CPSC 340: Machine Learning and Data Mining

Recommender Systems

Summer 2021

In This Lecture

1. Latent Factor Models for Images
2. Latent Factor Models for Netflix
   - Collaborative filtering
3. Latent Factor Models for Visualization

Last Few Lectures: Latent-Factor Models

- We've been discussing latent-factor models of the form:

$$f(W,Z) = \sum_{i=1}^{n}\sum_{j=1}^{d}(\langle w^j, z_i\rangle - x_{ij})^2$$

- We get different models under different conditions:
  - K-means: each z_i has one '1' and the rest are zero.
  - Least squares: we only have one variable (d=1) and the z_i are fixed.
  - PCA: no restrictions on W or Z.
    - Orthogonal PCA: the rows w_c have a norm of 1 and have an inner product of zero.
  - NMF: all elements of W and Z are non-negative.

Variations on Latent-Factor Models

- We can use all our tricks for linear regression in this context:

$$f(W,Z) = \sum_{i=1}^{n}\sum_{j=1}^{d} |\langle w^j, z_i\rangle - x_{ij}| + \frac{\lambda_1}{2}\sum_{i=1}^{n}\sum_{c=1}^{k} z_{ic}^2 + \frac{\lambda_2}{2}\sum_{j=1}^{d}\sum_{c=1}^{k} |w_{cj}|$$

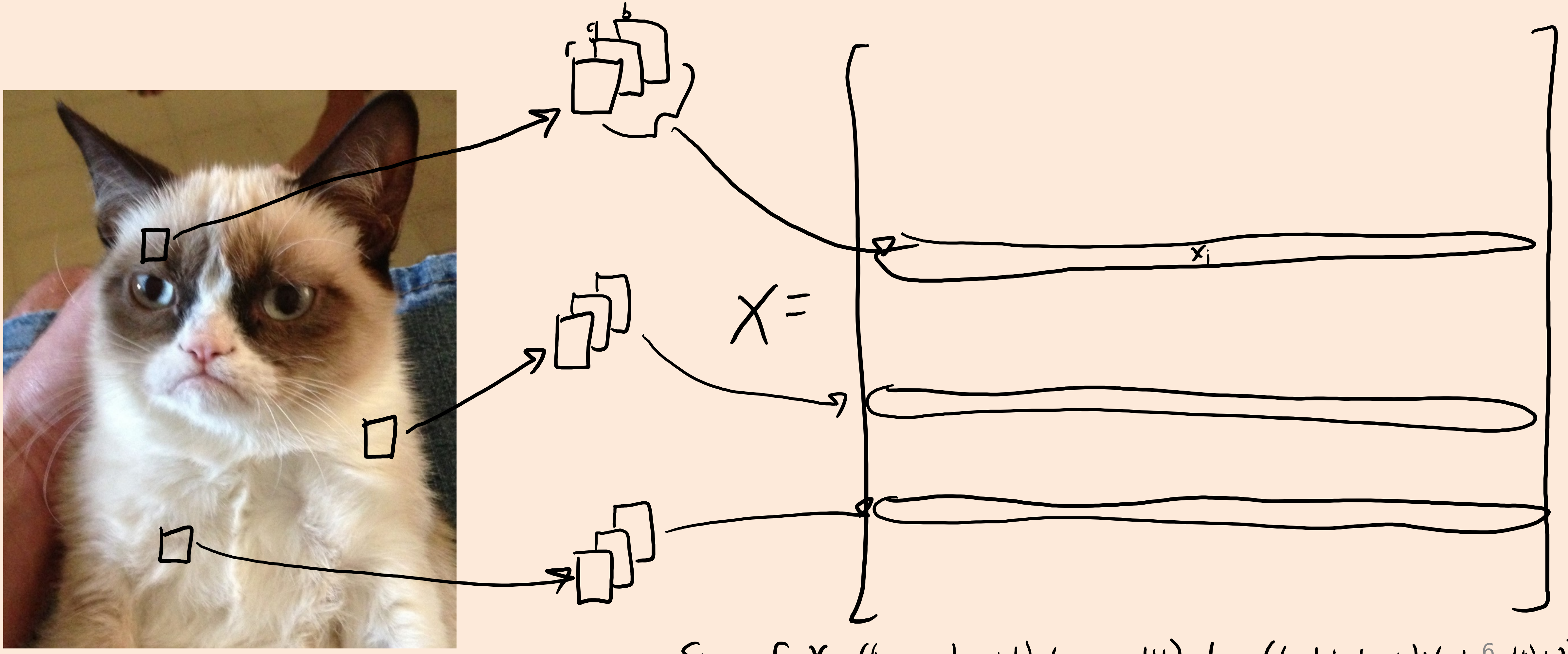
- Absolute loss gives robust PCA that is less sensitive to outliers.
- We can use L2-regularization.
  - Though only reduces overfitting if we regularize both 'W' and 'Z'.
- We can use L1-regularization to give sparse latent factors/features.
- Can use change of basis to learn non-linear latent-factor models.

Application: Image Restoration

http://www.jmlr.org/papers/volume11/mairal10a/mairal10a.pdf

Latent-Factor Models for Image Patches

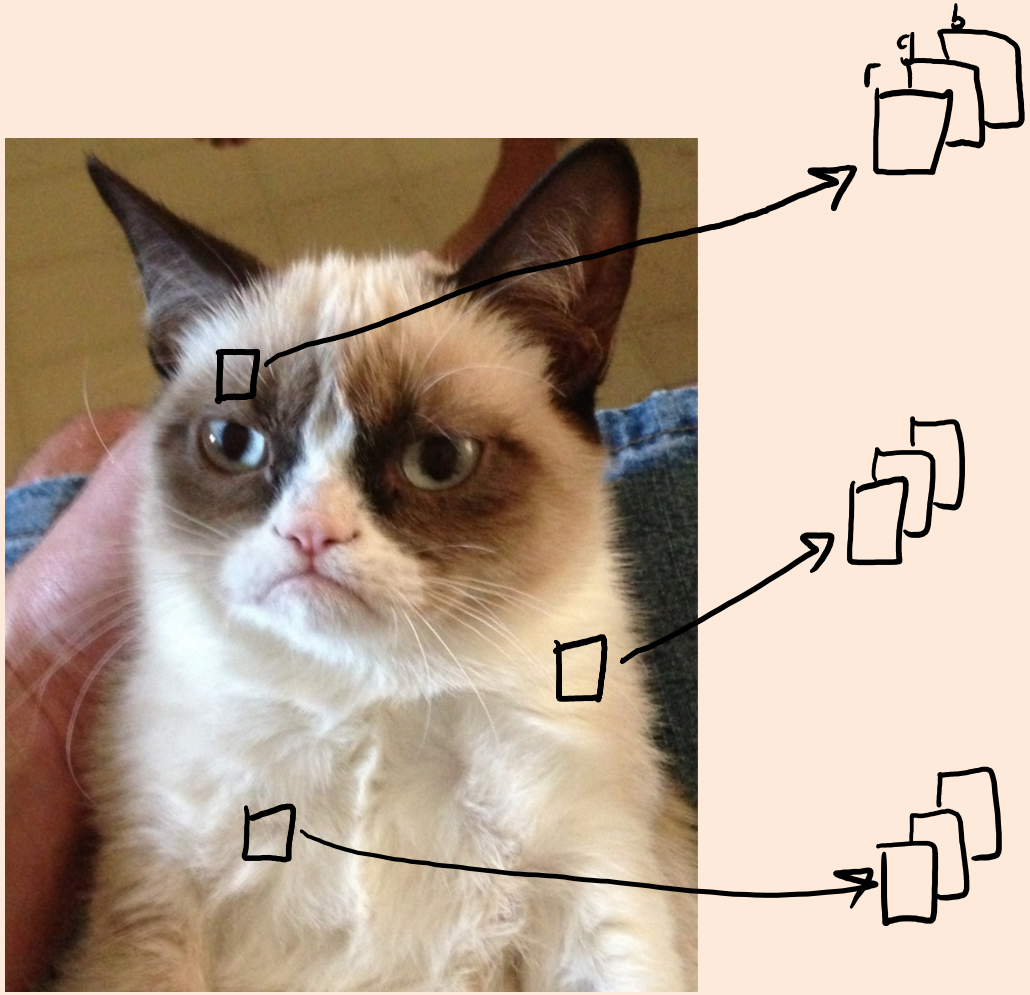
- Consider building latent-factors for general image patches:



Size of X: (image height)×(image width) by ((patch height)×(patch width)×3)

Latent-Factor Models for Image Patches

- Consider building latent-factors for general image patches:

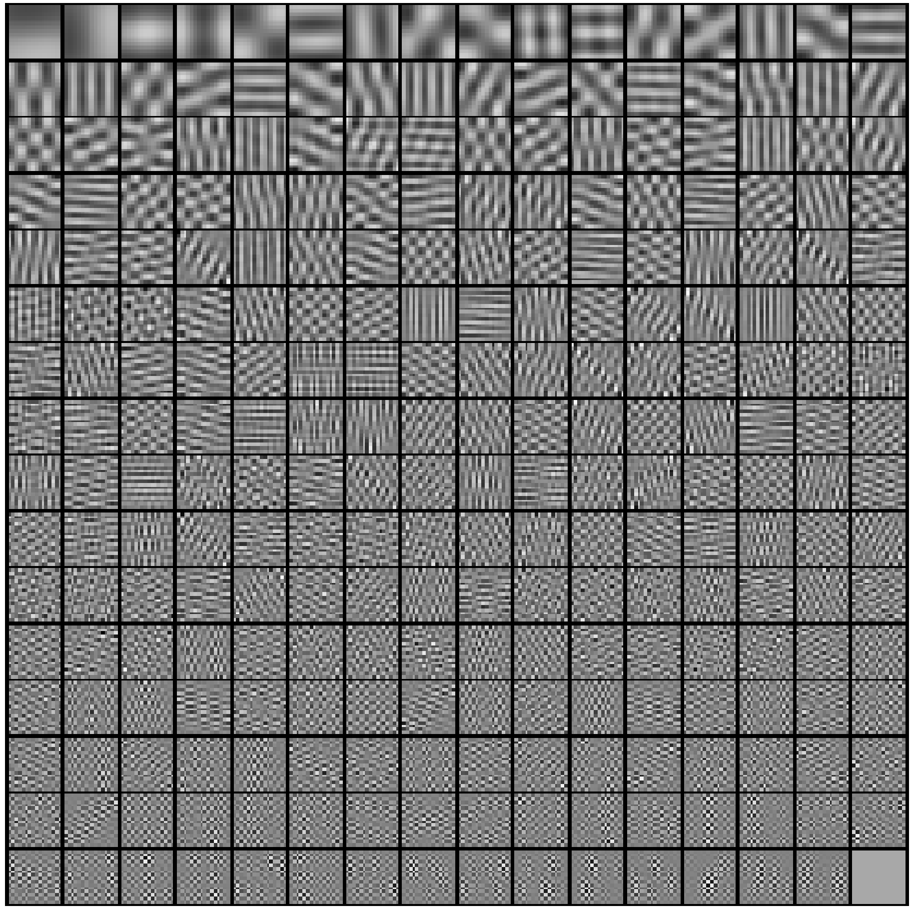


Typical pre-processing:

1. Usual variable centering
2. "Whiten" patches.

(remove correlations - bonus)

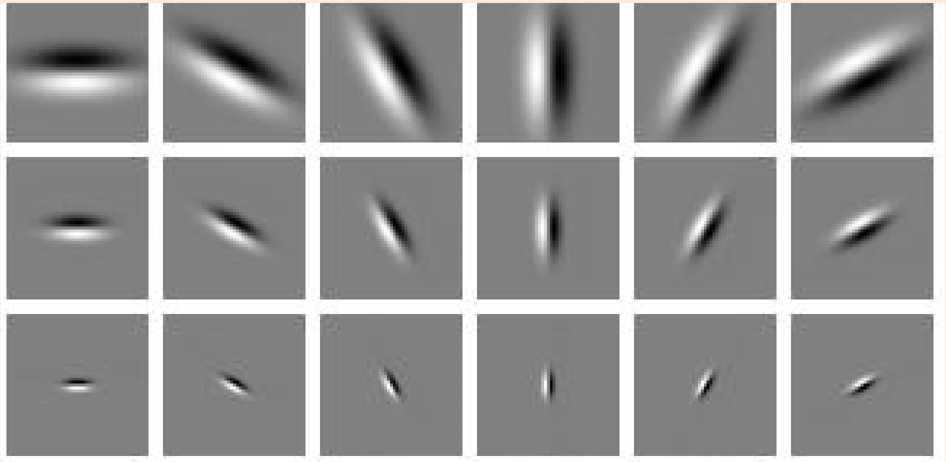
Latent-Factor Models for Image Patches



(b) Principal components.

Orthogonal bases don't seem right:
- Few PCs do almost everything.
- Most PCs do almost nothing.

Scientists say "simple cells" in visual cortex use:

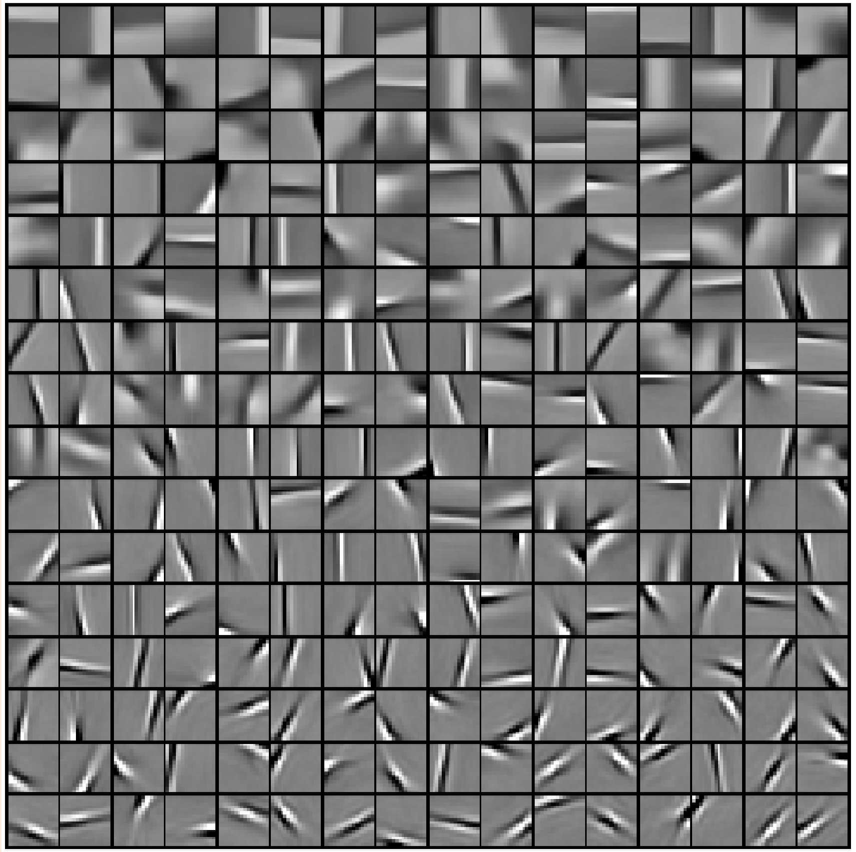


'Gabor' filters

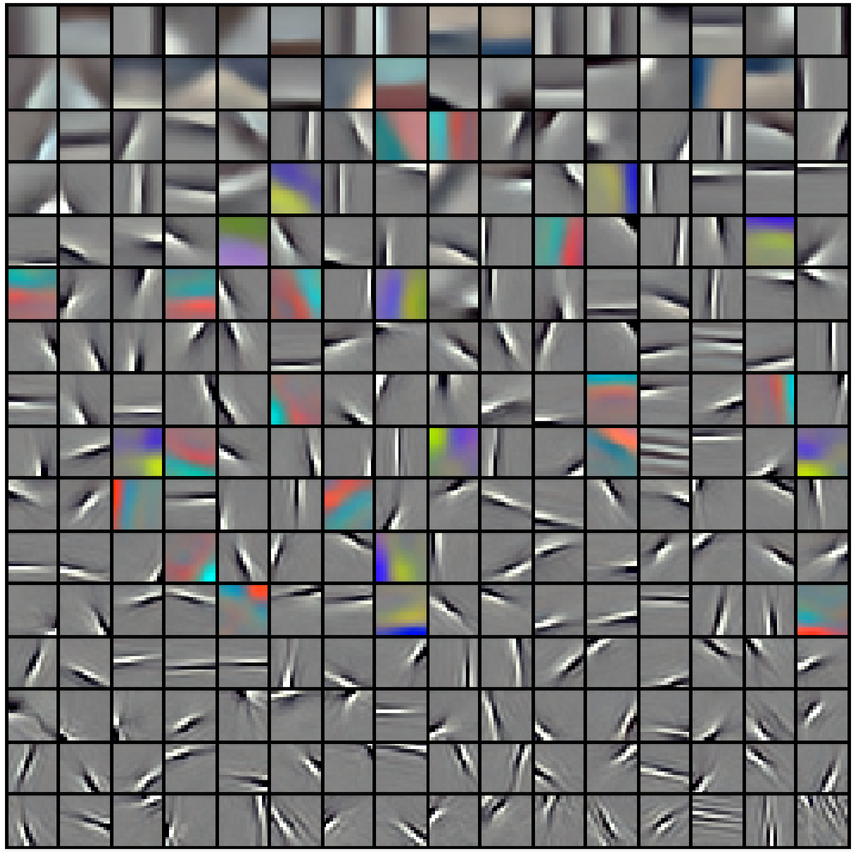
http://lear.inrialpes.fr/people/mairal/resources/pdf/review_sparse_arxiv.pdf
http://stackoverflow.com/questions/16059462/comparing-textures-with-opencv-and-gabor-filters

Latent-Factor Models for Image Patches

- Results from a "sparse" (non-orthogonal) latent factor model:



(a) With centering - gray.

