Probabilistic reasoning about plants, animals, objects, and people

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February 11, 2021

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"The mind is a neural computer, fitted by natural selection with combinatorial algorithms for causal and probabilistic reasoning about plants, animals, objects, and people. It is driven by goal states that served biological fitness in ancestral environments, such as food, sex, safety, parenthood, friendship, status and knowledge.

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Outline

1. What are relational probabilistic models and relational learning?
   - Relational Models
   - Knowledge Graphs
2. Learning Knowledge Graphs
3. Learning General Knowledge: Lifted Graphical Models
4. Bayesian $\Rightarrow$ Exchangeability $\Rightarrow$ Lifted Inference
5. Identity and Existence Uncertainty

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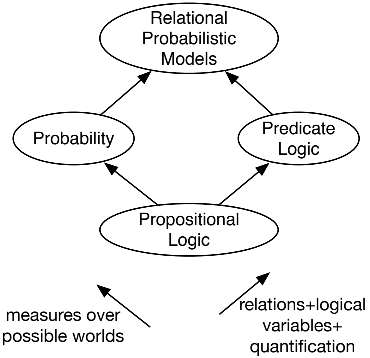

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What is a relational models?

Introductions to AI and machine learning typically start with learning from relations, e.g.:

Example	*Author*	*Thread*	*Length*	*Where_read*	*User_action*
e_1	*known*	*new*	*long*	*home*	*skips*
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What makes relational models in ML special is that the values are meaningless names. E.g., student #, product id, user id, movie id:

User	*Movie*	*Rating*	*Timestamp*
196	242	3	881250949
186	302	3	891717742
...	...	...	...

(Movielens 100k)

Names can be changed or exchanged with exactly same meaning.

Choosing Entities and Relations in Logic

First-order logical languages allow many different ways of representing facts.

E.g., How to represent: "Pen #7 is red."

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- $color(pen_7, red)$.
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- a single relation can be implicit $\longrightarrow$ triples: $(pen_7, color, red)$.

Triples are universal representations of relations

All relations can be represented in terms of triples:

	...	P_j	...
	...	...	...
r_i	...	v_{ij}	...
	...	...	...

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$prop(Entity, Property, Value)$ is the only relation needed:
$(Entity, Property, Value)$ triples, semantic network, entity relationship model, knowledge graphs, ...

Warning: Many knowledge graphs convert to triples naively

Projecting onto pairs loses information:

- For example:
  Air Canada flies from New York to Vancouver
  Air Canada flies from Vancouver to Los Angeles

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- These are true triples:
  $(Air\ Canada, Flies\ From, New\ York)$
  $(Air\ Canada, Flies\ To, Los\ Angeles)$
- However, Air Canada does not fly from New York to Los Angeles.
  The information about flights is lost!

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FB15K, a knowledge base commonly used in research papers, contains test cases:

- (Jade North, /sports/pro_athlete/teams./soccer/football_roster_position/position, Defender (association football))
  "*Jade North plays position defender.*"
- (Real Zaragoza, /soccer/football_team/current_roster./sports/sports_team_roster/position, Defender (association football))
  "*Real Zaragoza football club has position defender.*"

Predicting one position in the tuple given two others varies widely in difficulty!

Please look at a knowledge graph before you use it!

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- Words can have simple meanings but (almost all) entities are multi-faceted and complex.

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Typical: to learn probability of

- Boolean feature: sigmoid of a linear function
- discrete feature: softmax of a linear function

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— polyadic decomposition model (1927): two vector embeddings for each entity e ($E_0[e]$ and $E_2[e]$) and one for reach relation r ($E_1[r]$).

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- SimpleE: have an embedding for r^{-1} and learn to predict both (h, r, t) and $\left(t, r^{-1}, h\right)$
- SimpleE^{+} = SimplE with non-negative entity embeddings
  - can represent arbitrary relations
  - pointwise $\leq$ corresponds to implication
  - easy to explain what it learns

What/how embedding-based models learn

PD$^+$:

$$P((h,r,t)) = sigmoid(\sum_f E_0[h][f] * E_1[r][f] * E_2[t][f])$$

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- Negative values of $E_1[r][i]$ provide exceptions.

