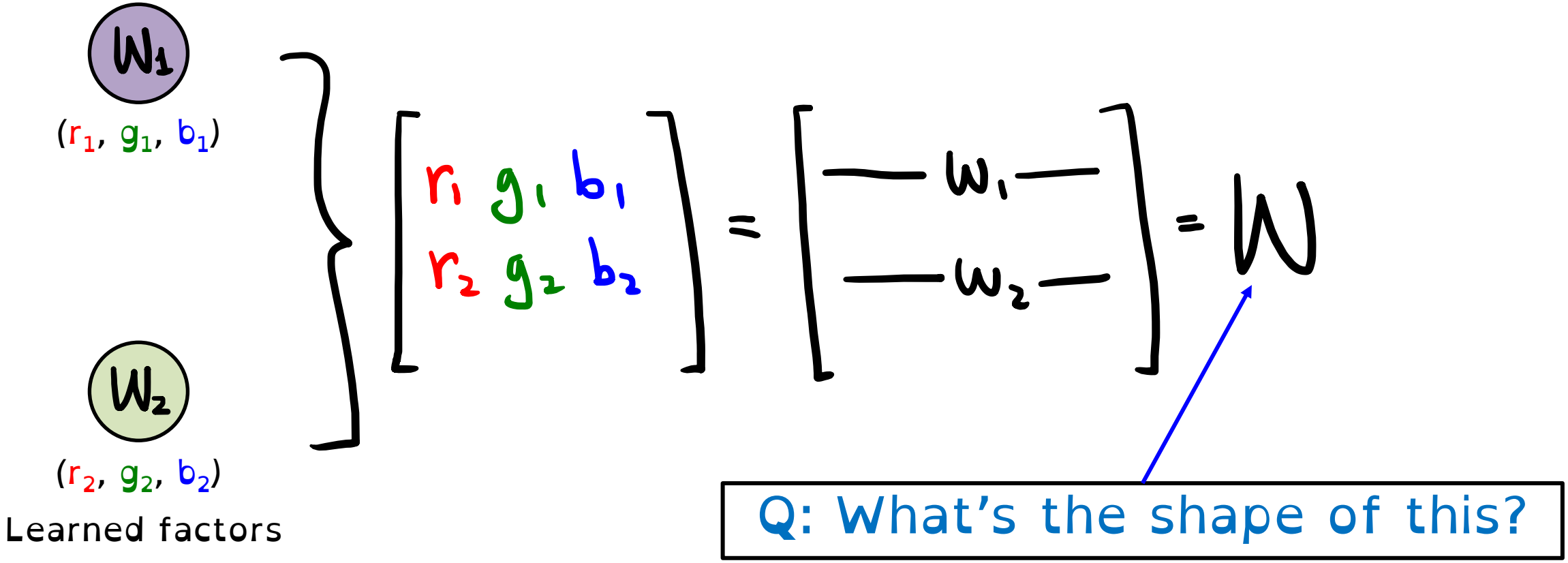
① $X_i \neq Z_i$

②

$$X_{i1} \approx z_{i1} r_1 + z_{i2} r_2$$
$$X_{i2} \approx z_{i1} g_1 + z_{i2} g_2$$
$$X_{i3} \approx z_{i1} b_1 + z_{i2} b_2$$

$$\rightarrow \begin{bmatrix} X_{i1} \\ X_{i2} \\ X_{i3} \end{bmatrix} \approx \begin{bmatrix} r_1 & r_2 \\ g_1 & g_2 \\ b_1 & b_2 \end{bmatrix} \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix}$$

Visualizing Factors



PCA Notation (MEMORIZE)

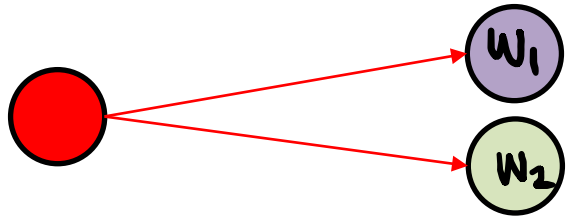
- PCA takes in a matrix 'X' and an input 'k', and outputs two matrices:

$$Z = \begin{bmatrix} - & z_1^T & - \\ - & z_2^T & - \\ & \vdots & \\ - & z_n^T & - \end{bmatrix} \begin{matrix} \} n \\ \\ \\ \\ \end{matrix} \qquad W = \begin{bmatrix} | & | & & | \\ w^1 & w^2 & \cdots & w^d \\ | & | & & | \end{bmatrix} \begin{matrix} \} k \\ \\ \\ \\ \end{matrix}$$

$$= \begin{bmatrix} - & w_1^T & - \\ - & w_2^T & - \\ & \vdots & \\ - & w_k^T & - \end{bmatrix} \begin{matrix} \} k \\ \\ \\ \\ \end{matrix}$$

k under Z; d under W; d under the last matrix

- Row c of W → w_c.
  - d-by-1 vector, called a "factor" or "principal component".
- Row i of Z → z_i.
  - k-by-1 vector, called "factor loadings" (or "features").
  - We have k factors, so this corresponds to k different "factor-ness" values
- Column j of W → w^j.
  - k-by-1 vector, index j of all the k "factors"
  - e.g. redness of w_1 and w_2



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Z: $n \times k$ (rows $z_1^T, z_2^T, \ldots, z_n^T$); W: $k \times d$ (rows $w_1^T, w_2^T, \ldots, w_k^T$); W: $k \times d$ (columns $w^1, w^2, \ldots, w^d$)

- With this notation, we can write our approximation of one x_{ij} as:

$$\hat{x}_{ij} = z_{i1} w_{1j} + z_{i2} w_{2j} + \cdots + z_{ik} w_{kj} = \sum_{c=1}^{k} z_{ic} w_{cj} = (w^j)^T z_i = \langle w^j, z_i \rangle$$

(NEW NOTATION)

- We can write approximation of the vector x_i as:

$$\hat{x}_i = \begin{bmatrix} \langle w^1, z_i \rangle \\ \langle w^2, z_i \rangle \\ \vdots \\ \langle w^d, z_i \rangle \end{bmatrix} = W^T z_i$$

$\hat{x}_i$: $d \times 1$; $W^T z_i$: $d \times k$ $k \times 1$

Different views (MEMORIZE)

- PCA approximates each x_{ij} by the inner product $< w^j, z_i >$.
- PCA approximates each x_i by the matrix-vector product $W^T z_i$.
- PCA approximates matrix 'X' by the matrix-matrix product ZW.

$$\underset{n \times d}{X} \approx \underset{n \times k}{Z} \underset{k \times d}{W}$$

  - PCA is also called a "matrix factorization" model.
  - Both 'Z' and 'W' are variables.