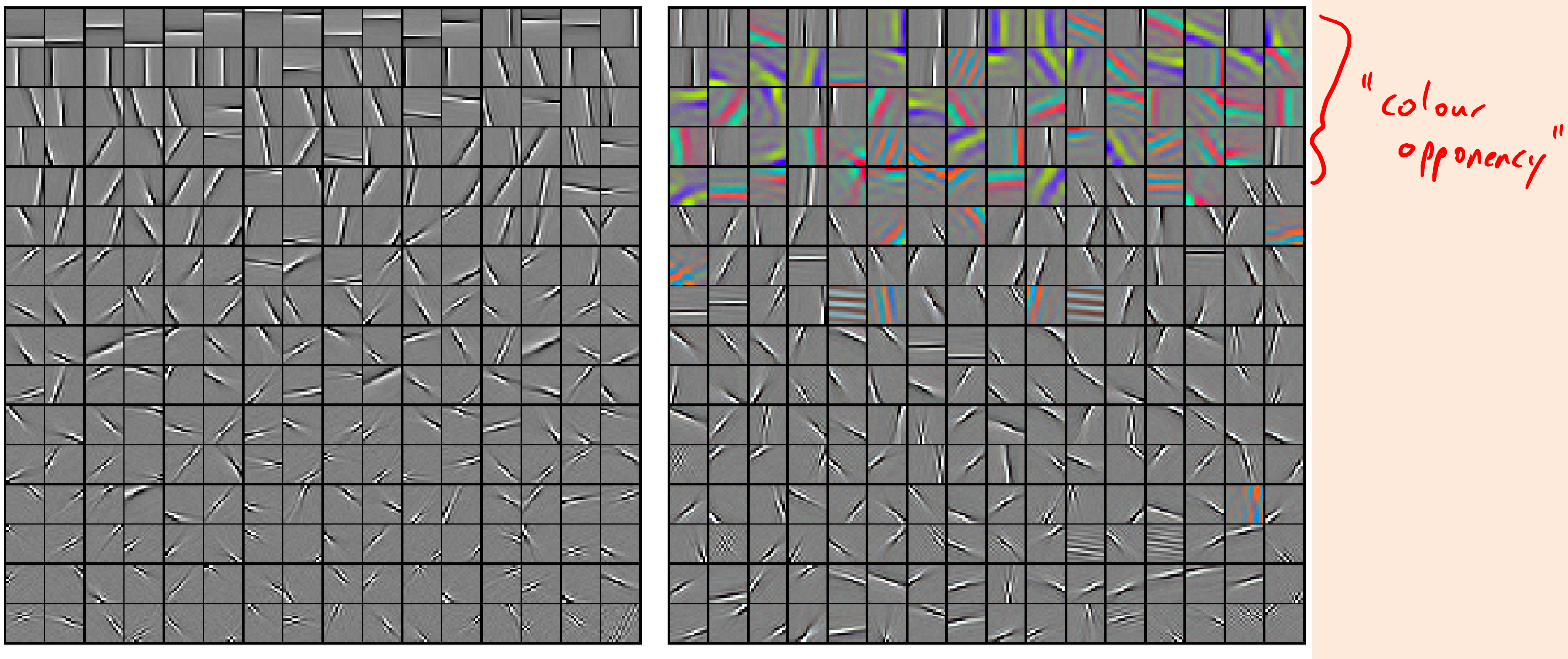


(b) With centering - RGB.

http://lear.inrialpes.fr/people/mairal/resources/pdf/review_sparse_arxiv.pdf

Latent-Factor Models for Image Patches

- Results from a "sparse" (non-orthogonal) latent factor model:

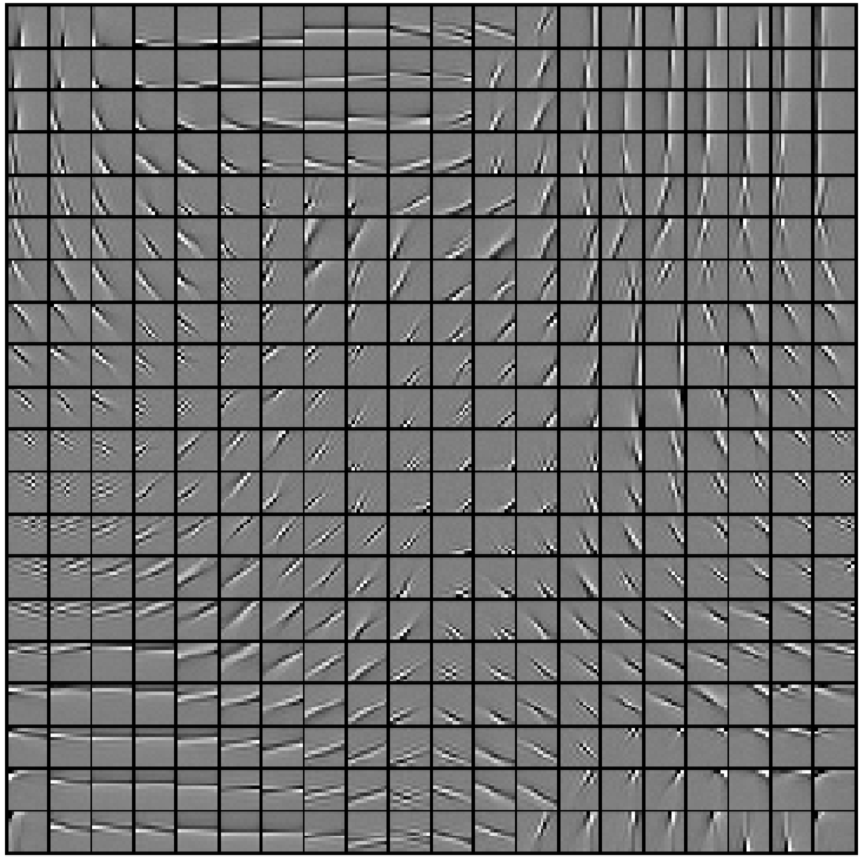


(c) With whitening - gray. (d) With whitening - RGB.

http://lear.inrialpes.fr/people/mairal/resources/pdf/review_sparse_arxiv.pdf

Recent Work: Structured Sparsity

- Basis learned with a variant of "structured sparsity":



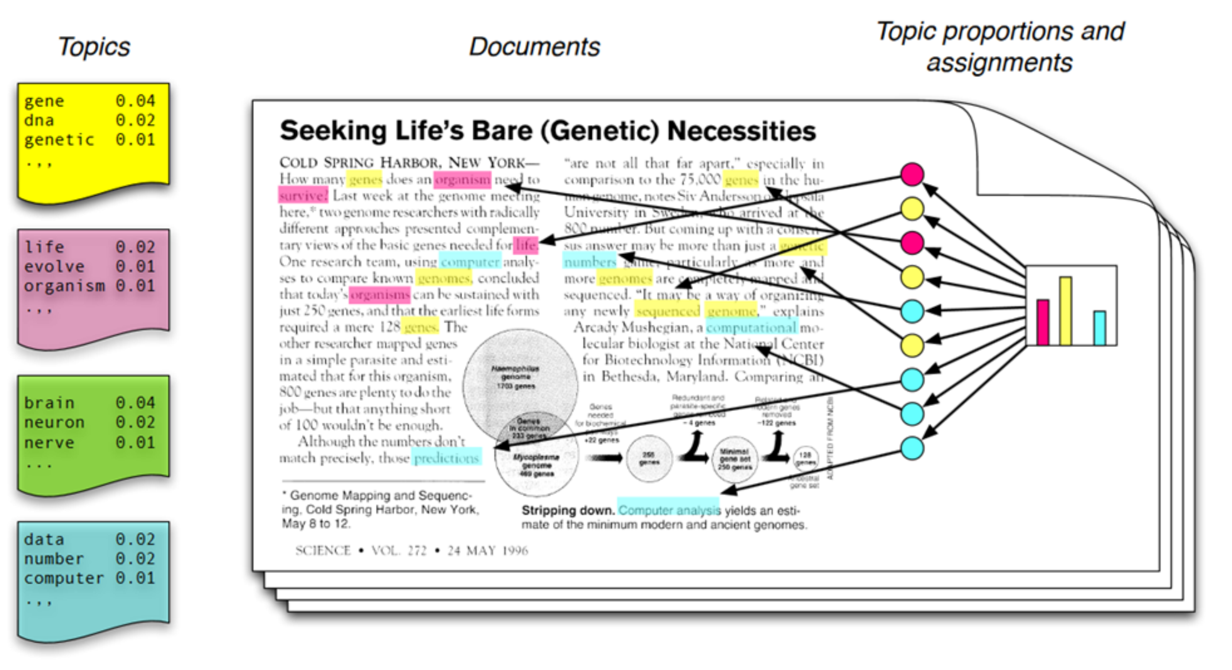
(b) With 4 × 4 neighborhood.

Similar to "cortical columns" theory in visual cortex.

http://lear.inrialpes.fr/people/mairal/resources/pdf/review_sparse_arxiv.pdf

Beyond NMF: Topic Models

- For modeling data as combinations of non-negative parts, NMF has been largely replaced by "topic models".
  - A "fully-Bayesian" model where sparsity arises naturally.
  - Most popular example is called "latent Dirichlet allocation" (CPSC 440).



http://menome.com/wp/wp-content/uploads/2014/12/Blei2011.pdf

When you win the Netflix Prize with simple latent factor models and not overcomplicated neural networks

Coming Up Next

LATENT FACTOR MODELS AND THE NETFLIX PRIZE

Recall: Netflix Show Recommendation

$x_{ij} :=$ user i's rating of show j
0 if never watched

x^1	x^2	x^3	x^4	x^5
5	4	2	2	5
2	0	1	0	4
3	1	0	1	3
4	3	1	0	2
1	1	1	5	0

Q: I've never watched Space Force… would I like it?

