UBC Physics 102 Section 951
Midterm Test 1
July 7, 2003

Instructions:

1. Do not open this test until told to do so.
2. This test is closed book. You may NOT bring any material in with you. Calculators are allowed. A formula sheet is provided on the last page of this test.
3. Print your name and student number on ALL pages.
4. Show all your work to justify your answers.
5. Use the back of the page if you need more space.
6. If you use pencil there will be NO changes to your mark once your paper has been handed back.

Marks:

	Q1	Q2	Q3	Q4	TOTAL
Mark					
Max	7	8	7	8	30

Last Name: **Solution**

First Name: **Key**

Student Number: 00000000

Name: ____________________

Student Number: ____________________

/7 1. Identify the following greek letters (1/2 point each):

(Example: theta θ)

lambda	λ	alpha	α		
phi	ϕ	Phi (uppercase)	Φ	mu	μ
epsilon	ϵ	delta	δ	rho	ρ
gamma	γ	omega	ω	tau	τ
pi	π	beta	β	sigma	σ

/8 2. Natural carbon (atomic mass = 12.0 u/particle) contains 1 part per 770 billion of radioactive ${}^{14}_{6}\mathrm{C}$ with a half-life of 5730 years.

(a) (4 points) How long will a 100 g sample of natural carbon take on average before there is only one ${}^{14}_{6}\mathrm{C}$ nucleus left?

Answer: First we need the original number of ${}^{14}_{6}\mathrm{C}$ nuclei, N_0:

$$\begin{aligned} N_0 &= 0.100 \text{ kg} \times \frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}} \times \frac{1 \text{ particle}}{12.0 \text{ u}} \times \frac{1}{770 \times 10^9} \text{①} \\ &= 6.52 \times 10^{12} \text{ particles.} \end{aligned}$$

We want to know how long it takes to reach $N = 1$. To use the radioactive decay law we need the decay constant λ,

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{5730 \text{ yr}} = 1.21 \times 10^{-4} \text{ yr}^{-1}. \text{①}$$

Now we can solve $N = N_0 \, e^{-\lambda t}$ for t,

$$\begin{aligned} t &= \frac{1}{\lambda} \ln \frac{N_0}{N} \text{①} \\ &= \frac{1}{1.21 \times 10^{-4} \text{ yr}^{-1}} \ln \frac{6.52 \times 10^{12} \text{ particles}}{1 \text{ particle}} \text{①} \\ &= 2.44 \times 10^5 \text{ yr.} \end{aligned}$$

(b) (2 points) How does the answer in (a) change if the sample is 1.00 kg?

Answer: This time we have ten times as much carbon so we also have ten times as many $^{14}_{6}C$ particles,

$$N_{0,(b)} = 10N_{0,(a)} = 6.52 \times 10^{13} \text{ particles.} ①$$

λ is the same as in (a) so

$$\begin{aligned} t &= \frac{1}{\lambda} \ln \frac{N_0}{N} \\ &= \frac{1}{1.30 \times 10^{-4} \text{ yr}^{-1}} \ln \frac{6.52 \times 10^{13} \text{ particles}}{1 \text{ particle}} \\ &= 2.63 \times 10^5 \text{ yr.} ① \end{aligned}$$

(c) (2 points) What does this tell you about the limits of carbon dating?

Answer: Increasing the sample size by an order of magnitude only gives a small increase in the time until depletion ①. After the sample is depleted of $^{14}_{6}C$ carbon dating is useless. So carbon dating can only date samples that are less than about a quarter of a million years old ①. Beyond that we need to use radio-isotopes with longer half-lives.

/7 3. Identify the following SI units (1 point each):

(Example: kilogram kg)

megaohm	MΩ	hertz	Hz
microwatt	μW	milliamp	mA
meter	m	centisecond	cs
		nanojoule	nJ

/8 4. Answer these problems in as much detail as possible. Be sure to include all your work. Partial credit will be given even if the answer is not fully correct. If you can not solve a problem because you are missing information, explain the steps you would take to solve it if you could.

(a) (2 points) Express the radioactive decay law, $N = N_0\, e^{-\lambda t}$, in terms of the half-life $T_{1/2}$.

Answer: The half-life is $T_{1/2} = \ln(2)/\lambda$ ① so

$$N = N_0\, e^{-\ln(2)t/T_{1/2}}. ①$$

(b) (2 points) Rewrite your answer for part (a) in powers of 2. (Ie. find A and B in $N = A\, 2^B$. Hint: $e^{\ln x} \equiv x$.)

Answer:

$$N = N_0\, e^{-\ln(2)t/T_{1/2}} ① = N_0 \left(e^{\ln 2}\right)^{-t/T_{1/2}} = N_0\, 2^{-t/T_{1/2}} ①.$$

(c) (2 points) From part (b) isolate the time t, in terms of the base-2 logarithm, $\log_2$. (Ie. find C and D in $t = C \log_2 D$. If part (b) is incomplete, use $N = A\, 2^{Bt}$.)

Answer: If we take the base-2 logarithm of (b) we get

$$\log_2 \frac{N}{N_0} = -\frac{t}{T_{1/2}} \quad \ldots \text{OR} \ldots \quad \log_2 \frac{N}{A} = Bt. \text{①}$$

So, isolating t gives

$$\begin{aligned} t &= -T_{1/2} \log_2 \frac{N}{N_0} \quad \ldots \text{OR} \ldots t = \frac{1}{B} \log_2 \frac{N}{A} \text{①} \\ &= T_{1/2} \log_2 \frac{N_0}{N}. \end{aligned}$$

(d) (2 points) How many half-lives have elapsed if one eighth of the radioactive element remains?

Answer: We know $N = \frac{N_0}{8}$①. If we plug that into our equation for t we find

$$\begin{aligned} t &= T_{1/2} \log_2 \frac{N_0}{N} \\ &= T_{1/2} \log_2 8 \\ &= 3\, T_{1/2}. \end{aligned}$$

So, 3 half-lives have elapsed.①

OR…if we couldn't solve for t we can still work this out. If the amount of remaining radioactive element declines by half with every half-life then it has one eighth of the original after 3 half-lives because $1/8 = (1/2) \times (1/2) \times (1/2) = (1/2)^3$.②

TA Marking guide

- Deduct 1/2 point for each error in units or significant digits (in the final answer).
- Exercise your own judgment. Try to encourage creativity.

END OF EXAM

(Did you print your name and student number on every page?)