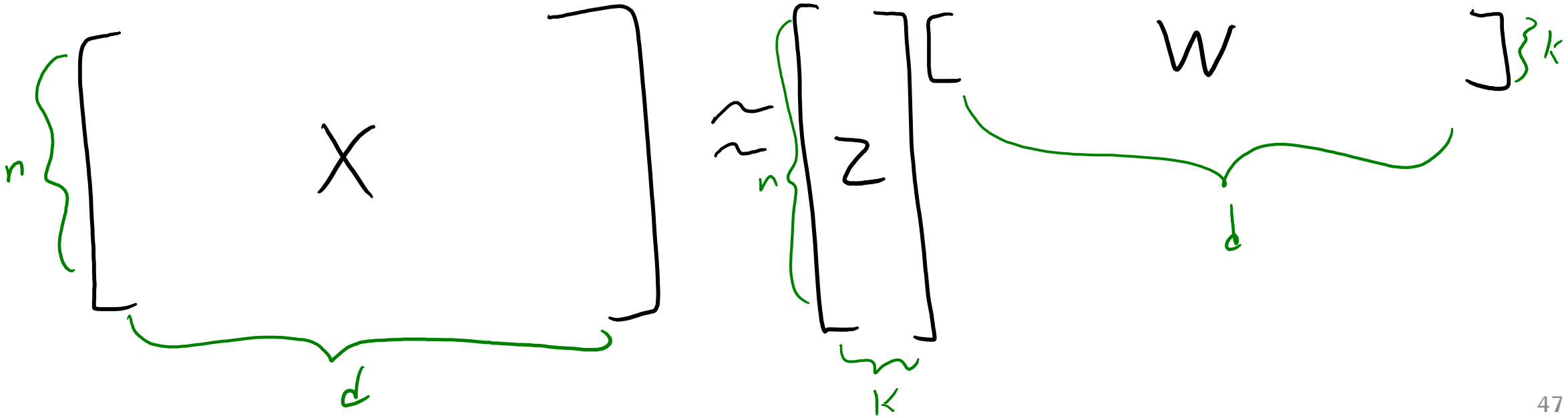
- This can be viewed as a "change of basis" from x_i to z_i values.
  - The "basis vectors" are the rows of W, the w_c.
  - The "coordinates" in the new basis of each x_i are the z_i.

Coming Up Next

APPLICATIONS OF PCA

PCA Applications

- Applications of PCA:
  - Dimensionality reduction: replace 'X' with lower-dimensional 'Z'.
    - If k << d, then compresses data.
    - Often better approximation than vector quantization.



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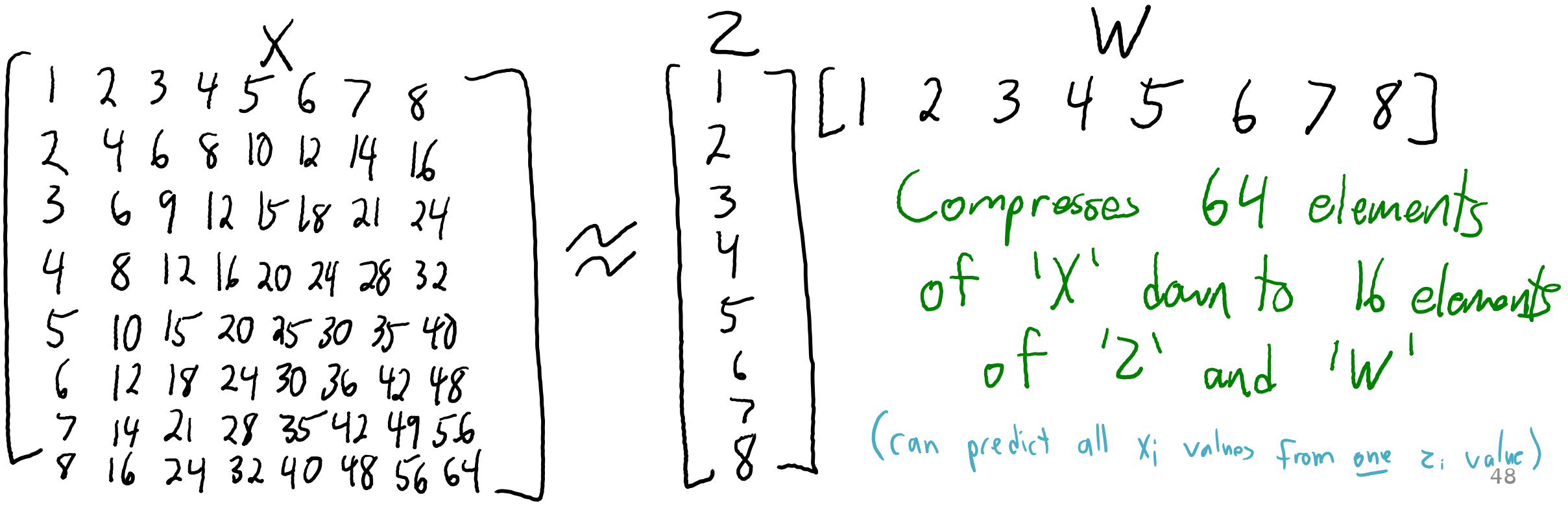