Recommender System Motivation: Netflix Prize

- Netflix Prize:
  - 100M ratings from 0.5M users on 18k movies.
  - Grand prize was $1M for first team to reduce squared error by 10%.
  - Started on October 2nd, 2006.
  - Netflix's system was first beat October 8th.
  - 1% error reduction achieved on October 15th.
  - Steady improvement after that.
    - ML methods soon dominated.
  - One obstacle was 'Napolean Dynamite' problem:
    - Some movie ratings seem very difficult to predict.
    - Should only be recommended to certain groups.

"The Room": 3.7/10 on IMDb

Lessons Learned from Netflix Prize

- Prize awarded in 2009:
  - Ensemble method that averaged 107 models.
  - Increasing diversity of models more important than improving models.

- Winning entry (and most entries) used collaborative filtering:
  - Methods that only looks at ratings, not features of movies/users.
- A simple collaborative filtering method that does really well (7% error):
  - "Regularized matrix factorization". Now adopted by many companies.

http://bits.blogs.nytimes.com/2009/09/21/netflix-awards-1-million-prize-and-starts-a-new-contest/?_r=0

Two Approaches to Recommender Systems

1. Content-based filtering (supervised).
2. Collaborative filtering (unsupervised).

What is Content-Based Filtering?

- Supervised learning:
  - Extract features x_i of users and items, building model to predict rating y_i given x_i.
  - Apply model to prediction for new users/items.
- Example: Gmail's "important messages" (personalization with "local" features).

What is Collaborative Filtering?

- “Unsupervised” learning
  (have label matrix ‘Y’ but no features):
    - We only have labels y_{ij} (rating of user ‘i’ for movie ‘j’).
  - Example: Amazon recommendation algorithm.

$$Y = \underbrace{\begin{bmatrix} ? & 4 & 3 & 2 & 3 & 3 \\ 2 & 1 & ? & 5 & ? & 5 \\ ? & 1 & ? & 5 & 5 & 5 \\ 2 & 3 & 3 & ? & ? & ? \end{bmatrix}}_{\text{movie}}$$

user

Course:CPSC522/Restricted Boltzmann Machines for Collaborative Filtering

< Course:CPSC522

[hide] Contents

Restricted Boltzmann Machines for Collaborative Filtering

In this page, we introduce two methods for collaborative filtering: Singular Vector Decomposition (SVD) [1] and Restricted Boltzmann Machines (RBM).[2]

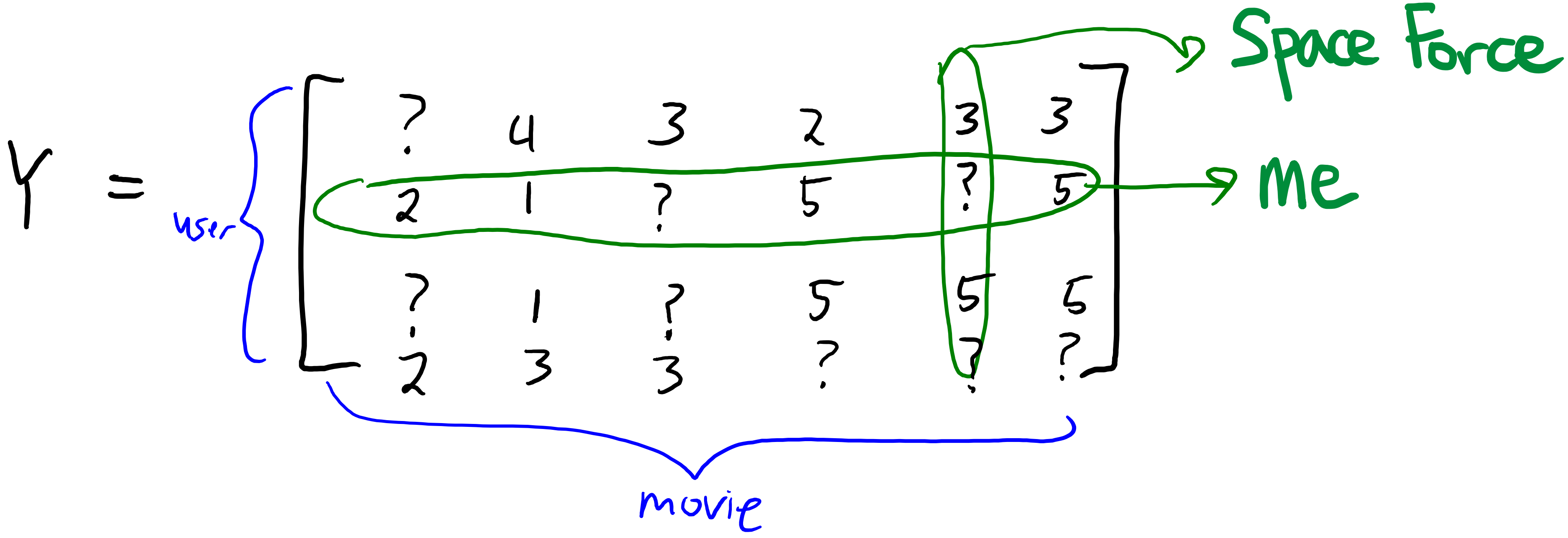
Principal Author: Nam Hee Gordon Kim

Coming Up Next

COLLABORATIVE FILTERING

Collaborative Filtering Problem

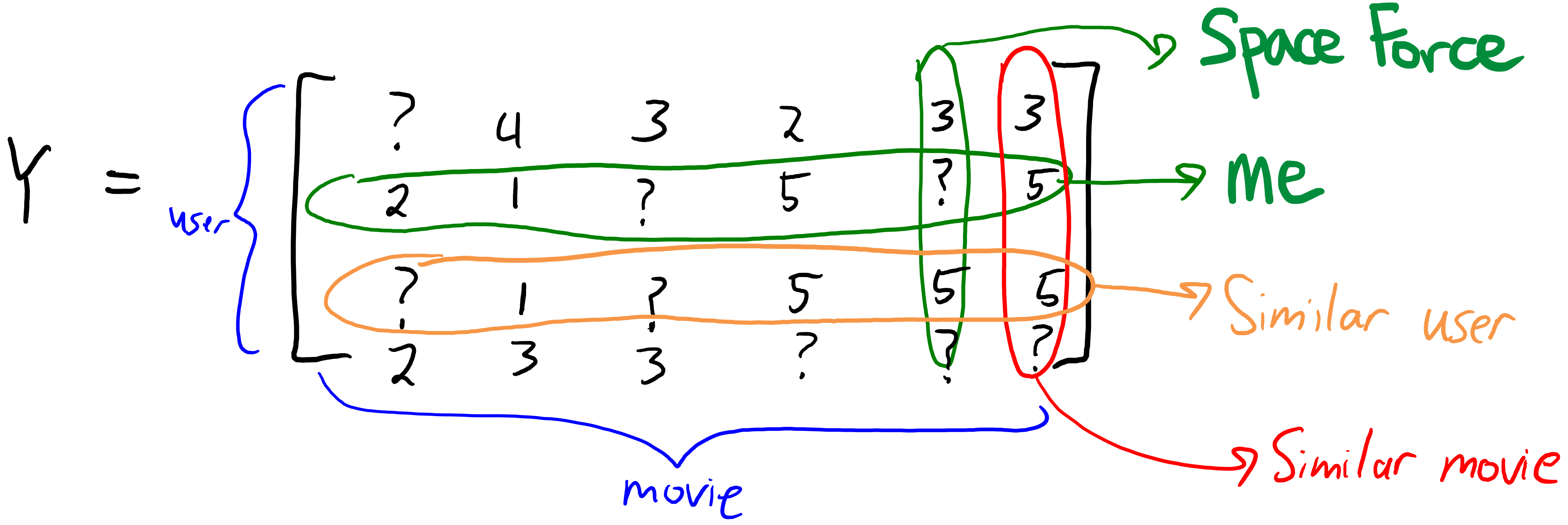
- Collaborative filtering is ‘filling in’ the user-item matrix:



- We have some ratings available with values {1,2,3,4,5}.
- We want to predict ratings “?” by looking at available ratings.