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- Which is better depends on the goals and how success is measured.

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- Ideally we would try to do both; learn about specific entities and general knowledge.

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- Evaluating predictions when only positive examples are provided
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Challenges of learning knowledge graphs

- Evaluating predictions when only positive examples are provided
  Consider the following relations:
  - Married to — each person related to 0 or 1 other persons (with a few exceptions)
  - Friend of — each person related to tens or hundreds of others
  - Knows about — each person might know about hundreds or thousands of others. Some people my be known by millions or billions of others.
  - Would get along with — almost everyone gets along with almost everyone else, but with some exceptions.

Beware of ranking

- Most knowledge graphs only contain positive information.
- How can we evaluate a prediction?
  (?, Plays Position, Defender)
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- Problem #1: is it not good for answers for which there is no answer or many answers:
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- Challenge: design a good evaluation scheme. Log-likelihood seems reasonable, but requires knowledge of negations.

Predicting Properties

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- Requires aggregation: some models provide implicit aggregation, and some you can use whatever aggregation you want. We need better models of aggregation.

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If we have relations with multiple arguments:

- We could convert them to triples by reifying ... but the reified entities have very few data points (number of arguments of original relations)
- Design embedding-based model that work directly with original relations
- Allow them to be inferred from other relations

Outline

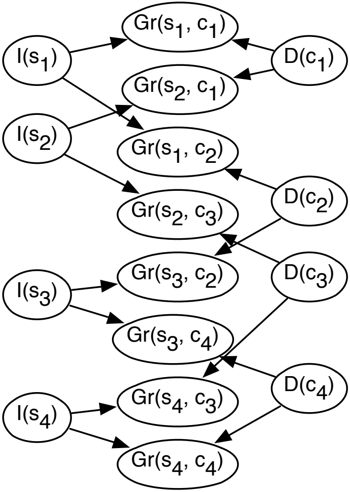
1. What are relational probabilistic models and relational learning?
   - Relational Models
   - Knowledge Graphs
2. Learning Knowledge Graphs
3. Learning General Knowledge: Lifted Graphical Models
4. Bayesian $\Rightarrow$ Exchangeability $\Rightarrow$ Lifted Inference
5. Identity and Existence Uncertainty

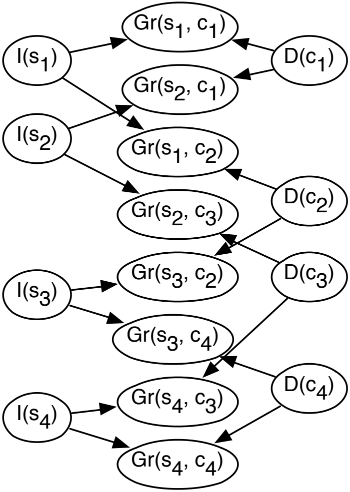
Example: Predicting Relations

Student	*Course*	*Grade*
s_1	c_1	A
s_2	c_1	C
s_1	c_2	B
s_2	c_3	B
s_3	c_2	B
s_4	c_3	B
s_3	c_4	?
s_4	c_4	?

- Students s_3 and s_4 have the same averages, on courses with the same averages.
- Which student would you expect to better?