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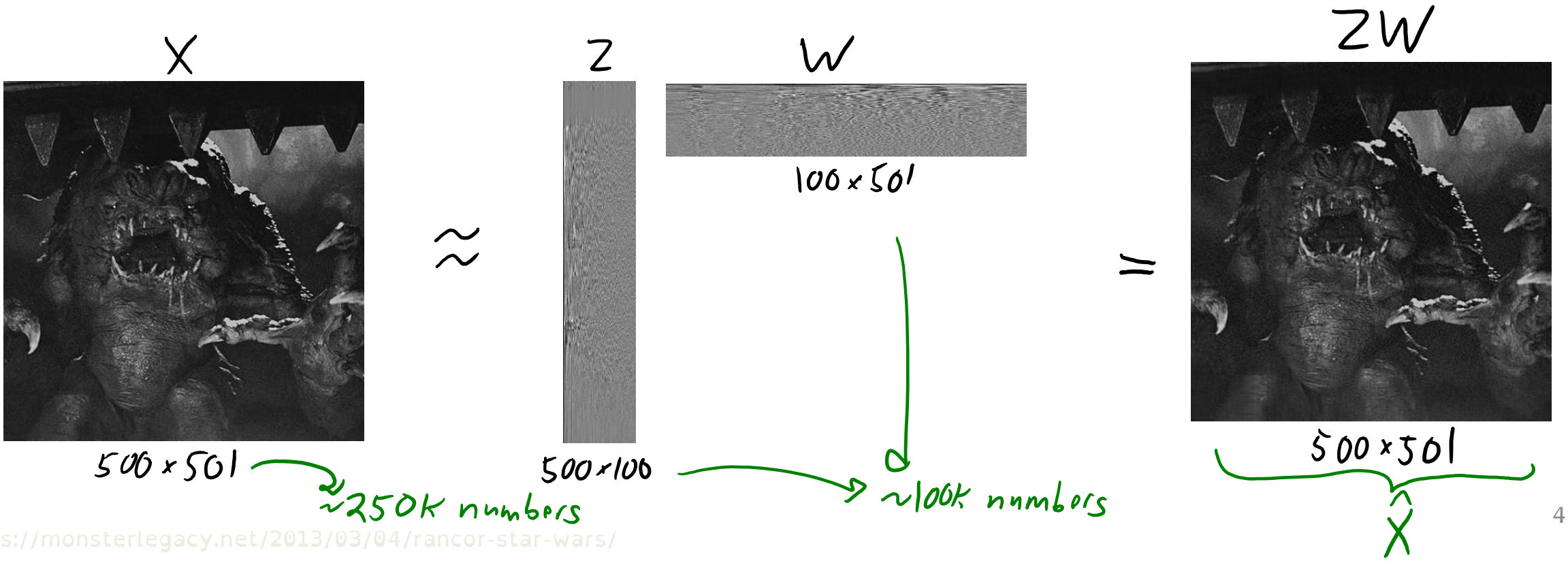

https://monsterlegacy.net/2013/03/04/rancor-star-wars/

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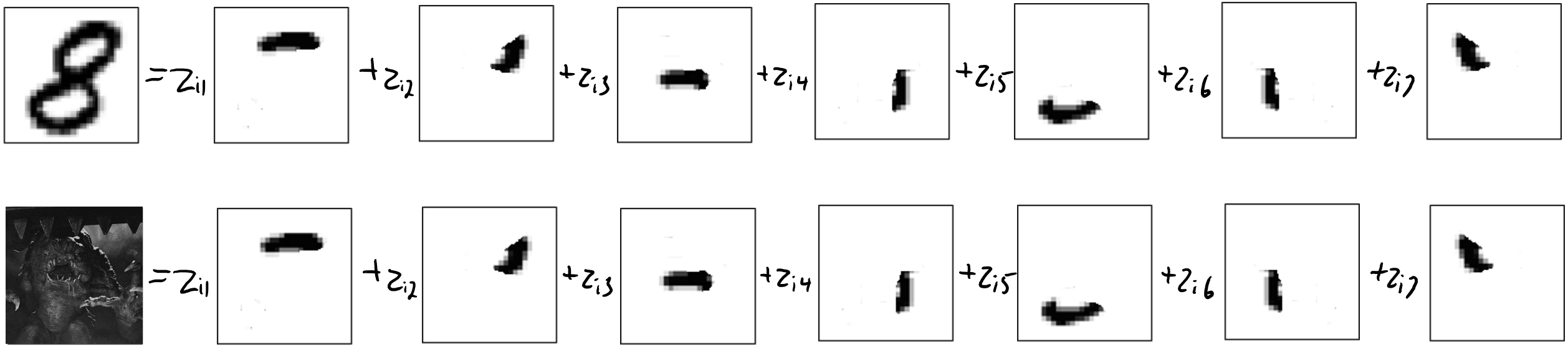
PCA Applications

- Applications of PCA:
  - Outlier detection: if PCA gives poor approximation of x_i, could be 'outlier'.
    - Though due to squared error PCA is sensitive to outliers.



PCA Applications

- Applications of PCA:
  - Partial least squares: uses PCA features as basis for linear model.

Compute approximation $X \approx ZW$

Now use Z as features in a linear model:

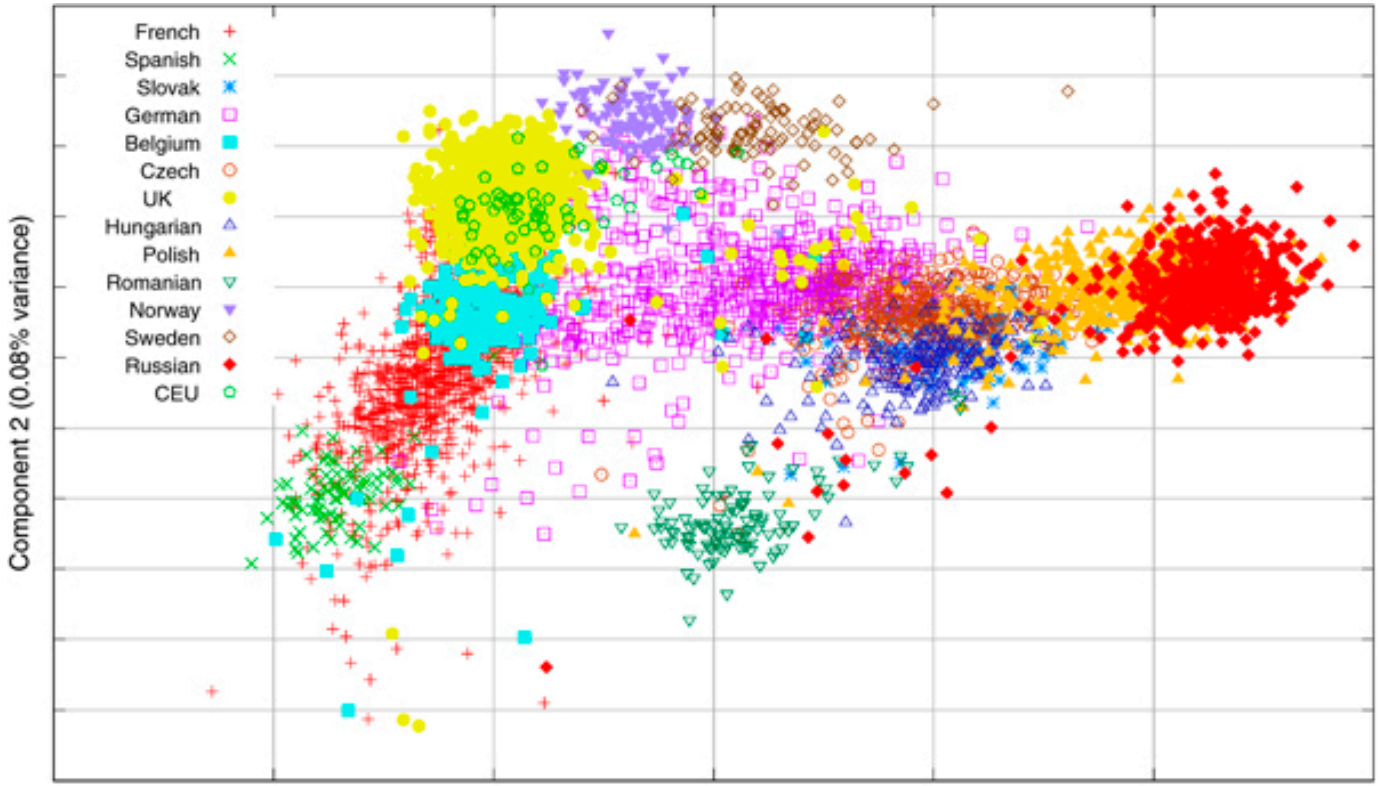
$$y_i = v^T z_i$$

linear regression weights 'v' trained under this change of basis.

lower-dimensional than original features so less overfitting

PCA Applications

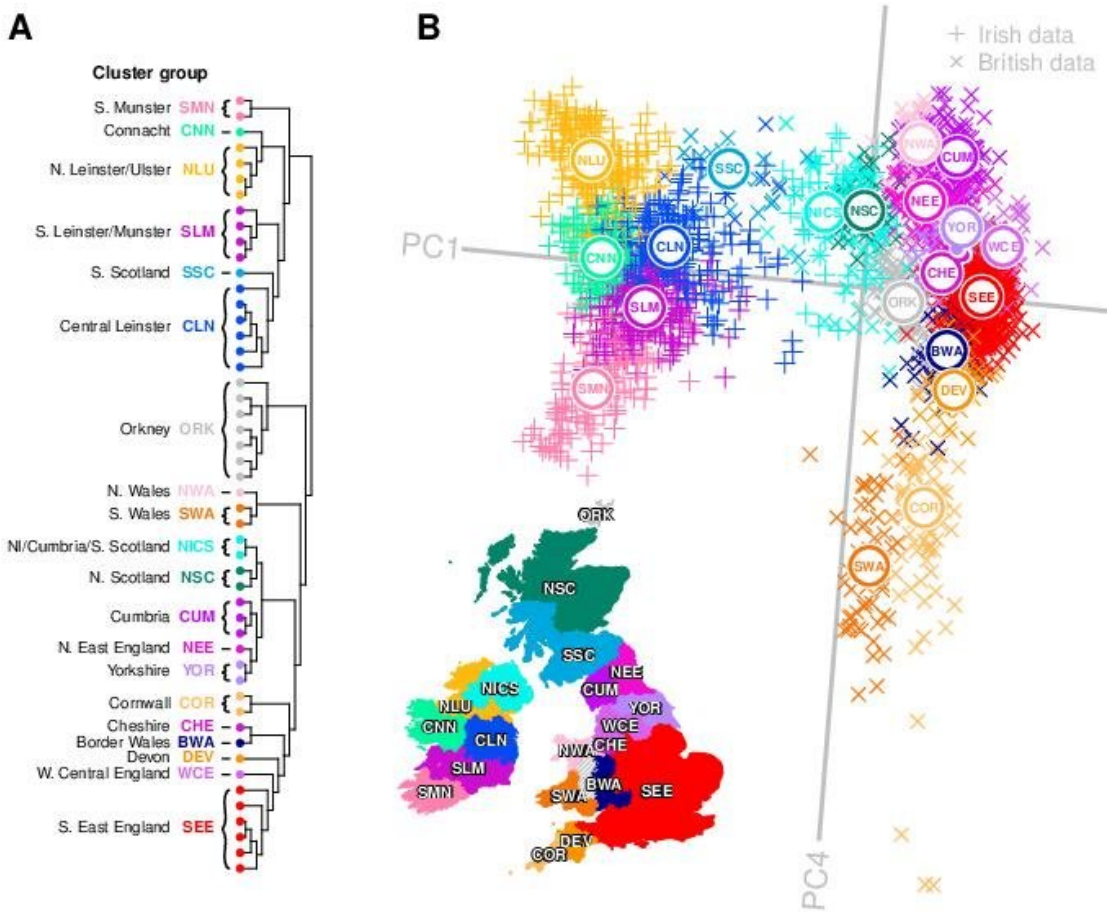
- Applications of PCA:
  - Data visualization: plot z_i with k = 2 to visualize high-dimensional objects.



http://infoproc.blogspot.ca/2008/11/european-genetic-substructure.html

PCA Applications

- Applications of PCA:
  - Data visualization: plot z_i with k = 2 to visualize high-dimensional objects.
    - Can augment other visualizations:



https://www.sciencedaily.com/releases/2018/01/180125140943.htm

PCA Applications

- Applications of PCA:
  - Data interpretation: we can try to assign meaning to latent factors w_c.
    - Hidden "factors" that influence all the variables.