Collaborative Filtering Problem

- Collaborative filtering is 'filling in' the user-item matrix:

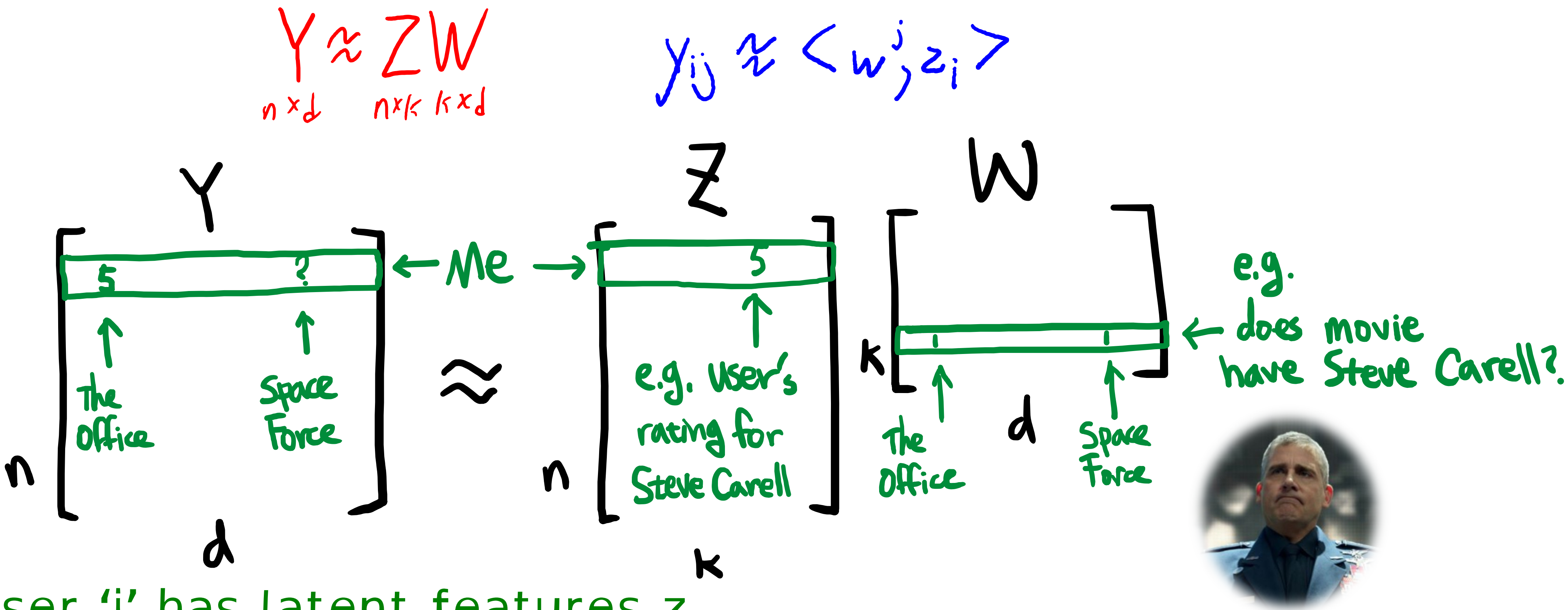


- What rating would I give to "Space Force"?
  - Why is this not completely crazy? We may have similar users and movies.

Q: Can we use latent factors to solve this?

Matrix Factorization for Collaborative Filtering

- Our standard latent-factor model for entries in matrix 'Y':



- User 'i' has latent features z_i.
- Movie 'j' has latent features w^j.

Matrix Factorization for Collaborative Filtering

- Our standard latent-factor model for entries in matrix ‘Y’:

$$\underset{n \times d}{Y} \approx \underset{n \times k}{Z}\,\underset{k \times d}{W} \qquad y_{ij} \approx \langle w^j, z_i \rangle$$

- User ‘i’ has latent features z_i. → z_{ic} could mean "likes Nicolas Cage"
- Movie ‘j’ has latent features w^j. → w_{jc} could mean "has Nicholas Cage"

- Idea:
  1. Learn Y ≈ ZW based on ________________
  2. Reconstruct: $\hat{Y}$ = ZW

Latent Factor Loss Function for CF

- Our loss functions sums over available ratings 'R':

$$f(Z,W) = \sum_{(i,j)\in R} (\langle w^j, z_i \rangle - y_{ij})^2 + \frac{\lambda_1}{2}\|Z\|_F^2 + \frac{\lambda_2}{2}\|W\|_F^2$$

$$Z, W \in \underset{Z,W}{\text{argmin}} \{ f(Z,W) \}$$

- And we add L2-regularization to both types of features.
  - Regularized PCA on the available entries of Y.
  - Typically fit with SGD. (WHY?)
- This simple method gives you a 7% improvement on the Netflix problem.

Adding Global/User/Movie Biases

- Our standard latent-factor model for entries in matrix 'Y':

$$\hat{y}_{ij} = \langle w^j, z_i \rangle$$

- Sometimes we don't assume the y_{ij} have a mean of zero:
  - We could add bias β reflecting average overall rating:

$$\hat{y}_{ij} = \beta + \langle w^j, z_i \rangle$$

  - We could also add a user-specific bias β_i and item-specific bias β_j.

$$\hat{y}_{ij} = \beta + \beta_i + \beta_j + \langle w^j, z_i \rangle$$

    - Some users rate things higher on average, and movies are rated better on average.
    - These might also be regularized.

Beyond Accuracy in Recommender Systems

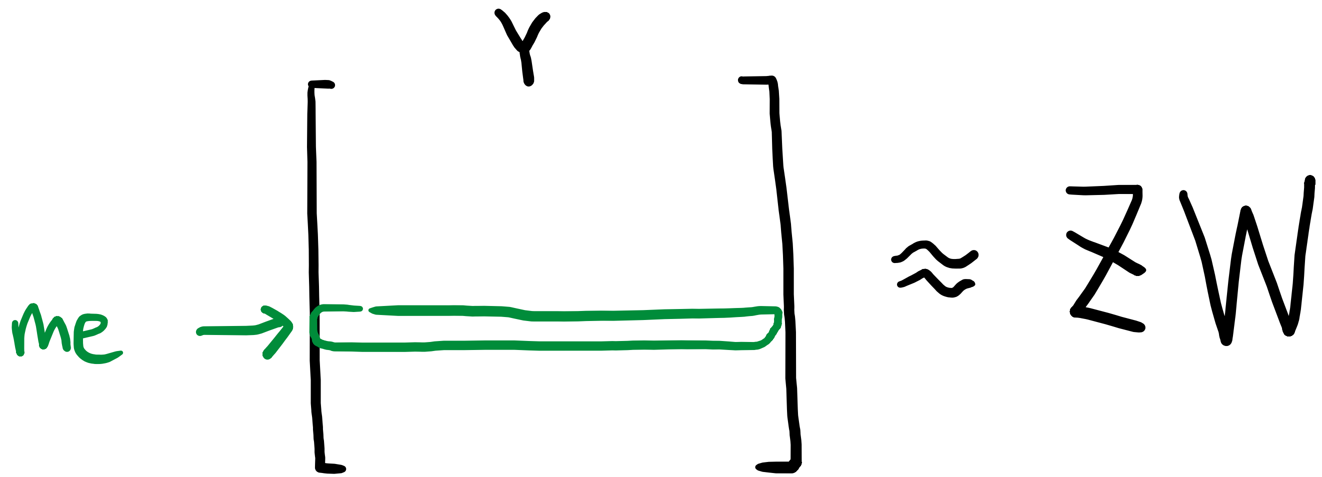
- Winning system of Netflix Challenge was never adopted.
- Other issues important in recommender systems:
  - Diversity: how different are the recommendations?
    - If you like 'Battle of Five Armies Extended Edition', recommend Battle of Five Armies?
    - Even if you really really like Star Wars, you might want non-Star-Wars suggestions.
  - Persistence: how long should recommendations last?
    - If you keep not clicking on 'Hunger Games', should it remain a recommendation?
  - Trust: tell user *why* you made a recommendation.
    - Quora gives explanations for recommendations.
  - Social recommendation: what did your friends watch?
  - Freshness: people tend to get more excited about *new/surprising* things.
    - Collaborative filtering does not predict well for new users/movies.
      - New movies don't yet have ratings, and new users haven't rated anything.

Content-Based vs. Collaborative Filtering

- Collaborative filtering: latent factors approach (Part 4):

$$\hat{y}_{ij} = \langle w^j, z_i \rangle$$

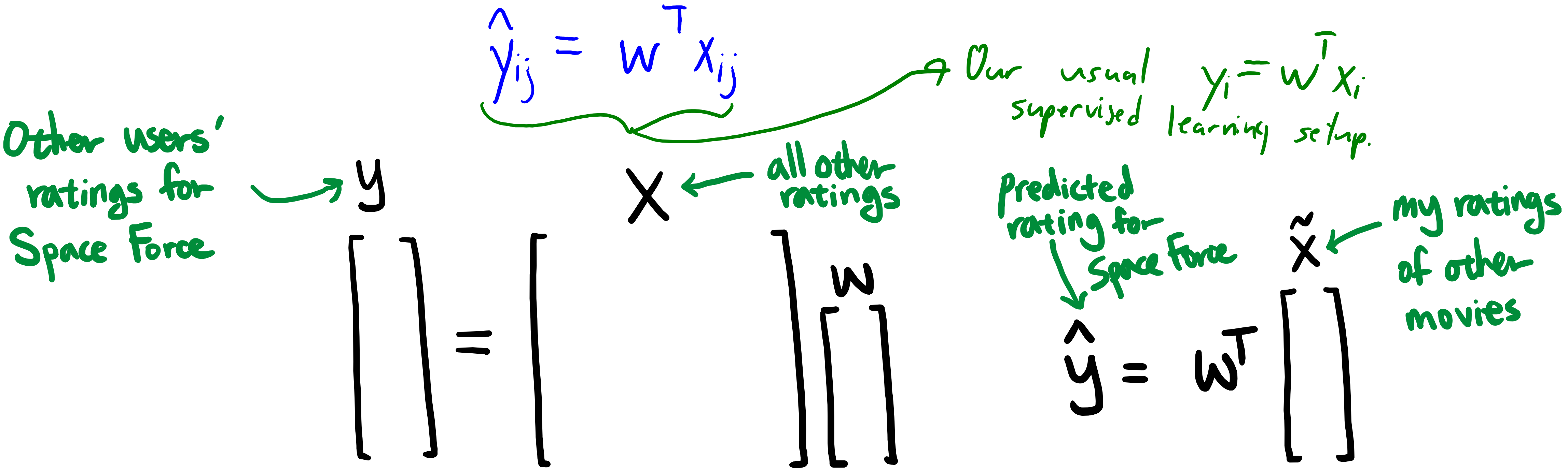
"hidden" features of movie (w^j) — "hidden" features of user (z_i)



Q: Can collaborative filtering predict preference of a new user/movie?