From Relations to Bayesian Belief Networks



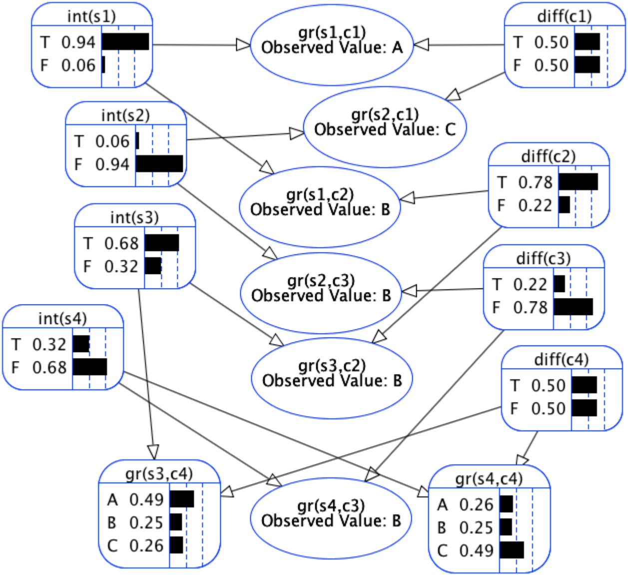


$I(S)$	$D(C)$	$Gr(S,C)$ A	B	C
true	*true*	0.5	0.4	0.1
true	*false*	0.9	0.09	0.01
false	*true*	0.01	0.09	0.9
false	*false*	0.1	0.4	0.5

$P(I(S)) = 0.5$
$P(D(C)) = 0.5$

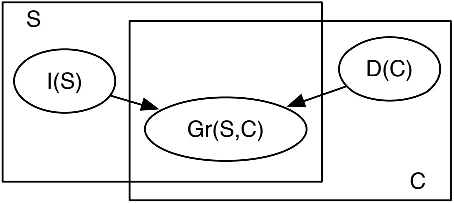
"parameter sharing"

Example: Predicting Relations



`http://artint.info/code/aispace/grades.xml`

Plate Notation



- S, C logical variable representing students, courses
- the set of entities of a type is called a population
- $I(S)$, $Gr(S, C)$, $D(C)$ are parametrized random variables

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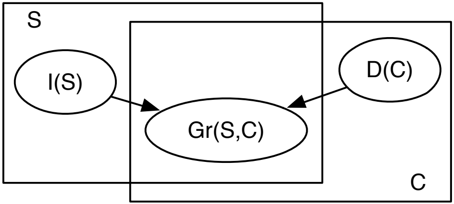

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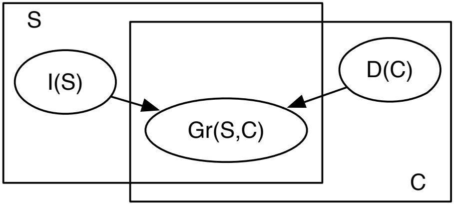

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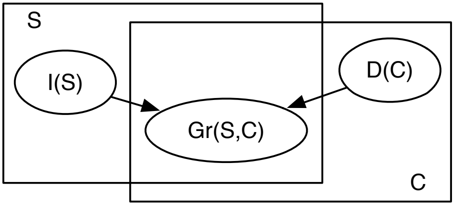

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- all instances share the same structure and parameters

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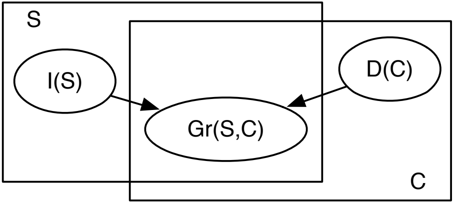

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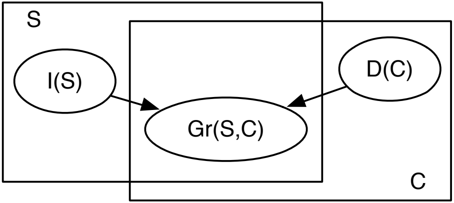

- If there were 1000 students and 100 courses:
  Grounding contains
  - 1000 $I(s)$ variables
  - 100 $D(c)$ variables
  - 100000 $Gr(s, c)$ variables

  total: 101100 variables
- To define the probabilities:
  1 for $I(S)$, 1 for $D(C)$, 8 for $Gr(S, C) = 10$ parameters.

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- MLNs and RLR are identical when "everything else" is observed.
- Also: relational dependency networks: directed models that induce a Markov chain.