Trait	Description
Openness	Being curious, original, intellectual, creative, and open to new ideas.
Conscientiousness	Being organized, systematic, punctual, achievement-oriented, and dependable.
Extraversion	Being outgoing, talkative, sociable, and enjoying social situations.
Agreeableness	Being affable, tolerant, sensitive, trusting, kind, and warm.
Neuroticism	Being anxious, irritable, temperamental, and moody.

"Most Personality Quizzes Are Junk Science. I Found One That Isn't."

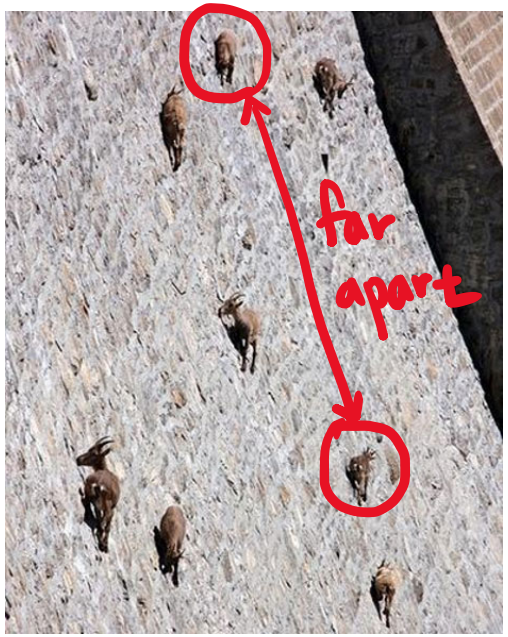
https://new.edu/resources/big-5-personality-traits

Coming Up Next

GEOMETRIC INTUITION FOR PCA

Overhead Map and Latent-Factor Models

- Consider these crazy goats trying to get some salt:
  - Ignoring height gives poor approximation of goat location.



Top-down view of a dam

- But the "goat space" is basically a two-dimensional plane.
  - Better k=2 approximation: define 'W' so that combinations give the plane.

www.momtastic.com/webecoist/2010/11/07/some-fine-dam-climbing-goats-scaling-steep-vertical-wall
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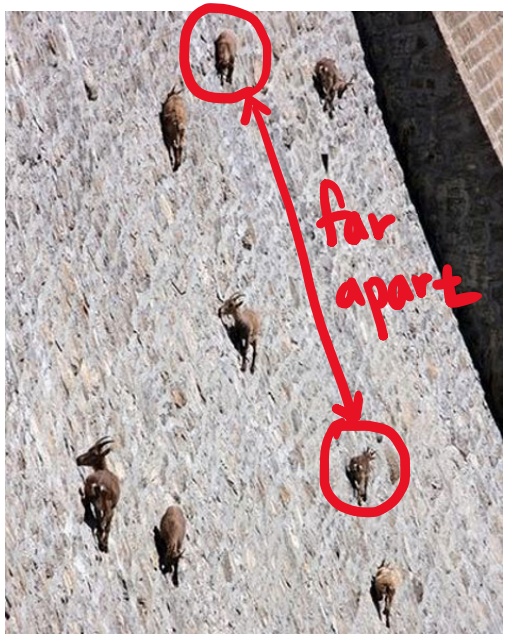

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Overhead Map and Latent-Factor Models

- A goat i's location in the world can be described by 3 coordinates:

$$X_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{bmatrix} \begin{matrix} \leftarrow \text{"x" coordinate} \\ \leftarrow \text{"y" coordinate} \\ \leftarrow \text{"z" coordinate} \end{matrix}$$

- The overhead view approximates these 3 coordinates with only 2:

$$z_i = \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} \begin{matrix} \leftarrow \text{"x" coordinate} \\ \leftarrow \text{"y" coordinate} \end{matrix}$$

- Our k=2 latent factors are the following:

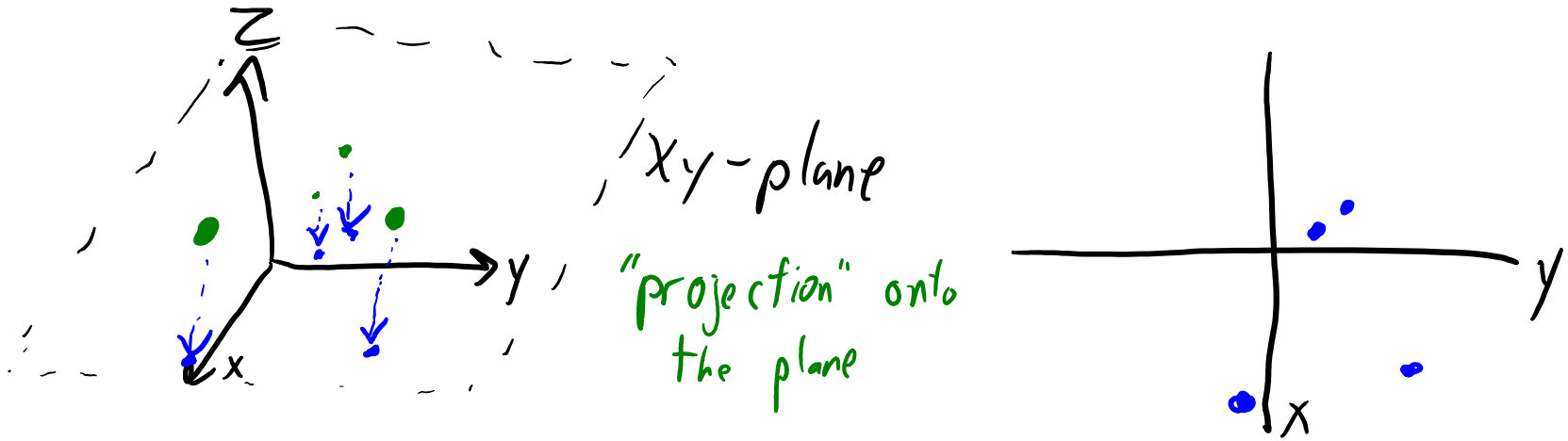
$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- So our approximation of x_i is:

$$\hat{x}_i = z_{i1} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + z_{i2} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