Content-Based vs. Collaborative Filtering

- Content-based filtering: supervised learning approach (Part 3):



  - Can predict on new users/movies, but can't learn about each user/movie.
    - Does the user like Steve Carell?
    - Does the movie have Steve Carell?

Hybrid Approaches

- Hybrid approaches combine content-based/collaborative filtering:
  - SVDfeature (won "KDD Cup" in 2011 and 2012).

$$\hat{y}_{ij} = \beta + \beta_i + \beta_j + w^T x_{ij} + \langle w^j, z_i \rangle$$

Average rating across all users/movies

Average rating for user 'i'

Average rating for movie 'j'

Linear model based on user/movie features x_{ij}.

Latent features z_i for user 'i' and latent features w_j for movie 'j'

Extra factors we learn for specific users and movies.

Standard supervised learning: can predict for new users/movies

  - Note that x_{ij} is a feature vector. Also, 'w' and 'w^j' are different parameters.

Stochastic Gradient for SVDfeature

- Common approach to fitting SVDfeature is stochastic gradient.
- Previously you saw stochastic gradient for supervised learning:
  - Choose a random example 'i'
  - Update parameters 'w' using gradient of example 'i'
- Stochastic gradient for SVDfeature (formulas as bonus):
  - Choose a random user 'i' and a random product 'j'
  - Update β, β_i, β_j, w, z_i, and w^j based on their gradient for this user-product.

Updated every time (β, w)

Social Regularization

- Many recommenders are now connected to social networks.
  - "Login using your Facebook account".

- Often, people like similar movies to their friends.

- Recent recommender systems use social regularization.
  - Add a "regularizer" encouraging friends' weights to be similar:

$$\frac{\lambda}{2} \sum_{(i,j) \in \text{"friends"}} \|z_i - z_j\|^2$$

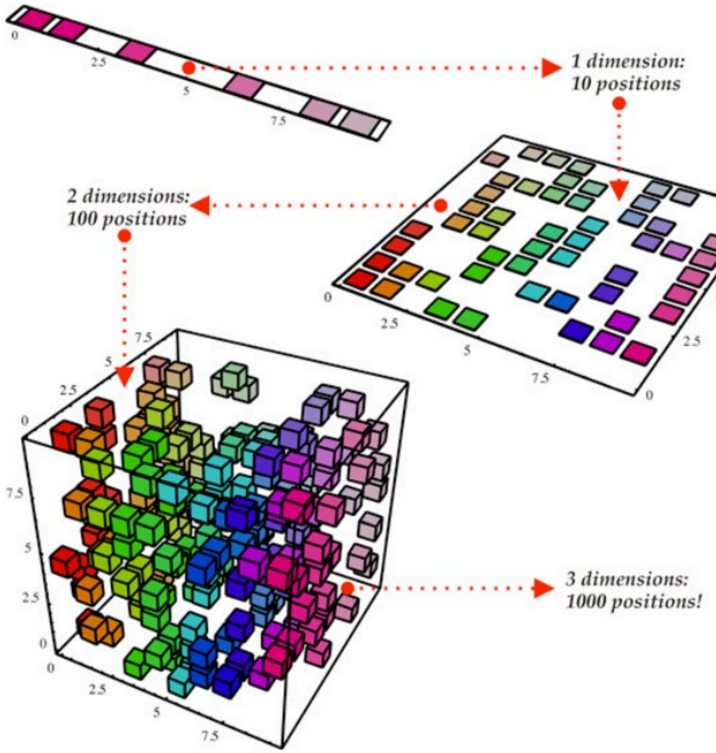
  - If we get a new user, recommendations are based on friend's preferences.

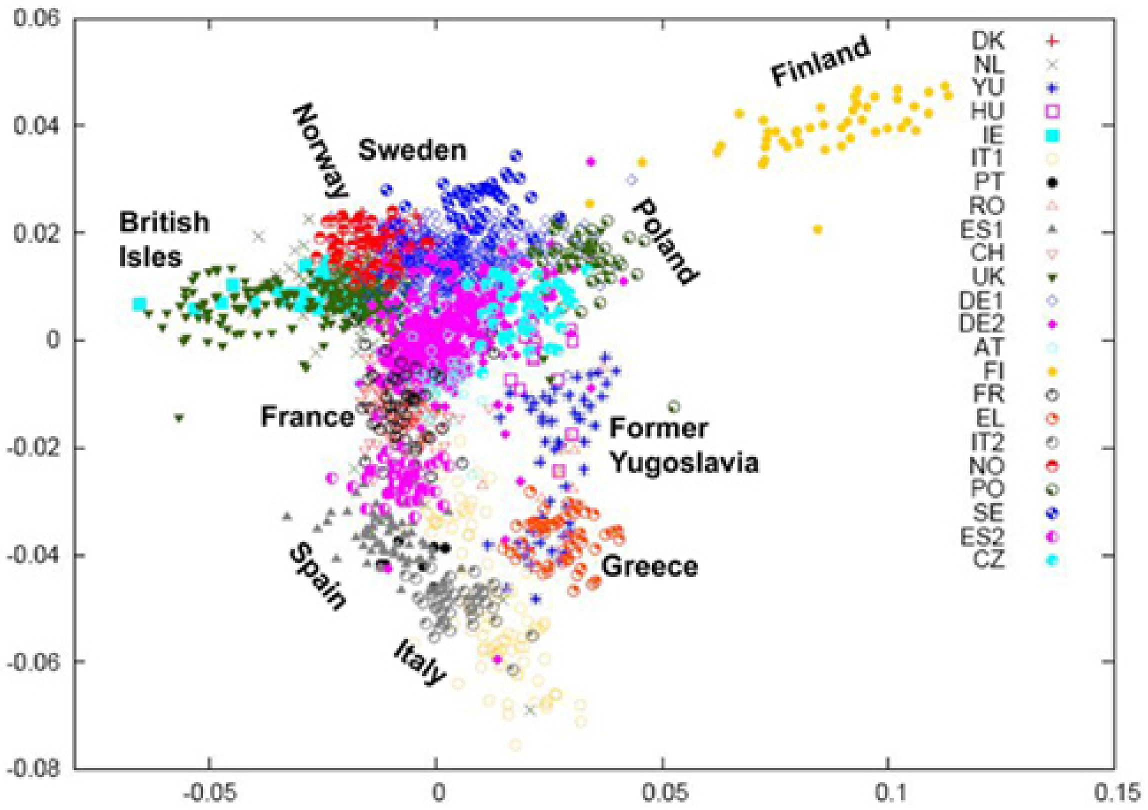
Coming Up Next

LATENT FACTOR MODELS FOR VISUALIZATION

Latent-Factor Models for Visualization

- PCA takes features x_i and gives k-dimensional approximation z_i.
- If k is small, we can use this to visualize high-dimensional data.

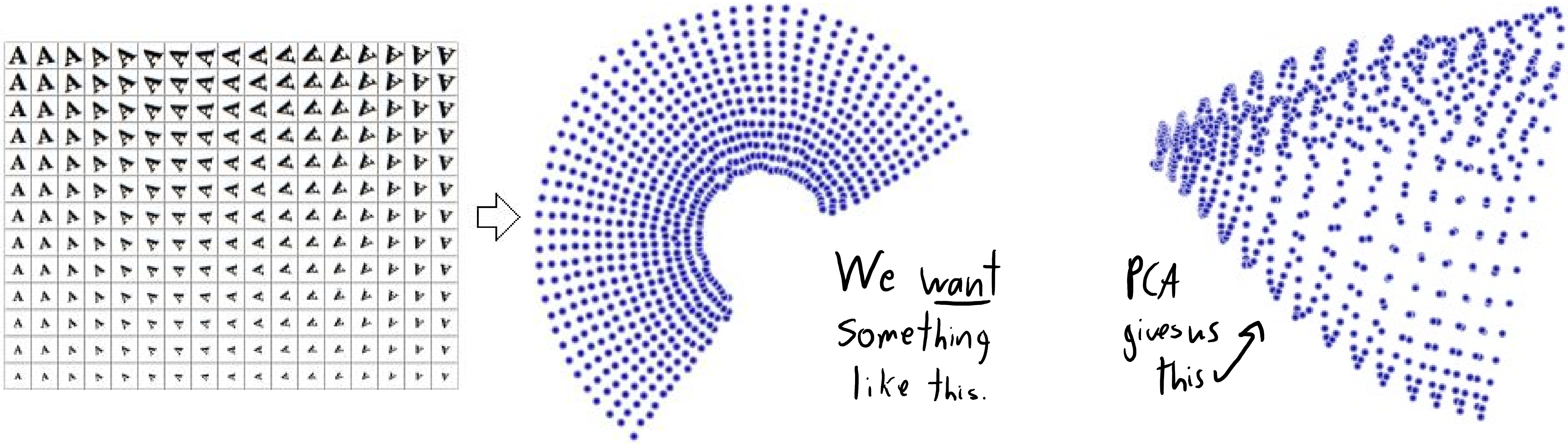




http://www.turingfinance.com/artificial-intelligence-and-statistics-principal-component-analysis-and-self-organizing-maps/
http://scienceblogs.com/gnxp/2008/08/14/the-genetic-map-of-europe/

Motivation for Non-Linear Latent-Factor Models

- But PCA is a parametric linear model
- PCA may not find obvious low-dimensional structure.