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$\alpha_2 : q \wedge r(X)$
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\end{aligned}$$

In RLR and in MLN, if all $r(a_i)$ are observed:

$$P(q \mid obs) = sigmoid(\alpha_0 + n_1\alpha_2 + n_1^2\alpha_7)$$

$r(X)$ is true for n_1 individuals

Danger of fitting to data without understanding the model

- Consider sigmoid of polynomials of degree 2:

  $$sigmoid(-0.01n^2 - 0.2n + 8)$$

  $$sigmoid(0.01n^2 - n + 16)$$

  Both go from ≈ 1 at $n = 10$ to ≈ 0 at $n = 30$.
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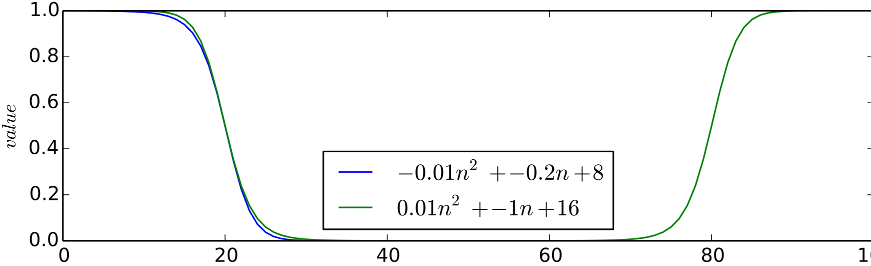

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- See Guy Van den Broeck's *Computers and Thought* lecture from IJCAI 2019.

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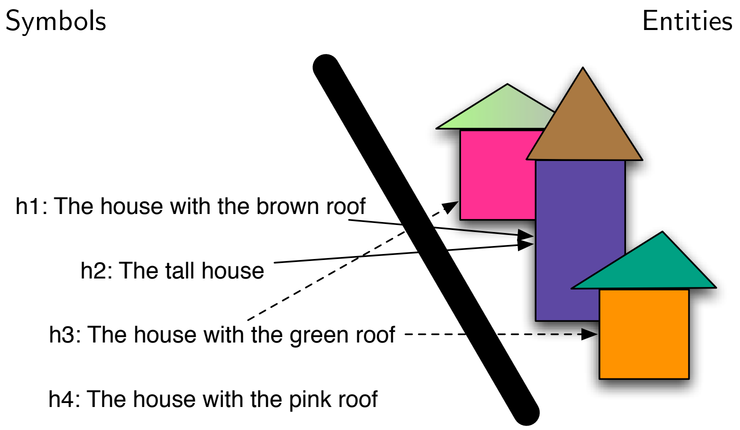
- If we also specified that there was no one else: there are between 1 and 3 people.
- We need knowledge graphs to (be able to) state "there are no more ..."

Correspondence Problem



c symbols and i entities $\longrightarrow c^{i+1}$ correspondences

Clarity Principle

Clarity principle: probabilities must be over well-defined propositions.

- What if an entity doesn't exist?
  - $house(h4) \land roof\_colour(h4, pink) \land \neg exists(h4)$

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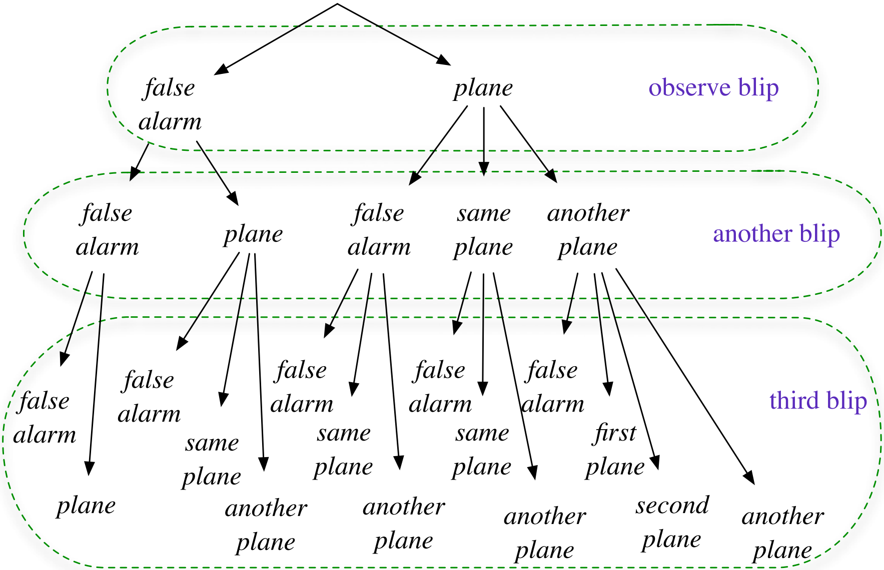
- What if more than one entity exists? Which one are we referring to?
  —In a house with three bedrooms, which is the second bedroom?