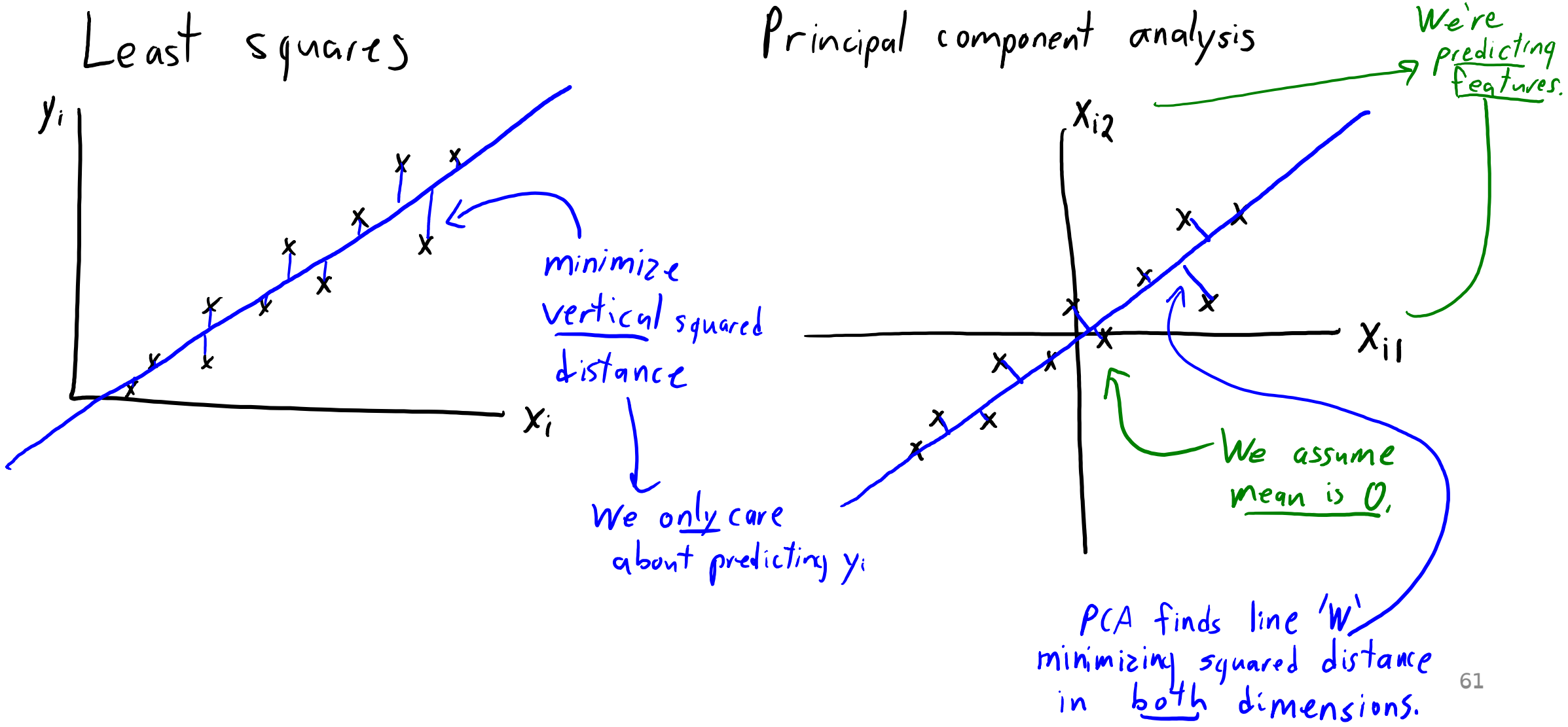
Overhead Map and Latent-Factor Models

- The "overhead map" approximation just ignores the "height".



  - This is a good approximation if the world is flat.
  - But it's a poor approximation if heights are different.