- We could use change of basis or kernels: but still need to pick basis.

https://en.wikipedia.org/wiki/Nonlinear_dimensionality_reduction

Multi-Dimensional Scaling

- PCA for visualization:
  - We're using PCA to get the location of the z_i values.
  - We then plot the z_i values as locations in a scatterplot.
- Multi-dimensional scaling (MDS):
  - Use gradient-based optimization to get z_i values.
    - "Gradient descent on the points in a scatterplot".
  - Needs a "cost" function saying how "good" the z_i locations are.
    - Traditional MDS cost function:

$$f(Z) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} \left(\|z_i - z_j\| - \|x_i - x_j\| \right)^2$$

sum over pairs of examples

distance in scatterplot

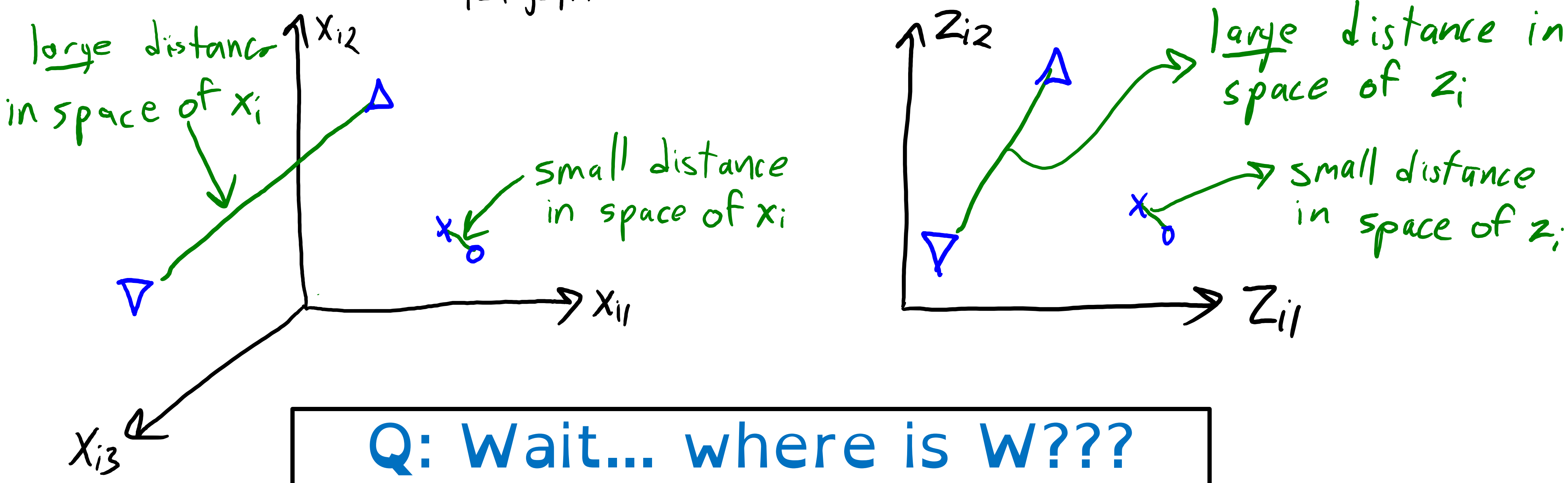
Distance between points in original 'd' dimensions

"Try to make scatterplot distances match high-dimensional distance"

Multi-Dimensional Scaling

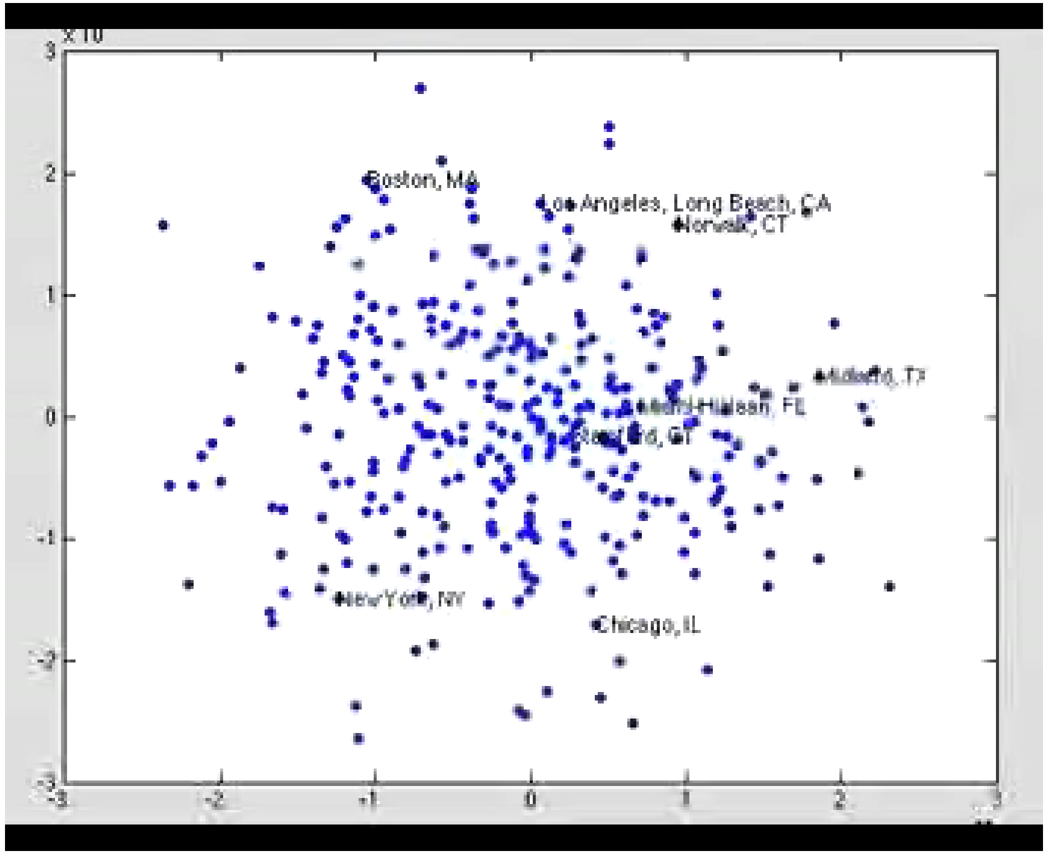
- Multi-dimensional scaling (MDS):
  - Optimize the locations of z_i values.

$$f(Z) = \sum_{i=1}^{n} \sum_{j=i+1}^{n} (\|z_i - z_j\| - \|x_i - x_j\|)^2$$



Q: Wait... where is W???

MDS Method ("Sammon Mapping") in Action



- Unfortunately, MDS often does not work well in practice.

Summary

- Recommender systems try to recommend products.
- Collaborative filtering tries to fill in missing values in a matrix.
  - Matrix factorization is a common approach.
- Multi-dimensional scaling is a non-parametric latent-factor model.

- Next time: making a scatterplot by gradient descent.

Review Questions

- Q1: What is the difference between content-based filtering and collaborative filtering?
- Q2: How does a latent factor model predict the rating of a movie for a particular user?
- Q3: What is the downside of PCA when it comes to visualizing datapoints?

Digression: “Whitening”

- With image data, features will be very redundant.
  - Neighbouring pixels tend to have similar values.
- A standard transformation in these settings is “whitening”:
  - Rotate the data so features are uncorrelated.
  - Re-scale the rotated features so they have a variance of 1.

- Using SVD approach to PCA, we can do this with:
  - Get ‘W’ from SVD (usually with k=d).
  - $Z = XW^T$ (rotate to give uncorrelated features).
  - Divide columns of ‘Z’ by corresponding singular values (unit variance).

- Details/discussion here.

Motivation for Topic Models

- Want a model of the "factors" making up documents.
  - Instead of latent-factor models, they're called topic models.
  - The canonical topic model is latent Dirichlet allocation (LDA).

Suppose you have the following set of sentences:

- I like to eat broccoli and bananas.
- I ate a banana and spinach smoothie for breakfast.
- Chinchillas and kittens are cute.
- My sister adopted a kitten yesterday.
- Look at this cute hamster munching on a piece of broccoli.