Handling Number and Existence Uncertainty

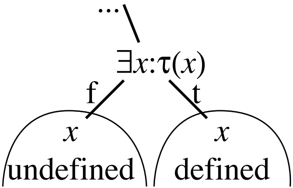
- distribution over the number of entities. For each number, reason about the correspondence.
- For each observation in sequence, hypothesize its correspondance e.g., if you observe a radar blip, there are three hypotheses:
  - the blip was produced by plane you already hypothesized
  - the blip was produced by another plane
  - the blip wasn't produced by a plane

Existence Example



First-order Semantic Trees

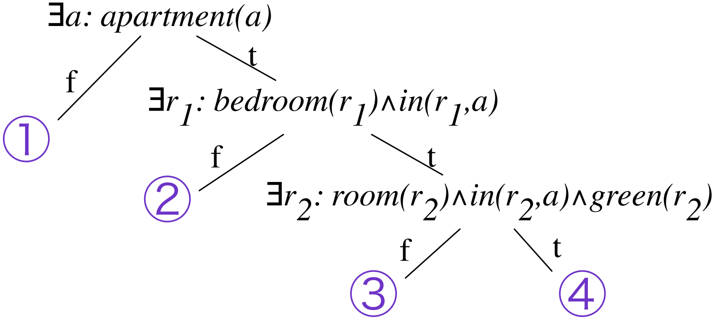
Split on quantified first-order formulae:



- The “true” sub-tree is in the scope of x
- The “false” sub-tree is not in the scope of x

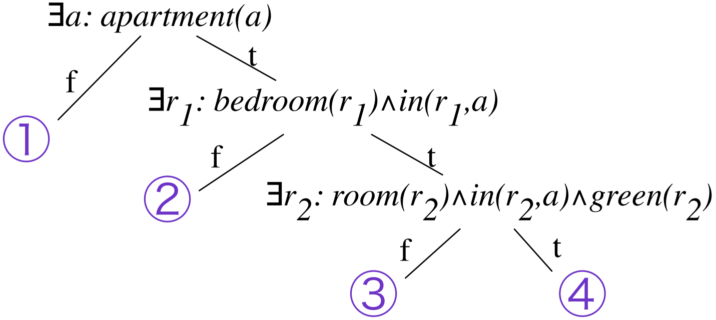
A logical generative model generates a first-order semantic tree.

First-order Semantic Tree (cont)



1. there is no apartment
2. there is no bedroom in the apartment
3. there is a bedroom but no green room
4. there is a bedroom and a green room

First-order Semantic Tree (cont)



① there is no apartment

② there is no bedroom in the apartment

③ there is a bedroom but no green room

④ there is a bedroom and a green room

All probabilities are over well-defined first-order formulae.

Conclusion

Challenge model and learn uncertainty about:

- Properties of entities
- Relationships among entities
- How properties and relations interrelate
- Identity (equality) of entities
- Existence (and number) of entities
- Interactions with time, ontologies, causality ...

Will you step up to this challenge? There is still lots to do!

What is now required is to give the greatest possible development to mathematical logic, to allow to the full the importance of relations, and then to found upon this secure basis a new philosophical logic, which may hope to borrow some of the exactitude and certainty of its mathematical foundation. If this can be successfully accomplished, there is every reason to hope that the near future will be as great an epoch in pure philosophy as the immediate past has been in the principles of mathematics. Great triumphs inspire great hopes; and pure thought may achieve, within our generation, such results as will place our time, in this respect, on a level with the greatest age of Greece.

– Bertrand Russell, Mysticism and Logic and Other Essays (1917)