PCA with d=2 and k =1



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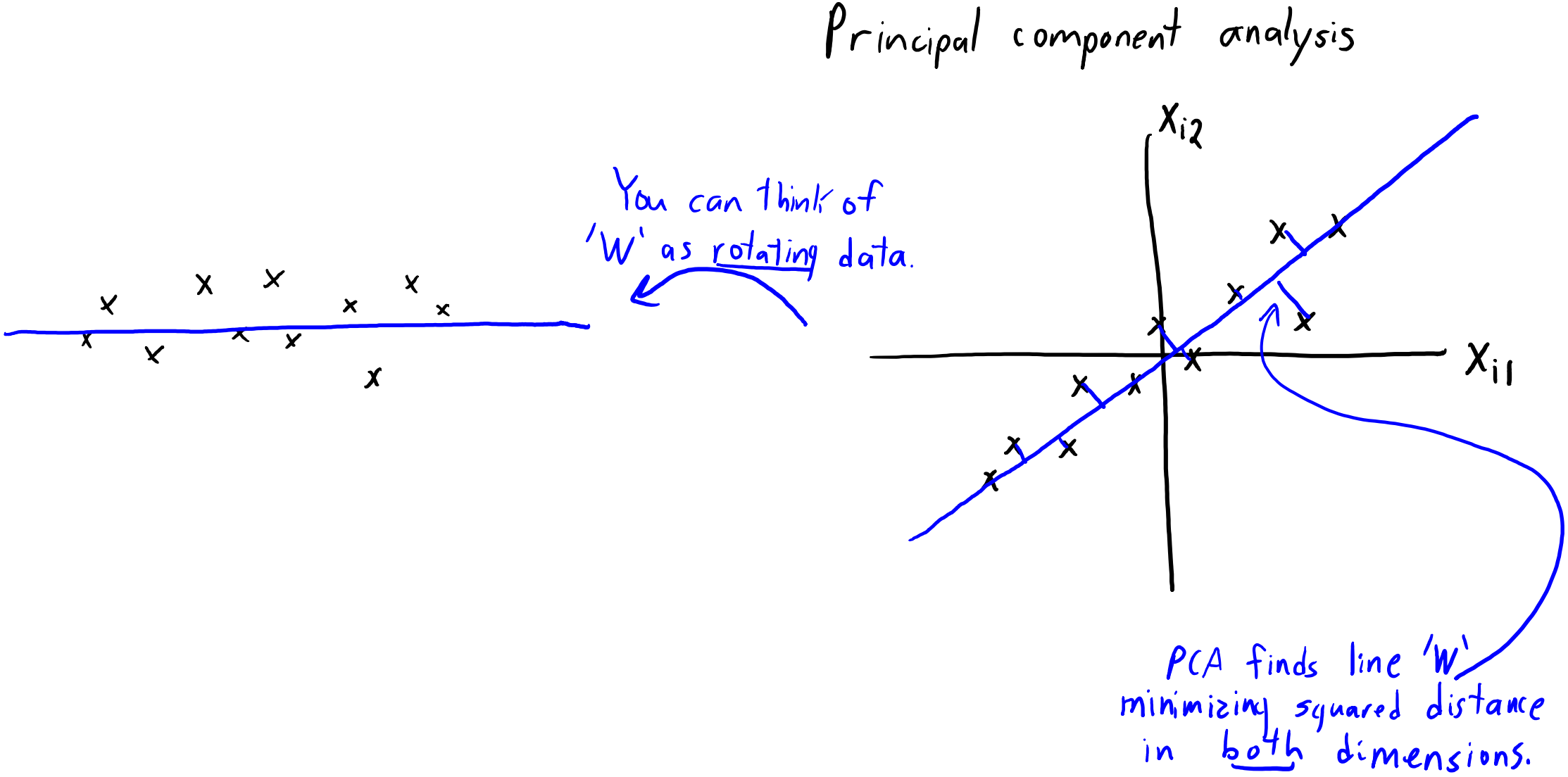
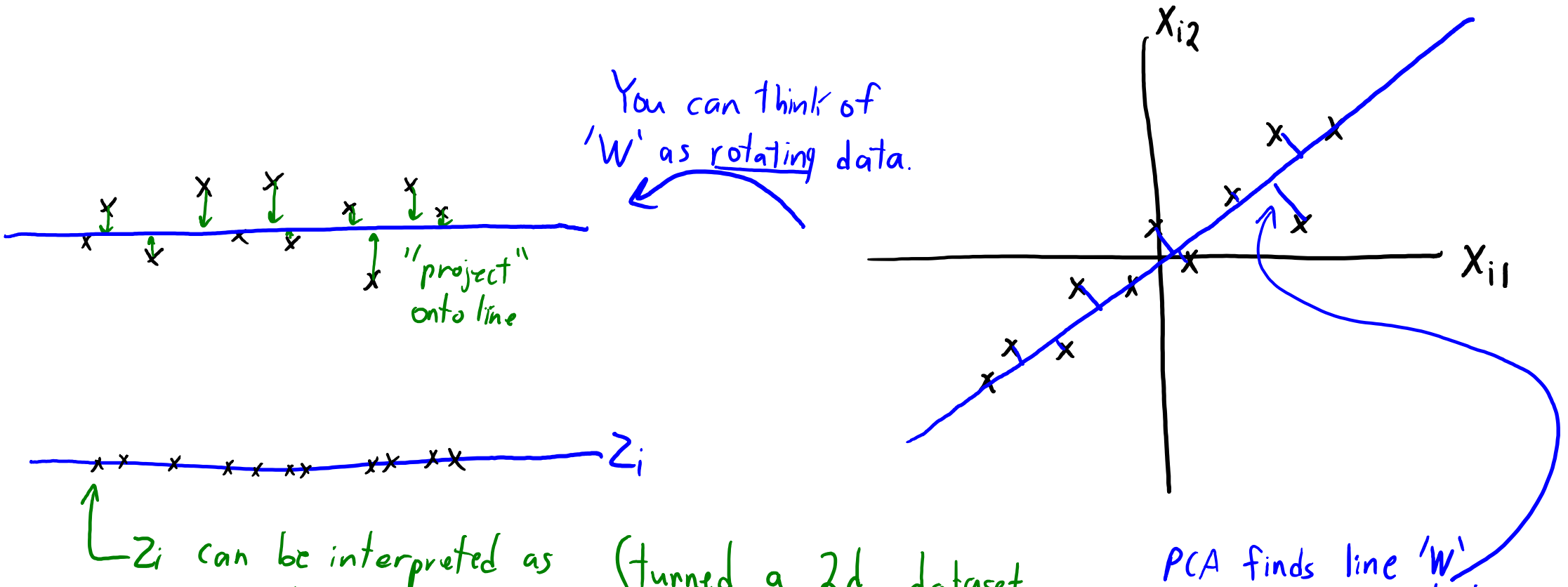