What is latent Dirichlet allocation? It's a way of automatically discovering **topics** that these sentences contain. For example, given these sentences and asked for 2 topics, LDA might produce something like

- **Sentences 1 and 2**: 100% Topic A
- **Sentences 3 and 4**: 100% Topic B
- **Sentence 5**: 60% Topic A, 40% Topic B
- **Topic A**: 30% broccoli, 15% bananas, 10% breakfast, 10% munching, ... (at which point, you could interpret topic A to be about food)
- **Topic B**: 20% chinchillas, 20% kittens, 20% cute, 15% hamster, ... (at which point, you could interpret topic B to be about cute animals)

http://blog.echen.me/2011/08/22/introduction-to-latent-dirichlet-allocation/

Term Frequency – Inverse Document Frequency

- In information retrieval, classic word importance measure is TF-IDF.

- First part is the term frequency tf(t,d) of term ‘t’ for document ‘d’.
  - Number of times “word” ‘t’ occurs in document ‘d’, divided by total words.
  - E.g., 7% of words in document ‘d’ are “the” and 2% of the words are “Lebron”.

- Second part is document frequency df(t,D).
  - Compute number of documents that have ‘t’ at least once.
  - E.g., 100% of documents contain “the” and 0.01% have “LeBron”.

- TF-IDF is tf(t,d)*log(1/df(t,D)).

Term Frequency – Inverse Document Frequency

- The TF-IDF statistic is tf(t,d)*log(1/df(t,D)).
  - It's high if word 't' happens often in document 'd', but isn't common.
  - E.g., seeing "LeBron" a lot it tells you something about "topic" of article.
  - E.g., seeing "the" a lot tells you nothing.

- There are *many* variations on this statistic.
  - E.g., avoiding dividing by zero and all types of "frequencies".

- Summarizing 'n' documents into a matrix X:
  - Each row corresponds to a document.
  - Each column gives the TF-IDF value of a particular word in the document.

Latent Semantic Indexing

- TF-IDF features are very redundant.
  - Consider TF-IDFs of "LeBron", "Durant", "Harden", and "Kobe".
  - High values of these typically just indicate topic of "basketball".

- We can probably compress this information quite a bit.

- Latent Semantic Indexing/Analysis:
  - Run latent-factor model (like PCA or NMF) on TF-IDF matrix X.
  - Treat the principal components as the "topics".
  - Latent Dirichlet allocation is a variant that avoids weird df(t,D) heuristic.

SVDfeature with SGD: the gory details

Objective: $\frac{1}{2}\sum_{(i,j)\in R}(\underbrace{\hat{y}_{ij} - y_{ij}}_{r_{ij}})^2$ with $\hat{y}_{ij} = \beta + \beta_i + \beta_j + w^T x_{ij} + (w^j)^T z_i$

Update based on random (i,j):

$$\beta = \beta - \alpha r_{ij}$$
$$\beta_i = \beta_i - \alpha r_{ij}$$
$$\beta_j = \beta_j - \alpha r_{ij}$$

Updates are the same, but 'β' is always update while β_i and β_j are only updated for the specific user + product

$w = w - \alpha r_{ij} x_{ij}$ ← Updated every time.

$z_i = z_i - \alpha r_{ij} w^j$

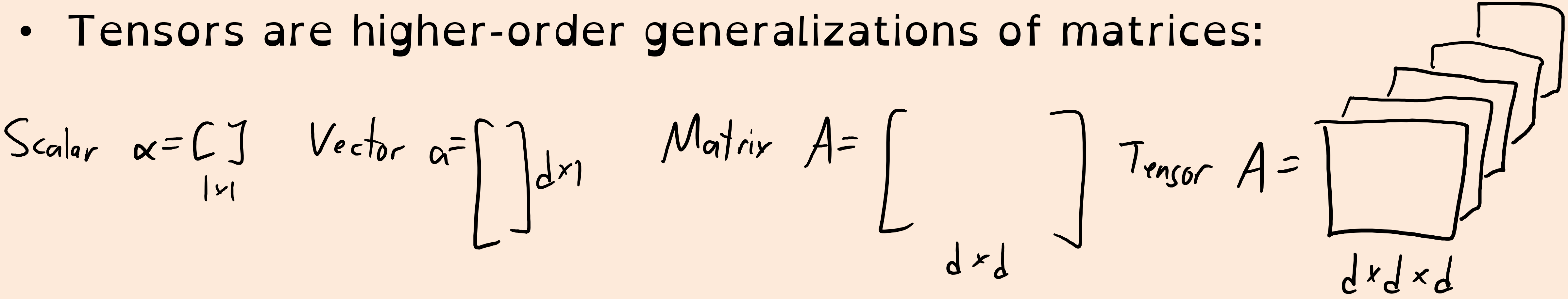
$w^j = w^j - \alpha r_{ij} z_i$

Updated for specific user and product.

(Adding regularization adds an extra term)

Tensor Factorization

- Tensors are higher-order generalizations of matrices:



- Generalization of matrix factorization is tensor factorization:

$$y_{ijm} \approx \sum_{c=1}^{k} w_{jc} z_{ic} v_{mc}$$

- Useful if there are other relevant variables:
  - Instead of ratings based on {user,movie}, ratings based {user,movie,group}.
  - Useful if you have groups of users, or if ratings change over time.

Field-Aware Matrix Factorization

- Field-aware factorization machines (FFMs):
  - Matrix factorization with multiple z_i or w_c for each example or part.
  - You choose which z_i or w_c to use based on the value of feature.
- Example from "click through rate" prediction:
  - E.g., predict whether "male" clicks on "nike" advertising on "espn" page.
  - A previous matrix factorization method for the 3 factors used:

$$w_{espn} w_{nike} + w_{espn} w_{male} + w_{nike} w_{male}$$

$$w_{espn}^{A} w_{nike}^{P} + w_{espn}^{G} w_{male}^{P} + w_{nike}^{G} w_{male}^{A}$$

  - FFMs could use:
    - wespnA is the factor we use when multiplying by a an advertiser's latent factor.
    - wespnG is the factor we use when multiplying by a group's latent factor.
- This approach has won some Kaggle competitions (link), and has shown to work well in production systems too (link).

Warm-Starting

- We've used data {X,y} to fit a model.
- We now have new training data and want to fit new and old data.

- Do we need to re-fit from scratch?

- This is the warm starting problem.
  - It's easier to warm start some models than others.

Easy Case: K-Nearest Neighbours and Counting

- K-nearest neighbours:
  - KNN just stores the training data, so just store the new data.
- Counting-based models:
  - Models that base predictions on frequencies of events.
  - E.g., naïve Bayes.
  - Just update the counts: $p(\text{"vicodin"} \mid \text{"spam"}) = \frac{\text{count of \{vicodin, spam\} in new and old data}}{\text{count of "spam" in new and old data}}$
  - Decision trees with fixed rules: just update counts at the leaves.

Medium Case: L2-Regularized Least Squares

- L2-regularized least squares is obtained from linear algebra:

$$w = (X^TX + \lambda I)^{-1}(X^Ty)$$

  - Cost is $O(nd^2 + d^3)$ for 'n' training examples and 'd' features.
- Given one new point, we need to compute:
  - X^Ty with one row added, which costs $O(d)$.
  - Old X^TX plus $x_ix_i^T$, which costs $O(d^2)$.
  - Solution of linear system, which costs $O(d^3)$.
  - So cost of adding 't' new data point is $O(td^3)$.

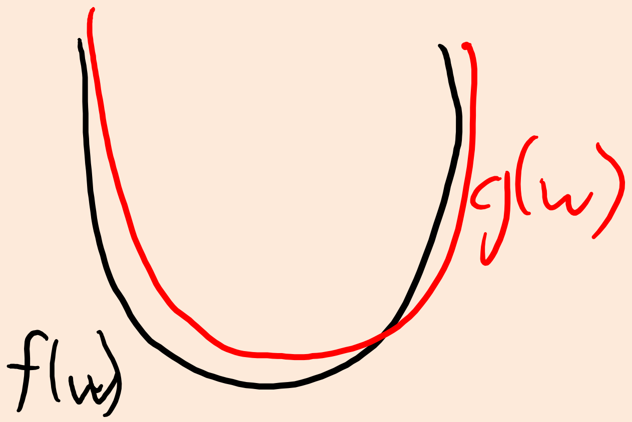
- With "matrix factorization updates", can reduce this to $O(td^2)$.
  - Cheaper than computing from scratch, particularly for large d.

Medium Case: Logistic Regression

- We fit logistic regression by gradient descent on a convex function.
- With new data, convex function f(w) changes to new function g(w).

$$f(w) = \sum_{i=1}^{n} f_i(w) \qquad g(w) = \sum_{i=1}^{n+1} f_i(w)$$

- If we don't have much more data, 'f' and 'g' will be "close".
  - Start gradient descent on 'g' with minimizer of 'f'.
  - You can show that it requires fewer iterations.



Hard Cases: Non-Convex/Greedy Models

- For decision trees:
  - "Warm start": continue splitting nodes that haven't already been split.
  - "Cold start": re-fit everything.

- Unlike previous cases, this won't in general give same result as re-fitting:
  - New data points might lead to different splits higher up in the tree.

- Intermediate: usually do warm start but occasionally do a cold start.

- Similar heuristics/conclusions for other non-convex/greedy models:
  - K-means clustering.
  - Matrix factorization (though you can continue PCA algorithms).