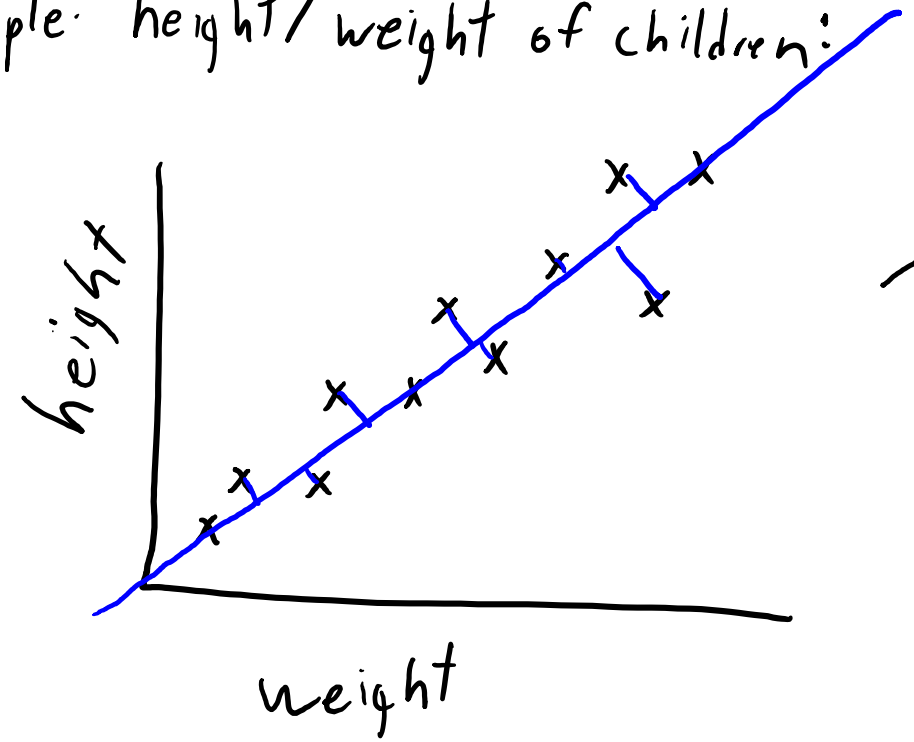
PCA with d=2 and k =1

Principal component analysis

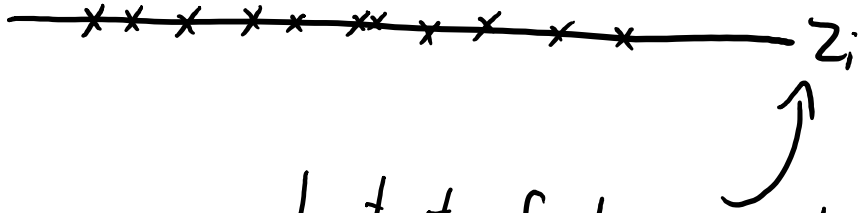


PCA with d=2 and k =1

Example: height/weight of children:



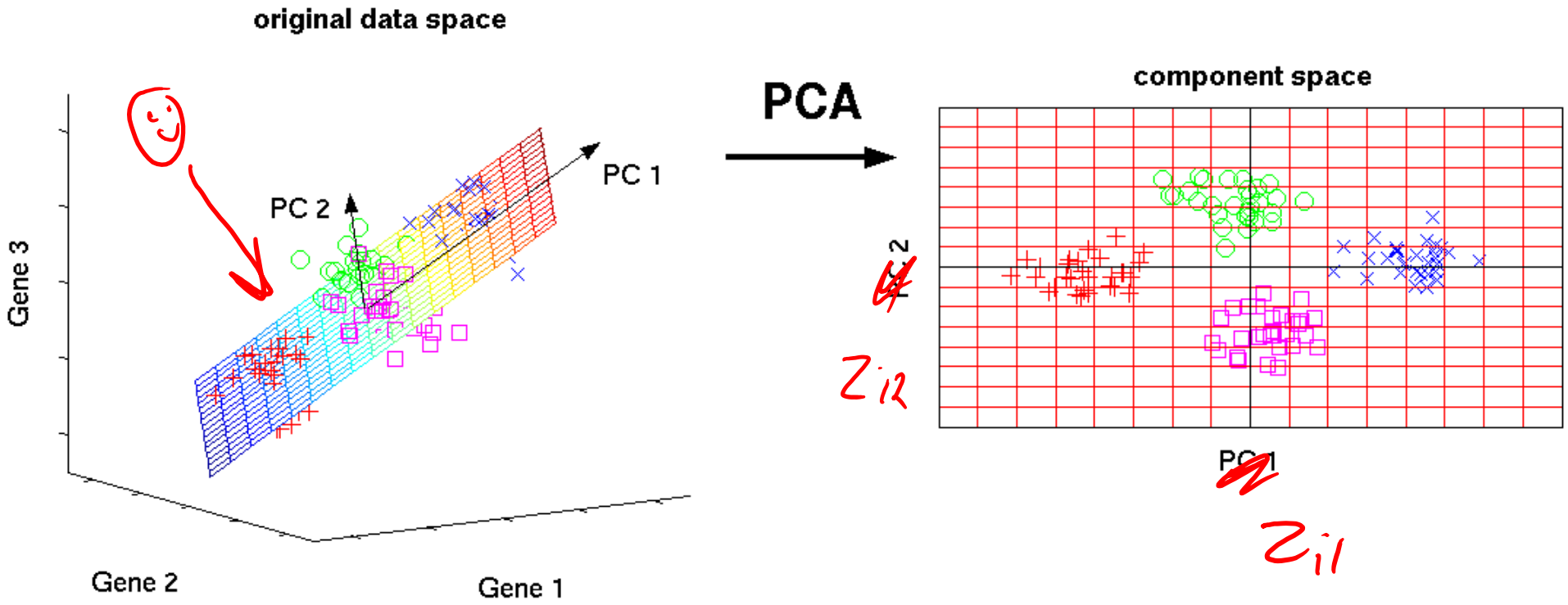
PCA with k=1



Latent factor could be viewed as measure of size.

PCA with d=3 and k=2.

- With d=3, PCA (k=1) finds line minimizing squared distance to x_i.
- With d=3, PCA (k=2) finds plane minimizing squared distance to x_i.



http://www.nlpca.org/fig_pca_principal_component_analysis.png

Summary

- MAP estimation directly models p(w | X, y).
  - Gives probabilistic interpretation to regularization.
- Losses for weird scenarios are possible using MLE/MAP:
  - Ordinal labels, count labels, censored labels, unbalanced labels.

- Latent-factor models:
  - Try to learn basis Z from training examples X.
  - Usually, the z_i are "part weights" for "parts" w_c.
  - Useful for dimensionality reduction, visualization, factor discovery, etc.
- Principal component analysis:
  - Writes each training examples as linear combination of parts.
    - We learn both the "parts" 'W' and the "features" Z.
  - We can view 'W' as best lower-dimensional hyper-plane.
  - We can view 'Z' as the coordinates in the lower-dimensional hyper-plane.

- Next time: PCA in 4 lines of code.